Many-body localization: response to thermal spots

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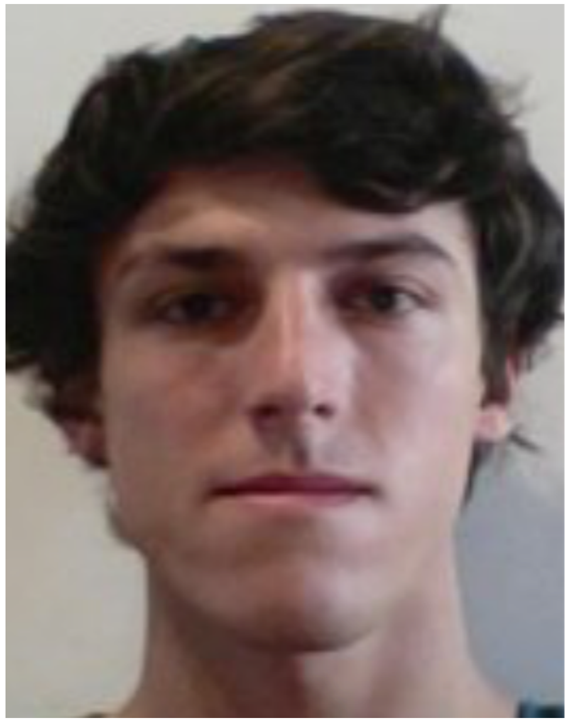
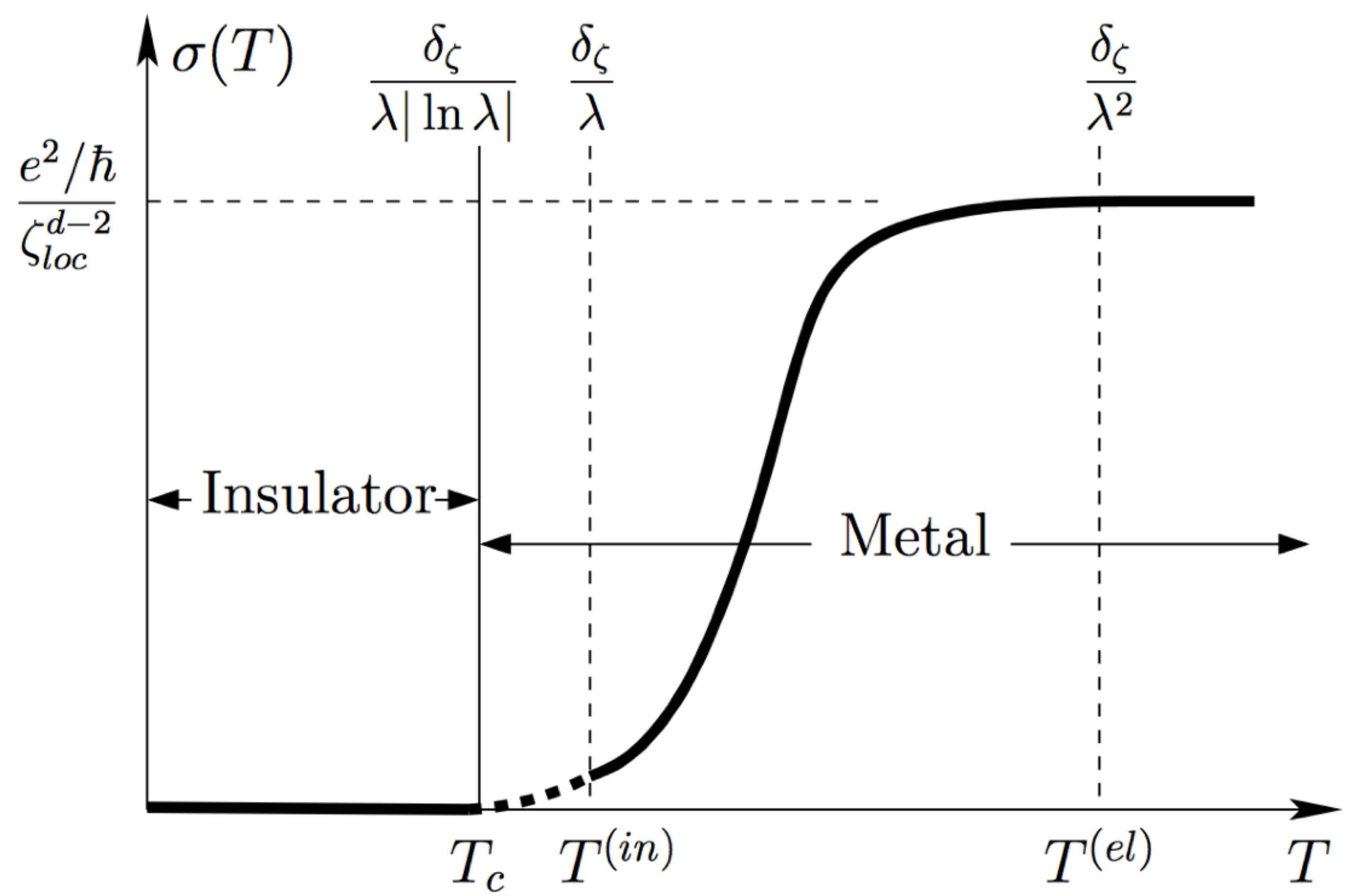


Thimothée
Thiery
(KUL)

Many-body localization (MBL)

Original point of view: **super-insulator**



From Basko et al. ‘06

Anderson localization with **interactions** among the electrons

Anderson, **Fleishman** ’80, **Gornyi**, **Mirlin**, **Polyakov** ’05, **Basko**, **Aleiner**, **Altshuler** ‘06

Many-body localization (MBL)

Current view: **ergodicity breaking**

Key features:
- non-integrable, interacting, 'generic'…
- no transport on any time scale
- ergodicity breaking

Ergodicity breaking:
- no thermalization: no flow towards a maximal entropy state,
- more constraints than the macroscopic conserved quantities

E.g. quantum quench:

