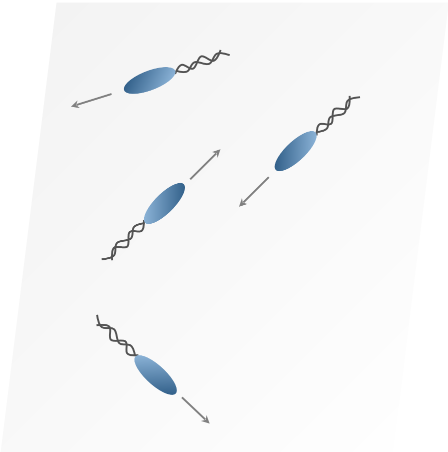
The long-ranged influence of disorder on active systems

Yariv Kafri
Technion - Israel Institute of Technology

Sunghan Ro, Mehran Kardar, Julien Tailleur,
Ydan Ben Dor, Eric Woillez, Alex Solon

(PRL 123, 048003 (21), PRE 100, 052620 (19))

Active Matter



Overdamped particles with self-propulsion -

$$\dot{\mathbf{r}} = \boldsymbol{\eta}(t) + v\hat{\mathbf{u}} + \text{interactions}$$

noise

self-propulsion along $\hat{\mathbf{u}}$
reset direction with rate α

Depending on details can lead to a host of different phases and phase diagrams

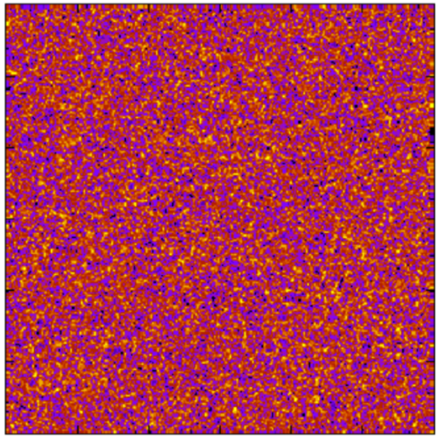
Active Matter - Example of Phase Diagram (Scalar)