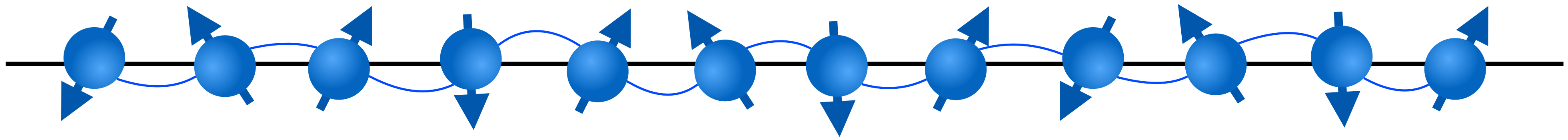
$E(T_L)$ $E(T_R)$

… wait … wait …

$E(T_L) + \mathcal{O}(\varepsilon)$ $E(T_R) + \mathcal{O}(\varepsilon)$

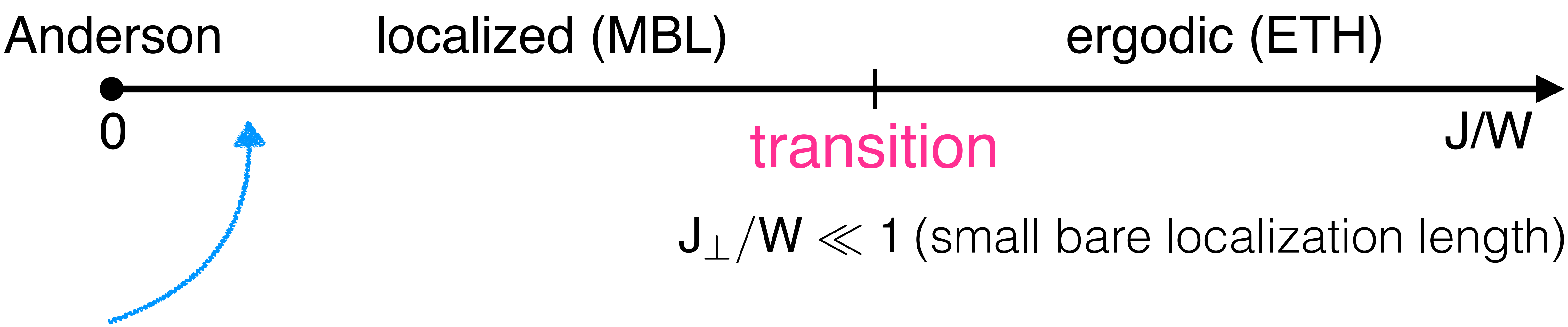
Many-body localization (MBL)

Example: **disordered spin chain**



$$H = \sum_i h_i \sigma_i^z + J_\perp(\sigma_i^+\sigma_{i+1}^- + \sigma_i^-\sigma_{i+1}^+) + J\sigma_i^z\sigma_{i+1}^z$$

$-W < h_i < W$ i.i.d. (W = disorder strength).



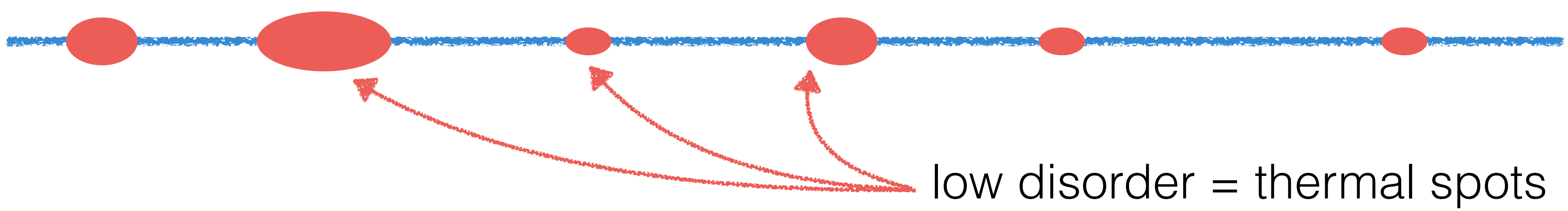
$J_\perp/W \ll 1$ (small bare localization length)

Gornyi et al. ’05, **Basko** et al. ’06,
Oganesyan et al. ’07, **Serbyn** et al. ’13, **Huse** et al ’14, **Imbrie** ’16, etc…

Plan of the talk

Mechanism for thermalization:

instability of the MBL phase to the inclusion of thermal spots



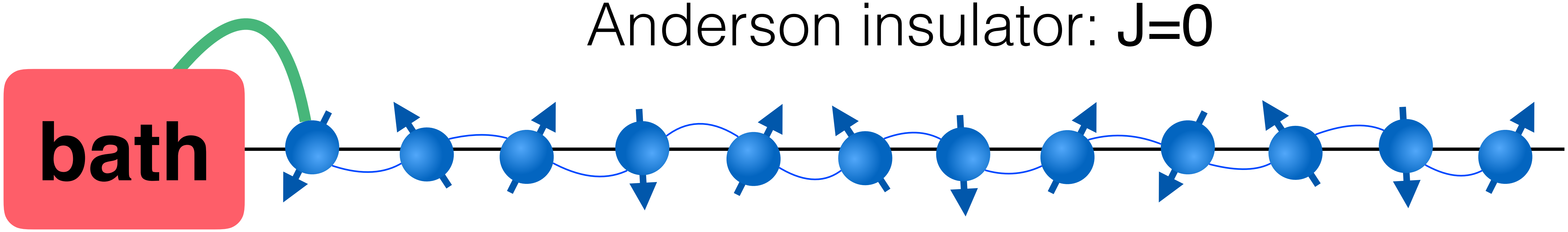
1) Response to a single spot (microscopic)
2) General considerations on the transition
3) Picture of the transition through a multi-scale analysis (RG)

Part I:

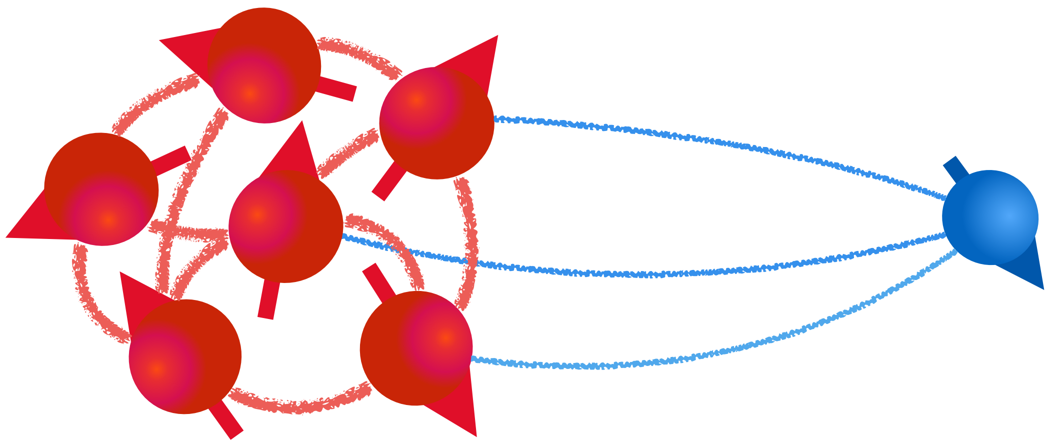
Single spot

Anderson insulator coupled to an imperfect bath

Anderson insulator: **J=0**



Imperfect bath = fixed (and small) number of spins:

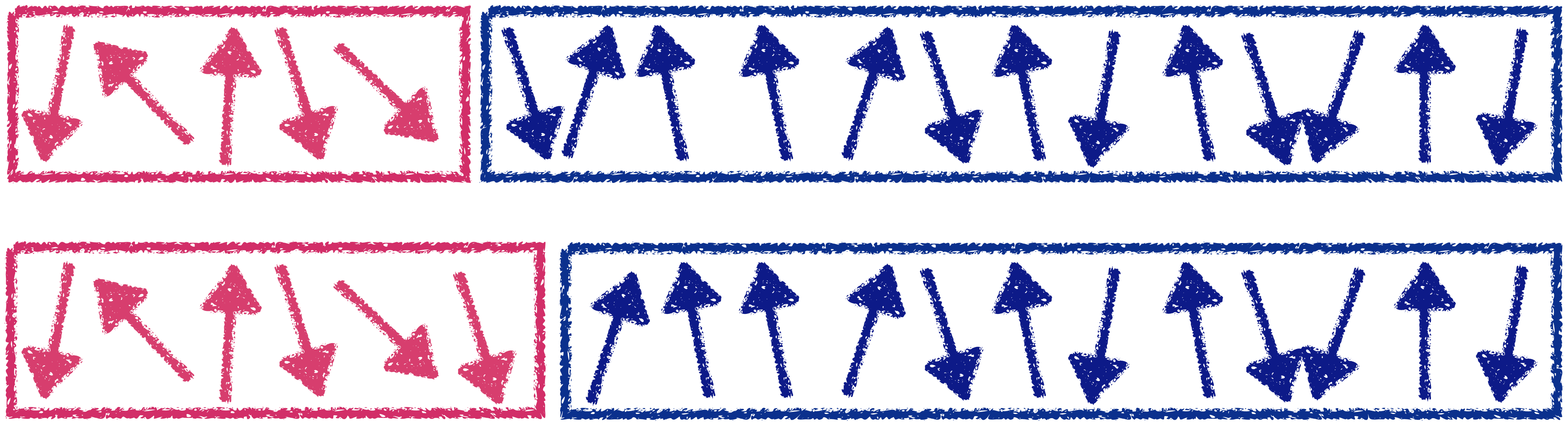


random matrix interaction

Minimal model for MBL with non-trivial phase diagram

Possibility of avalanches

The ergodic spot thermalizes the near spins…



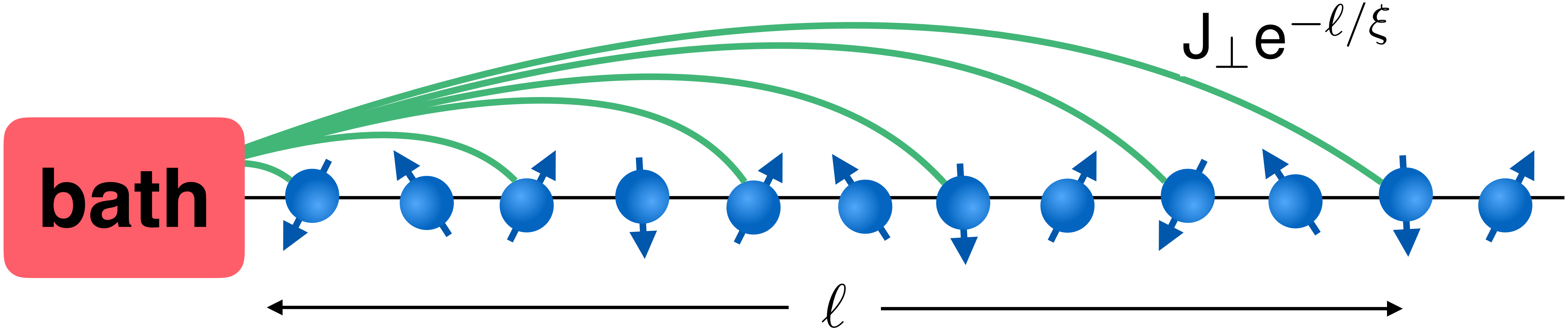
… and becomes a larger spot

Eventually, the full material could become thermal!

d=1 and large disorder: fallacy! (see **Imbrie** '12)

In general (including d>1): this can happen!

Move to the Anderson basis



ξ: localization length

$$H = H_b + \sum_{\ell \geq 1} h_\ell \sigma_\ell^z + J \sum_{\ell \geq 1} e^{-\ell/\xi} \sigma_b^x \sigma_\ell^x$$

$$H_b = R + R^\dagger, \quad R = \mathrm{GOE}(2^{L_b} \times 2^{L_b}), \quad L_b \text{ fixed}$$

Idealization: • h_ℓ no longer i.i.d.

• fluctuations around $J\, e^{-\ell/\xi}$

How many spins are thermalized?

$$\mathcal{G}(\ell) := \frac{\text{matrix element}}{\text{level spacing}} \sim \frac{e^{-\ell/\xi}\, e^{-s(T)(L_b+\ell)/2}}{e^{-s(T)(L_b+\ell)}}$$

$s(T=\infty) = \log(2)$ (entropy density)

random matrix assumption

$\mathcal{G}(\ell) < 1$: spin at ℓ is localized

$\mathcal{G}(\ell) > 1$: spin at ℓ is thermalized

cfr. **W. De Roeck** and **F. H.**, PRB '17, for more justifications

Upper bound on the localization length

Avalanche stops when $\mathcal{G}(\ell) < 1$, i.e. for

$$\ell \sim \frac{1}{\xi^{-1} - \frac{\log 2}{2}} \mathsf{L_b}$$

Write $\xi_c = 2/\log 2$

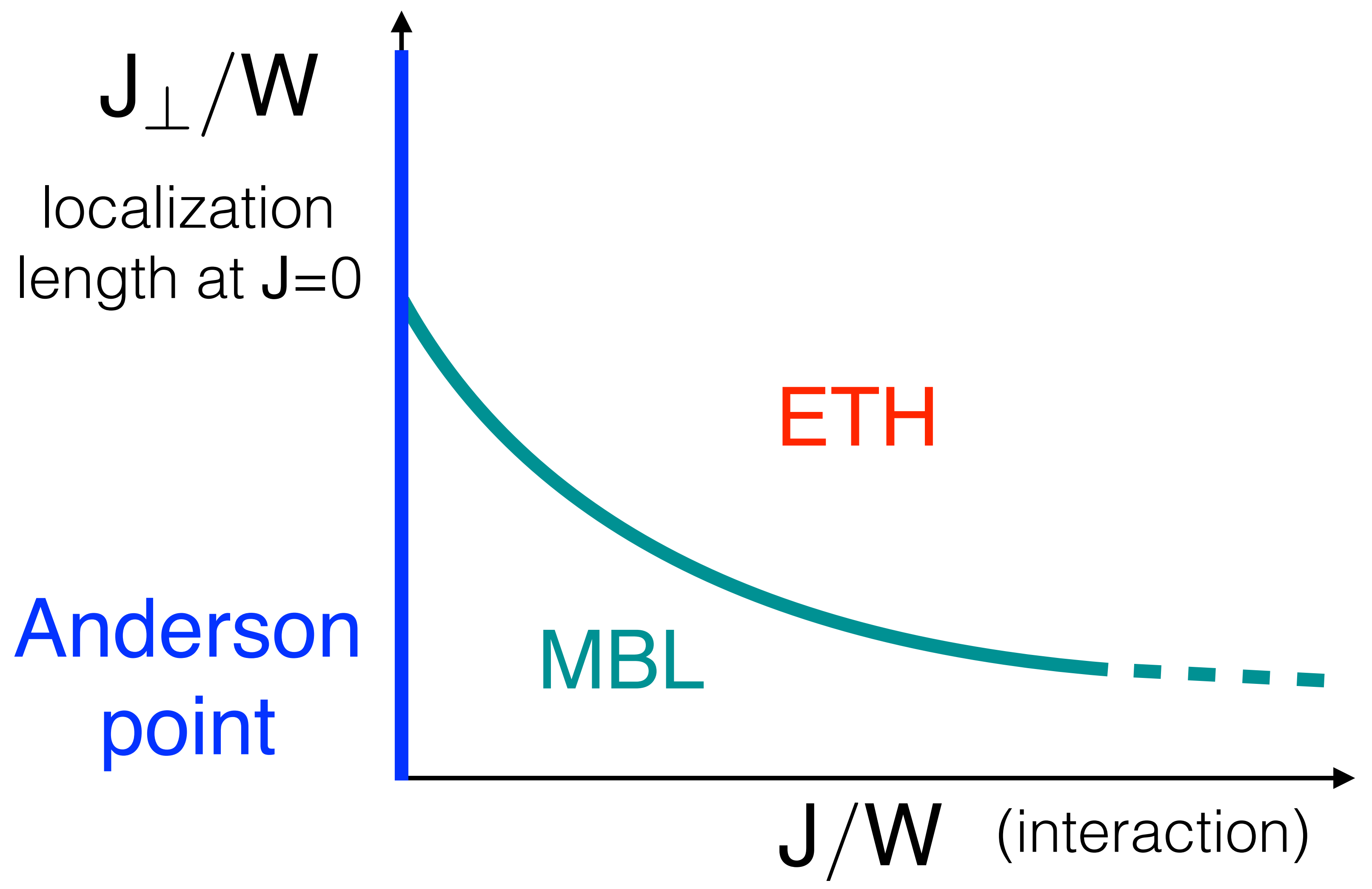
$\xi < \xi_c$: The avalanche will eventually stop

$\xi > \xi_c$: MBL is unstable

The value of ξ_c depends on the lattice:

- spins on both sides of the spot: $\xi_c = 1/\log 2$
- $d > 1$: $\xi_c = 0$

Expected phase diagram

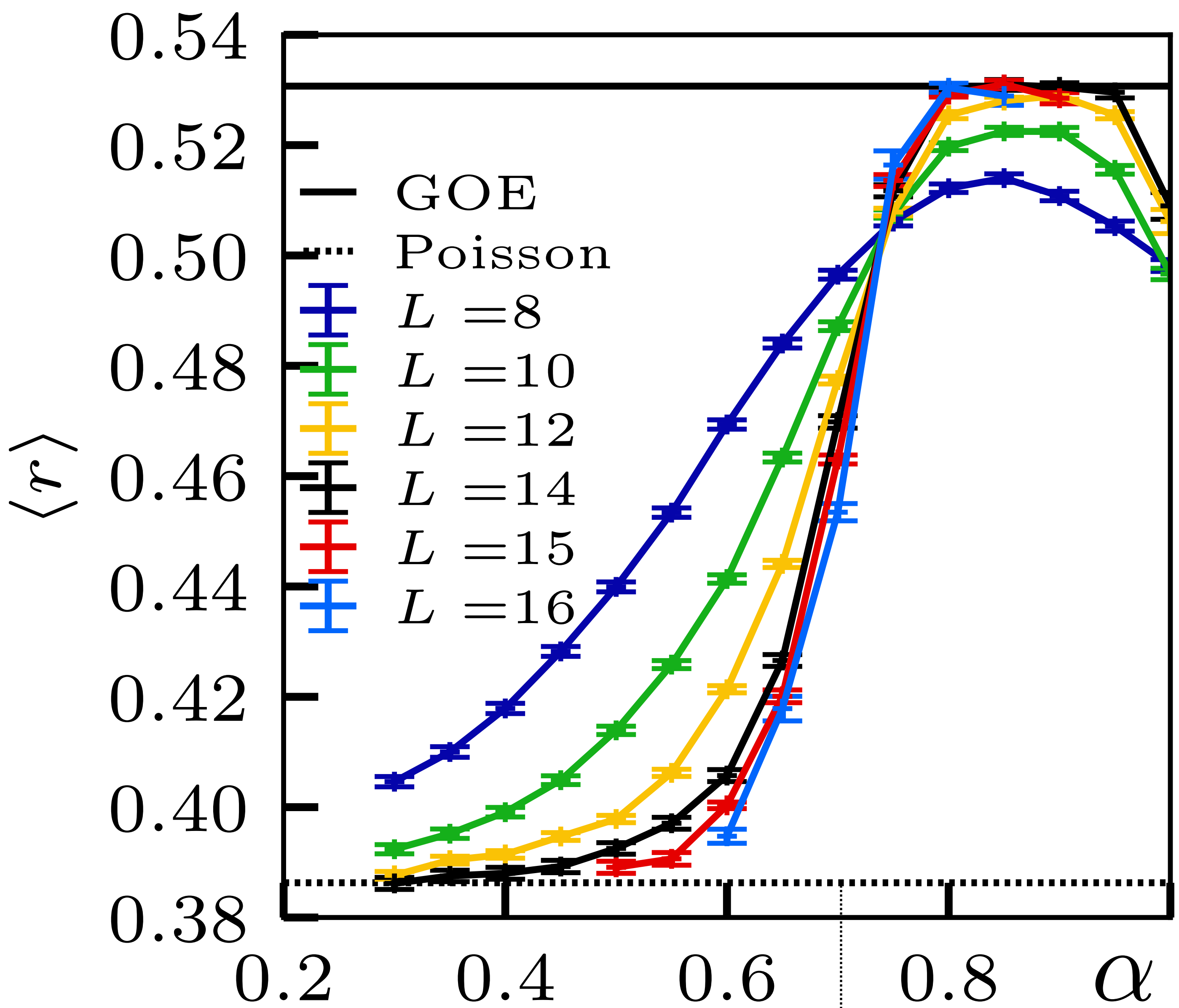


(qualitative picture)