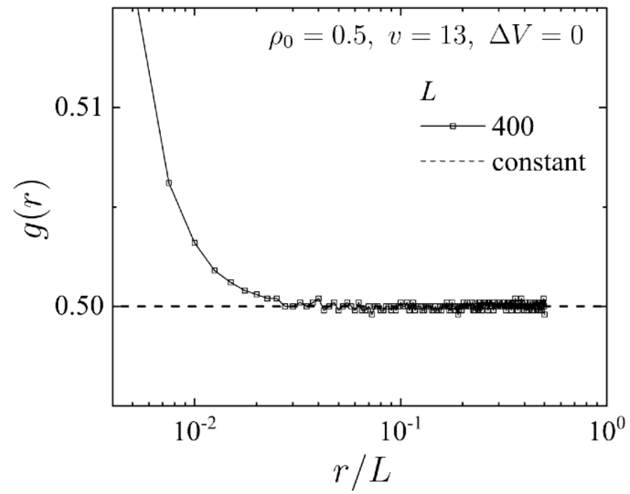
Density snapshot

Density pair correlation

Dilute

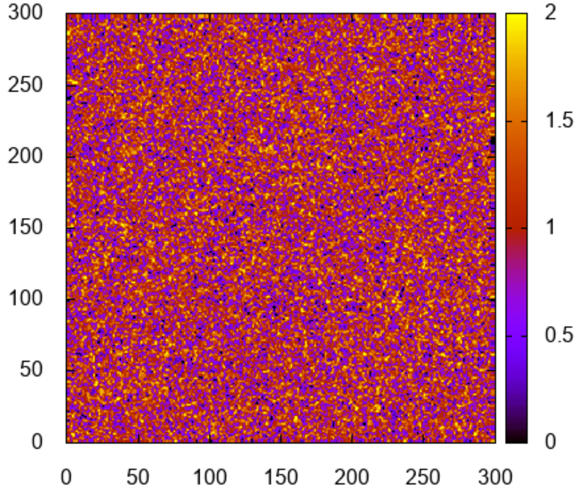
Common to all active system

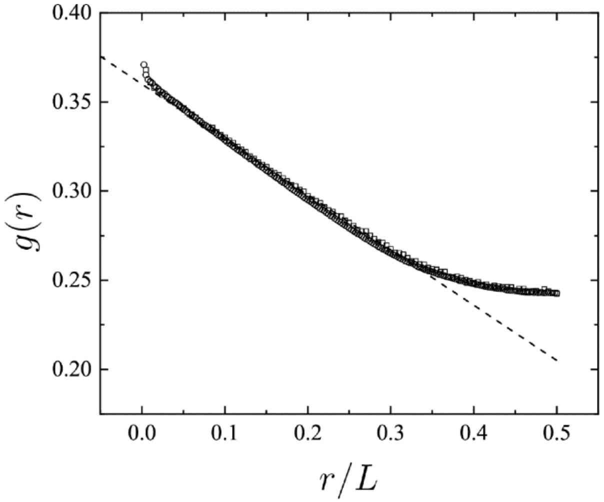




Dense

Scalar - Interactions do not align



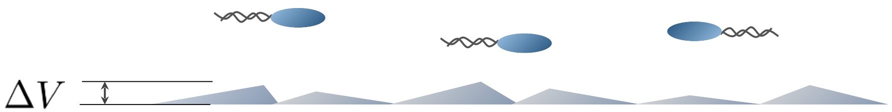


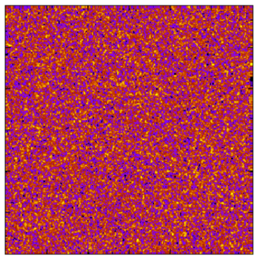
Cates and Tailleur,
Annu. Rev. Condens. Matter Phys. (2015)
Solon, et al., New. J. Phys. (2018), Tjhung et. al. (2019)

This Talk: Effects of Quenched Disorder

$$\dot{\mathbf{r}} = \boldsymbol{\eta}(t) + v\hat{\mathbf{u}} - \mu\nabla V(\mathbf{r}) + \text{interactions}$$

short-range correlations
CONSERVING FIELD!

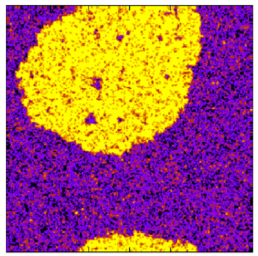




dilute?

Recap equilibrium:

correlations still short ranged
(Lorentzian squared)



fate of long range order?

lower critical dimension

$$d_c = 2$$

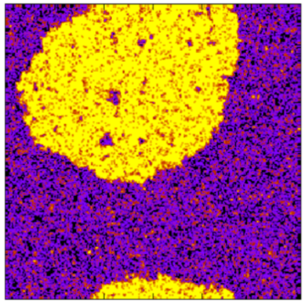
Y. Imry and S.-k. Ma, PRL (1975)
A. Aharony, Y. Imry, and S.-k. Ma, PRL (1976)
U. Glaus, PRB (1986)

?

other?

Show very different for active system:

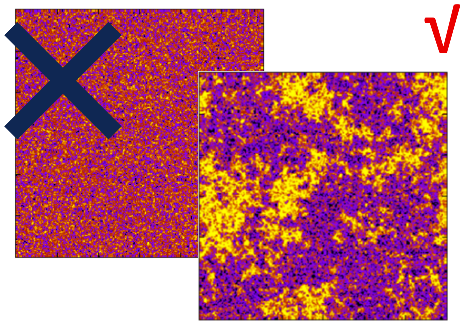
Fate of MIPS



$d_c = 4$

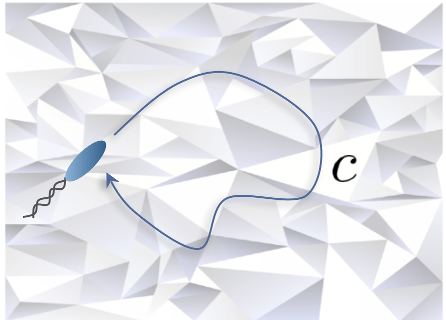
lower critical dimension

Structure factor



$S(q) \propto q^{-2}$

Steady-state current



$\overline{J^2}(c) \propto c$

comments:

- **True for potential disorder <u>including torque inducing disorder</u>**
- **True for any dilute active system**

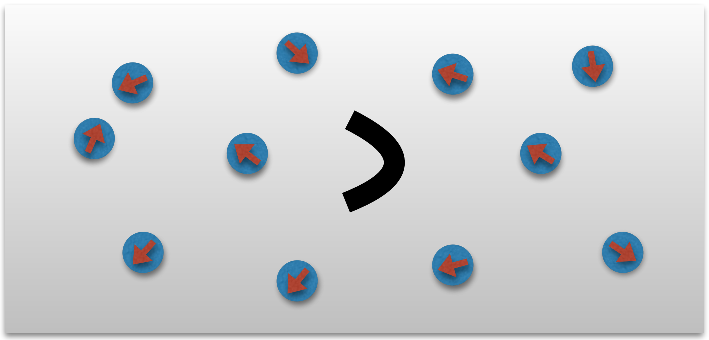
Outline:

- One speck of disorder
- Dilute systems
- Interacting systems
- One-dimensional case

- **One speck of disorder**

One Speck of Disorder

generally an **asymmetric** potential in active bath

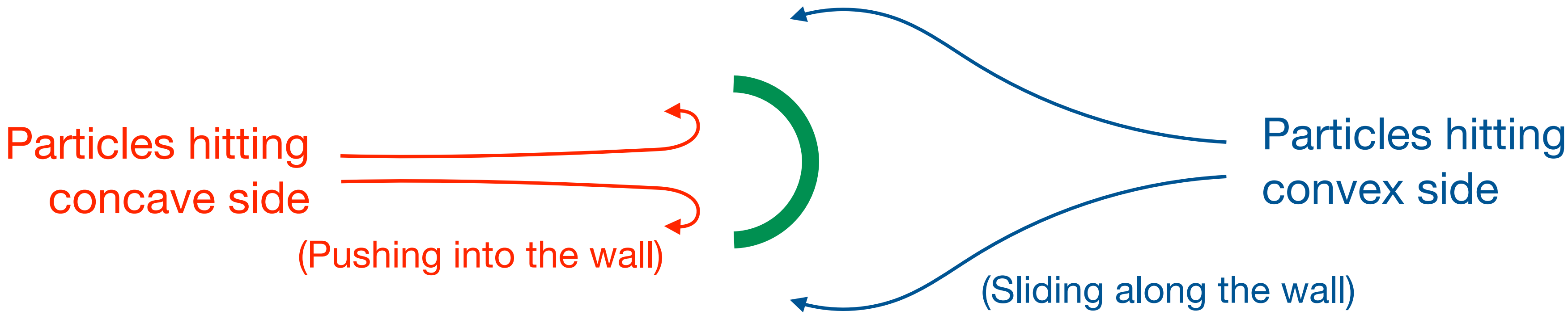


Breaking of time reversal symmetry

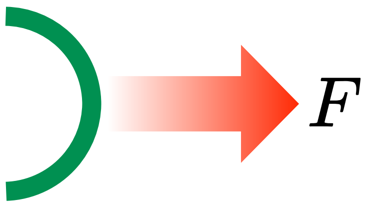
=

Current of active particle

Speck of disorder in an active fluid



Suggests that an active fluid also applies a nonzero net force on the potential

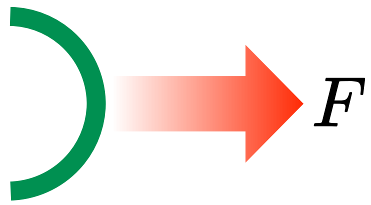


$$F = \int d^2 r \rho(\mathbf{r}) \nabla V(\mathbf{r})$$

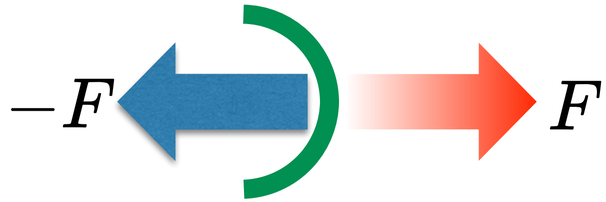
body

Same picture different words

An active fluid applies a nonzero net force
on an asymmetric body.