see also **M. Znidaric** and **M. Ljubotina**, PNAS ‘18

Numerical check

$L_b = 3 \;:\; H_{bath} = GOE(8 \times 8)$



$\alpha = e^{-1/\xi}$ $\alpha_c = 0.71$

$$r_i = \frac{\min\{\Delta E_i, \Delta E_{i+1}\}}{\max\{\Delta E_i, \Delta E_{i+1}\}}$$

$$\Delta E_i = E_i - E_{i-1}$$

$$\langle r_{GOE} \rangle = 0,53$$

$$\langle r_{Poisson} \rangle = 0,38$$

(Oganesyan et Huse '06)

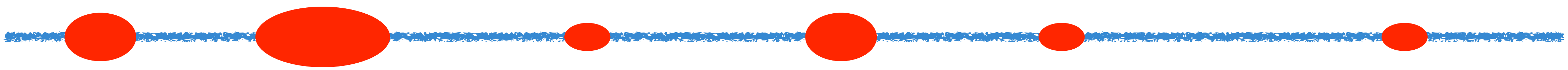
from D. Luitz, F. H. and W. De Roeck, PRL 117, 2017

Part II:

General ‘facts’ about the transition

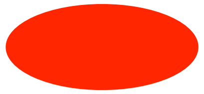
Spots all over the chain

Disorder fluctuations generate small baths (Griffiths regions):



Simplified model:

 : Anderson insulator with localization length ξ_1

 : resonant spot = imperfect bath

ε = density of n.n. resonances

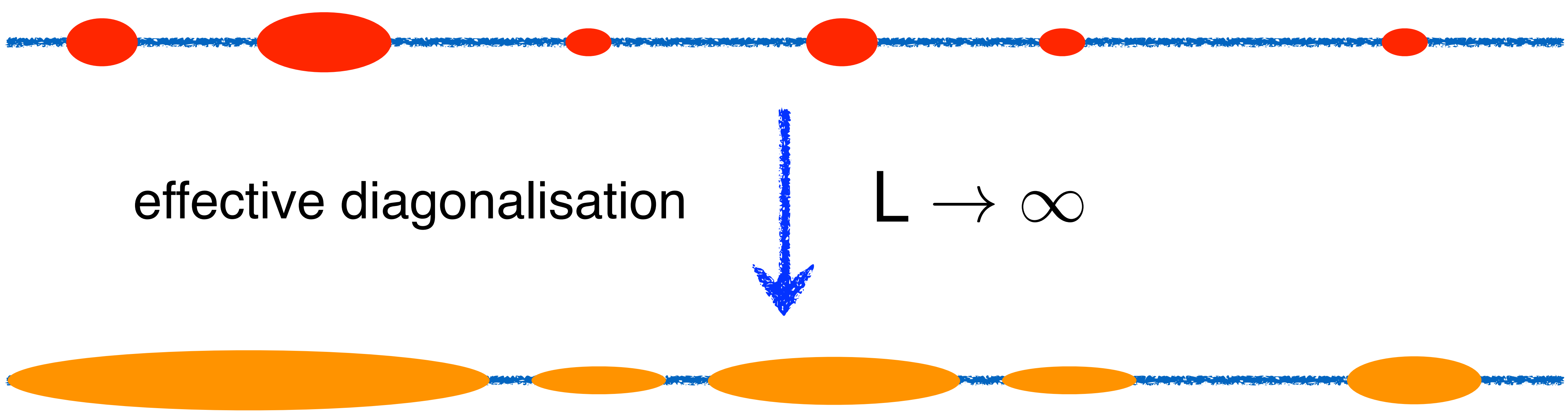
spot of size **k**: **k** consecutive resonances

some relation between ξ_1 and ε: $\xi_1 = 1/\log(K/\varepsilon)$

Effective diagonalisation

Classical and effective algorithm to deal with resonances

ε = density of **resonant** spots (inverse disorder strength)



$\rho(\varepsilon)$ = density of **thermal** spots

$\rho(\varepsilon) = 1$: the material is thermal

$\rho(\varepsilon) < 1$: the material is localized

Two basic assumptions

Hard task to find a good scheme. Some conclusions can first be drawn from general arguments. Let

$$\mathsf{T}(\mathsf{L}) \ = \ \text{the thermal region.} \qquad \text{Thus} \ \ \rho \ = \ |\mathsf{T}(\mathsf{L})|/\mathsf{L}$$

Two 'reasonable' assumptions:

A1: $\varepsilon \ \rightarrow \ \langle|\mathsf{T}(\mathsf{L})|\rangle_\varepsilon$ is continuous and non-decreasing

A2: $\mathsf{T}(\mathsf{L}) \subset \mathsf{T}(\mathsf{L}')$ if we enlarge the system from **L** to **L'**

Remark: **A2** does probably not hold true at the microscopic level (proximity effects). Neglected here.

consequences

Thermal density: $\rho(\mathsf{L}) = |\mathsf{T}(\mathsf{L})|/\mathsf{L}$

C1: For any ε, $\langle\rho(\mathsf{L})\rangle_\varepsilon \to \rho^\star(\varepsilon)$ as $\mathsf{L} \to \infty$

Follows from **A2** by Fekete's superadditivity lemma

C2: concentration around the mean:

$$\mathsf{P}(|\rho(\mathsf{L}) - \rho^\star(\varepsilon)| > \delta) \to 0 \qquad \forall\delta > 0 \quad \text{as} \quad \mathsf{L} \to \infty$$

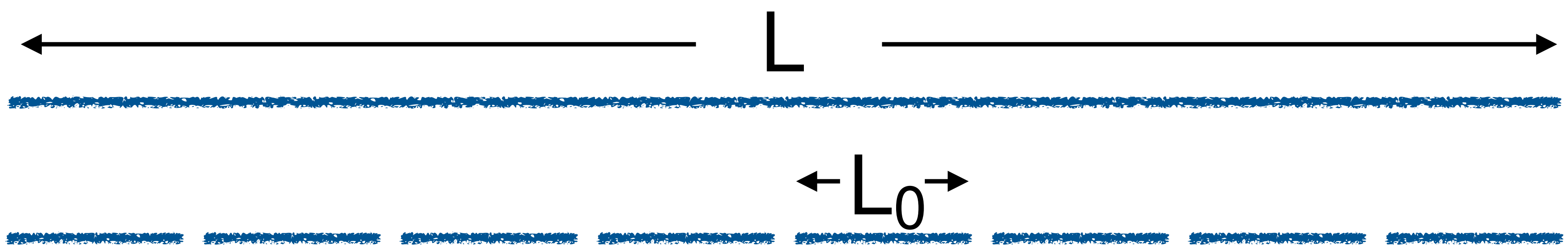
In particular, 2 possibilities at criticality:

1) MBL *with probability 1* if $\rho^\star(\varepsilon_c) < 1$

2) Thermal *with probability 1* if $\rho^\star(\varepsilon_c) = 1$

Why **C2**?

Compare with a systems cut into blocks of size **L_0**



L_0 large enough so that $\langle \rho(L_0) \rangle_\varepsilon \sim \rho^\star(\varepsilon)$ (by **C1**)

C2 holds true for the ‘block’ system, hence by **A2**,

$$P(\rho(L) - \rho^\star(\varepsilon) > -\delta) \to 0 \quad \text{as} \quad L \to \infty$$

concentration in the other direction: $\rho^\star(\varepsilon)$ is the average

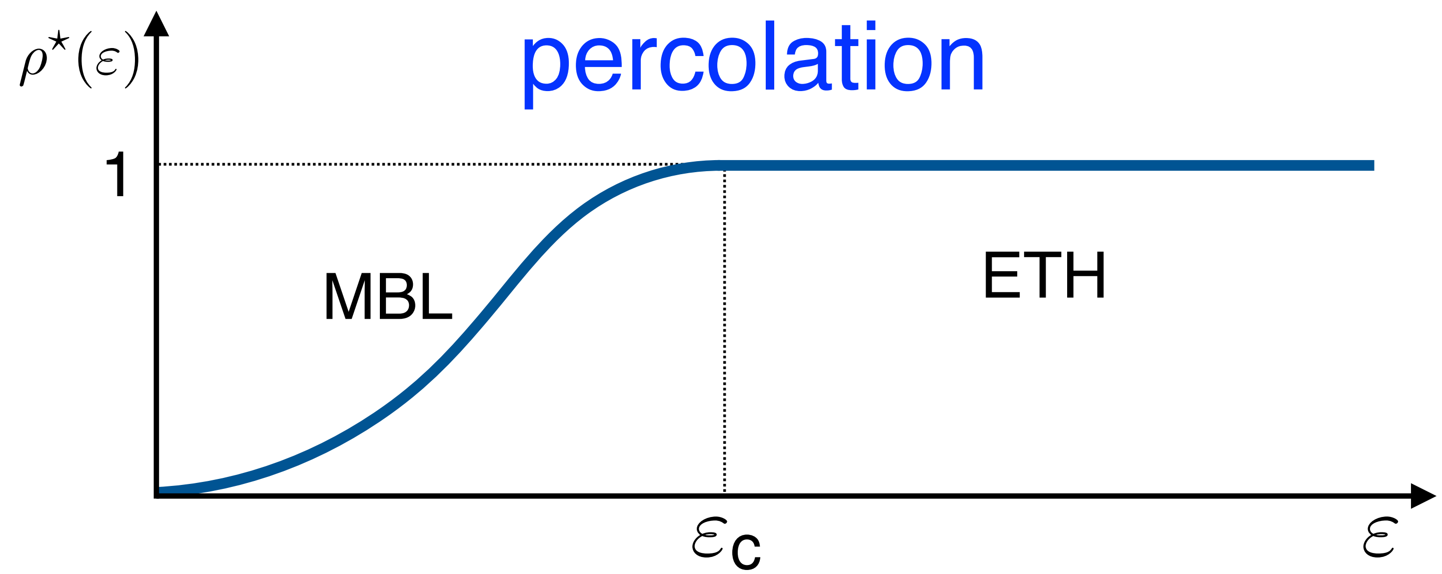
MBL fixed point

C3: $\varepsilon \rightarrow \rho^\star(\varepsilon)$ is left-continuous and non-decreasing

Follows by standard arguments from **A1** and **C1**

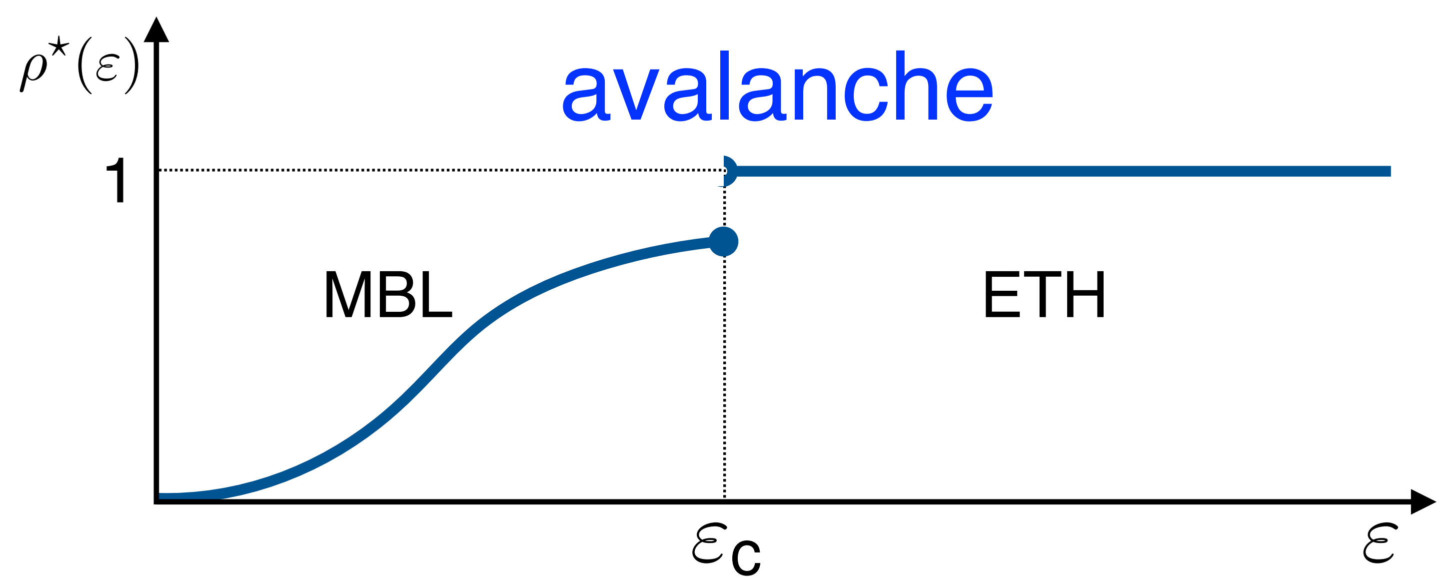
Two possibilities:

Thermal fixed point



No

MBL fixed point



Yes

because $\xi < \xi_c$, and $\xi \rightarrow \infty$ as $\rho \rightarrow 1$.