Body applies a nonzero net force
on the particles.



Force on particles generate flows

Can show:

PRL 2016
with Nikolai,Solon,Kardar,Tailleur,Voituriez

For a general potential (also with pairwise interactions)

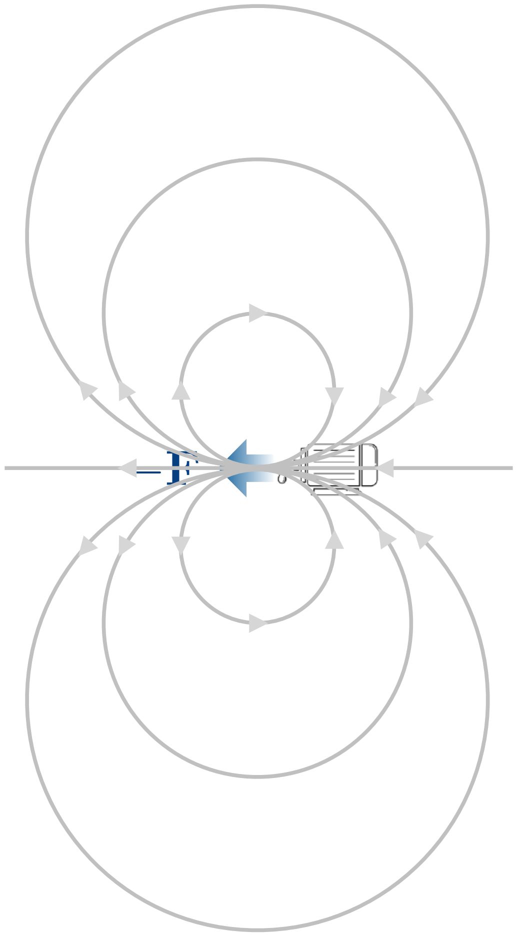
$$\mu F = -\int d^2 r J(\mathbf{r})$$

current density

Note - Force and current depend on potential/shape

Speck of disorder influence on density: (dilute system)

In *far field* disorder speck **acts like a pump in diffusive medium**



$\mathbf{J}(\mathbf{r})$

Steady-state equation

$$\nabla \cdot \mathbf{J} = -D_{\text{eff}} \nabla^2 \rho - \nabla \cdot [\mu \mathbf{F} \delta(\mathbf{r})] = 0$$

Point force (≒ dipole)

Density (2d)

$$\rho(\mathbf{r}) = \rho_b - \frac{\mu}{2\pi D_{\text{eff}}} \frac{\mathbf{r} \cdot \mathbf{F}}{r^2}$$