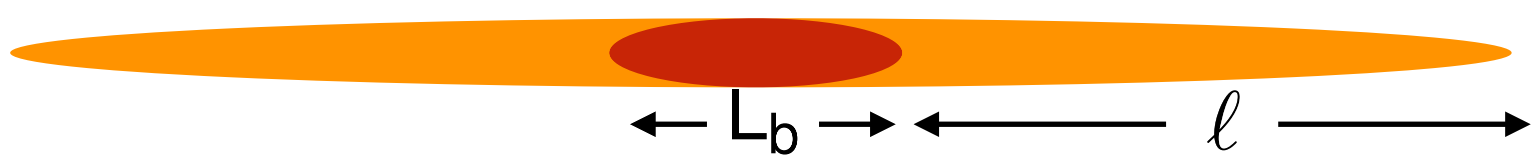
Part III:

Multi-scale analysis (RG)

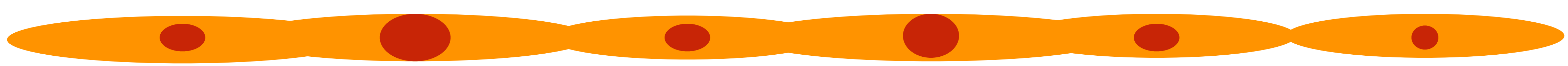
The need for more

- Develop a picture for how the transition happens
- Is $\xi_c = 1/\log 2$ still the critical localization length?

Avoid paradoxes: resonances percolate at some $\xi_\star < \xi_c$



$$\ell \sim \frac{L_b}{\xi_c - \xi_\star}$$



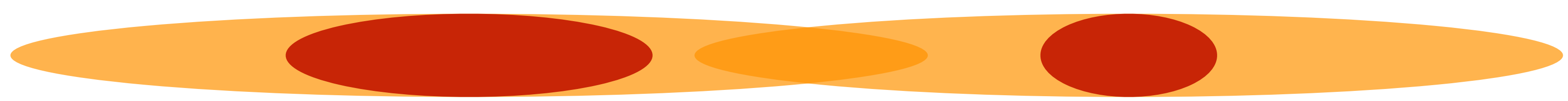
Impossible: $\rho_c < 1$!

- Finite size scalings, critical exponents and distributions…

Issues in finding a good effective scheme

main issue: interactions among spots

- Effective spot issuing from these two?



- Effective localization length for the large spot?

- What spots to deal with first?

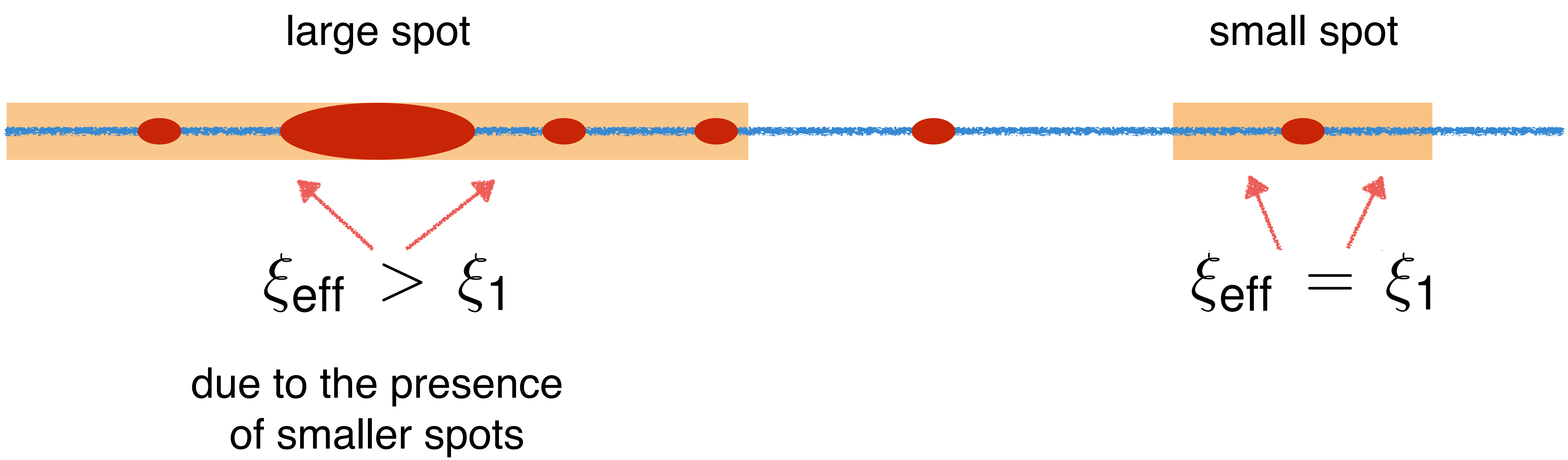
Several earlier proposals: **Vosk** and **Altman** '14, **Vosk**, **Huse** and **Altman** '15, **Potter**, **Vasseur** and **Parameswaran** '15, **Imbrie** '16, **Zhang**, **Zhao**, **Devakul** and **Huse** '16, **Dumitrescu**, **Vasseur** and **Potter** '17, **Goremykina**, **Vasseur** and **Serbyn** '18

Imbrie '16: main inspiration for our RG

Simplified RG scheme

Flow on a few effective parameters by some approximations

- Scale **k**: resonant spot of size **k**
- Deal with the smallest scales first (to avoid non-sense)
- 1st parameter: effective localization length ξ_k



Simplified RG scheme

Rule of **halted decay**: no decay through thermal regions

$$e^{-\ell/\xi_{\mathrm{eff}}} = e^{-(\ell-\ell_{\mathrm{th}})/\xi_0} \quad \Rightarrow \quad \xi_{\mathrm{eff}} = \xi_0 \frac{1}{1-\ell_{\mathrm{th}}/\ell}$$

ℓ_{th} : number of spins thermalized at previous scales : bare spots + collar

- 2nd parameter: effective response to thermal inclusions: length of the thermalized region by a **k** resonance:

$$\ell_k \sim \frac{k}{\xi_c - \xi_k}, \qquad \xi_c = 1/\log 2$$

- 3d parameter: thermal density from **k** resonant spots:

$$\rho_k \sim \varepsilon_k (k + \ell_k)$$

Simplified RG scheme

Flow on the parameters ξ_k, ℓ_k and ρ_k:

$$\begin{cases} \xi_{k+1}^{-1} = (1-\rho_k)\,\xi_k^{-1} & \text{rule of halted decay} \\ \ell_k = \dfrac{k}{\xi_c - \xi_k} & \text{thermal length for } \mathbf{k} \text{ resonant spots} \\ \rho_k = \varepsilon^k (k + \ell_k) & \text{thermal density from } \mathbf{k} \text{ resonant spots} \end{cases}$$

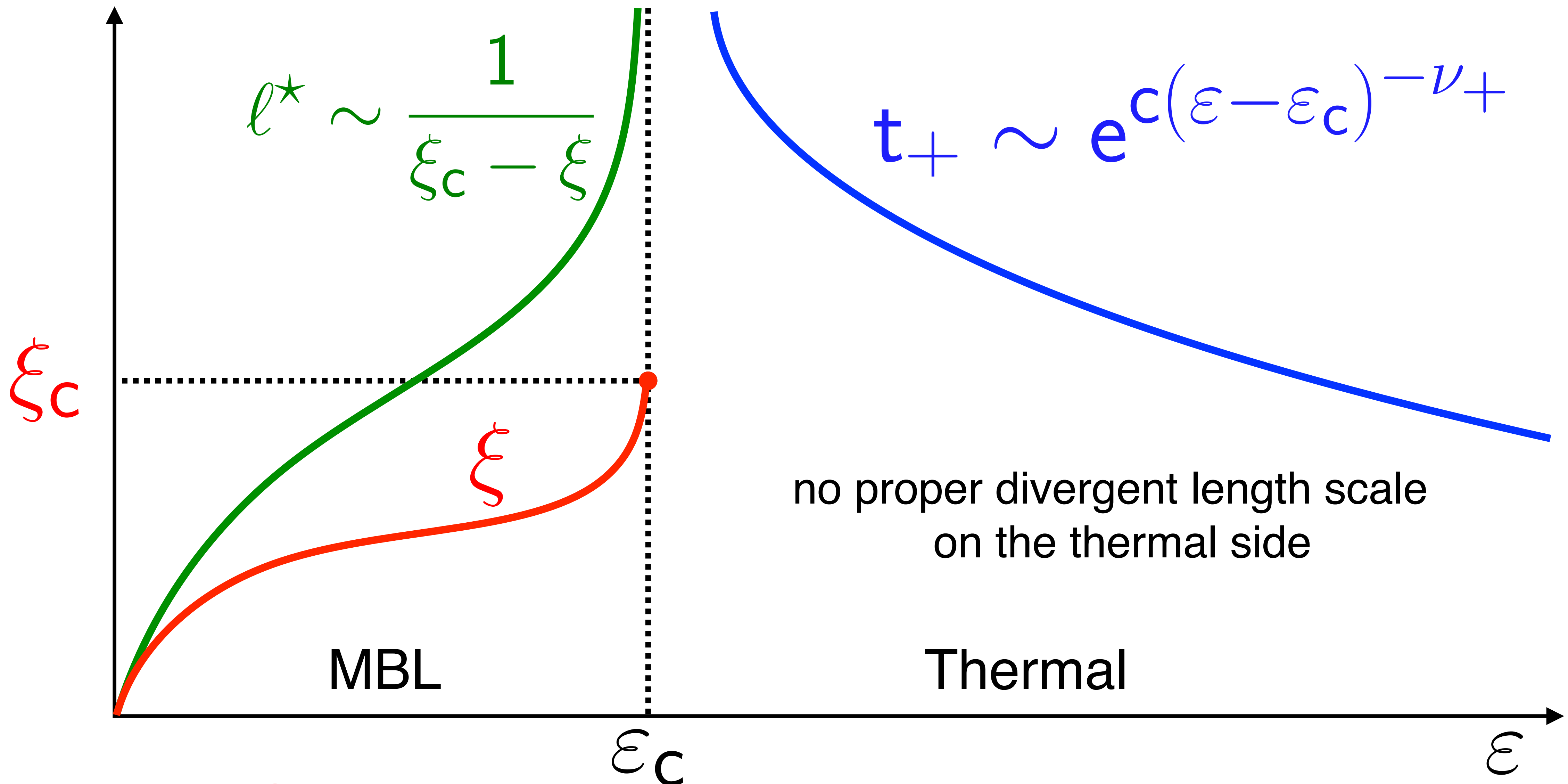
$\xi_k \to \xi_c < +\infty$ from the MBL side but ℓ_k diverges:

MBL : $\ell_k / k \to \ell^\star$ as $k \to \infty$

critical : $\ell_k / k \to \infty$ as $k \to \infty$

thermal : $\ell_k / k = \infty$ for some $k < \infty$ (avalanche)

Qualitative diagram



$$\xi_c = 1/\log 2$$

From T. Thiery, F. H., M. Mueller, W. De Roeck, PRL 121, 2018

Issues with the approximations

1. Fluctuations have been neglected. Finite size scalings and critical exponent (computed numerically) violate the rigorous Harris bound.

E.g.: $p(\varepsilon, L)$: probability that a system of size L is thermal

$$p(\varepsilon, L) \sim F_{\pm}(L/L_{\pm}(\varepsilon))$$

MBL side: $L_{-} \sim |\varepsilon_c - \varepsilon|^{-\nu_{-}}$

Thermal side: $L_{+} \sim |\varepsilon_c - \varepsilon|^{-\nu_{+}}$

At criticality: $p(\varepsilon_c, L) \sim L^{-\beta}$

Wrong exponents (Harris: $\nu_{\pm} \geq 2$) but polynomial behavior at criticality is correct.

Remark: no reason to expect $\nu_{+} = \nu_{-}$.

Issues with the approximations

2. Interactions among spot of the same size have been neglected. The thermal density ρ_k is underestimated

Sparse resonant structures (Cantor set like) may result in huge effective resonant regions

acts effectively as

As a result:

$$\varepsilon^{k} \to \varepsilon^{k^{\alpha(\varepsilon)}} \quad \text{with} \quad \alpha(\varepsilon) < 1$$

Microscopic effective scheme

Cure to these problems:

- Abandon the reduced description with a few parameters
- Instead, fix precise rules to deal with the effect of individual resonant spots, including interaction among spots. Cfr. **T. Thiery**, **M. Müller** and **W. De Roeck, arXiv:1711.09880**
- Solve the scheme numerically

Upshot:

- The overall picture of the simplified scheme is confirmed
- Critical exponents are agree with Harris bound

Conclusions

Instability of the MBL phase:

- A single imperfect bath can destabilize MBL
- Localized transition point, with finite loc. length
- Discontinuity of the thermal density at the transition (unlike percolation)
- Physical picture from RG, scale dependent loc. length
- Divergent response to the inclusion of thermal spots