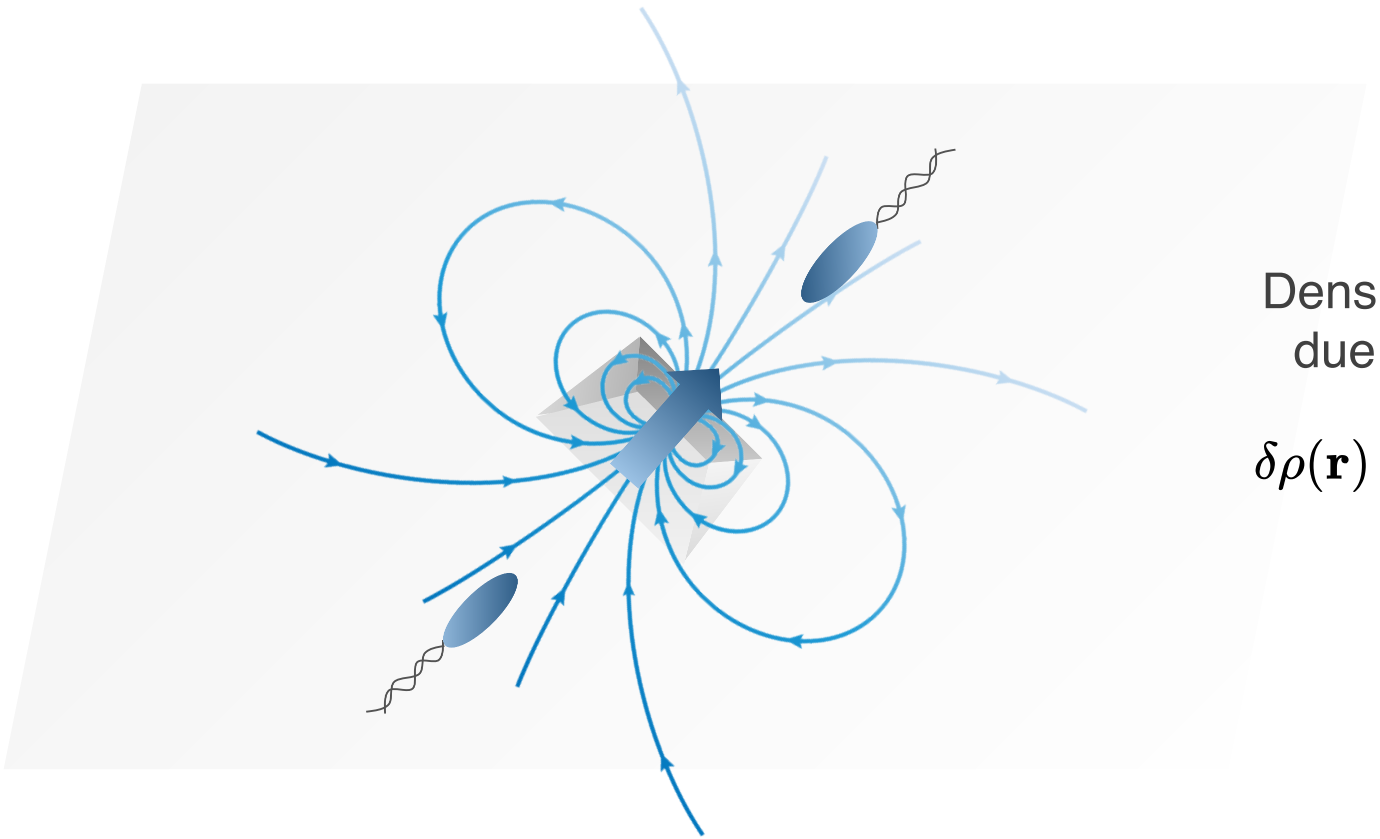
Diffusive current (2d)

$$\mathbf{J}(\mathbf{r}) = \frac{\mu}{2\pi} \left[\frac{\mathbf{F}}{r^2} - \frac{2(\mathbf{r} \cdot \mathbf{F})\mathbf{r}}{r^4} \right]$$

long-range density and current fields
(**non-local** distribution function)

F - **force acting on speck**

Speck in any dimension



Density fluctuation due to pumping

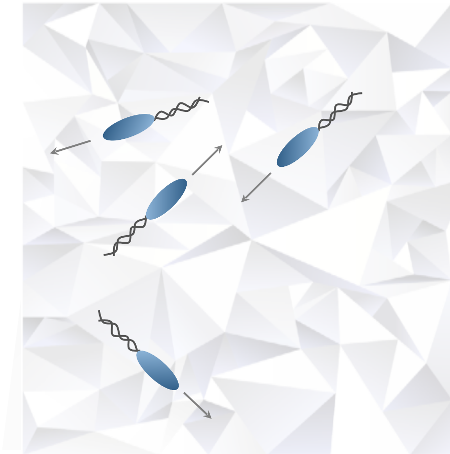
$$\delta\rho(\mathbf{r}) \propto \left|\mathbf{r} - \mathbf{r}_0\right|^{-(d-1)}$$

long-range density modulations
(non-local function of potential)

Can derive exact form even with pairwise interactions

Y. Baek, A. P. Solon, X. Xu, N. Nikola, and Y. Kafri, PRL (2018)
O. Granek, Y. Baek, Y. Kafri, and A. P. Solon, J. Stat. Mech. (2020)

- **Dilute Active System with Potential Disorder**



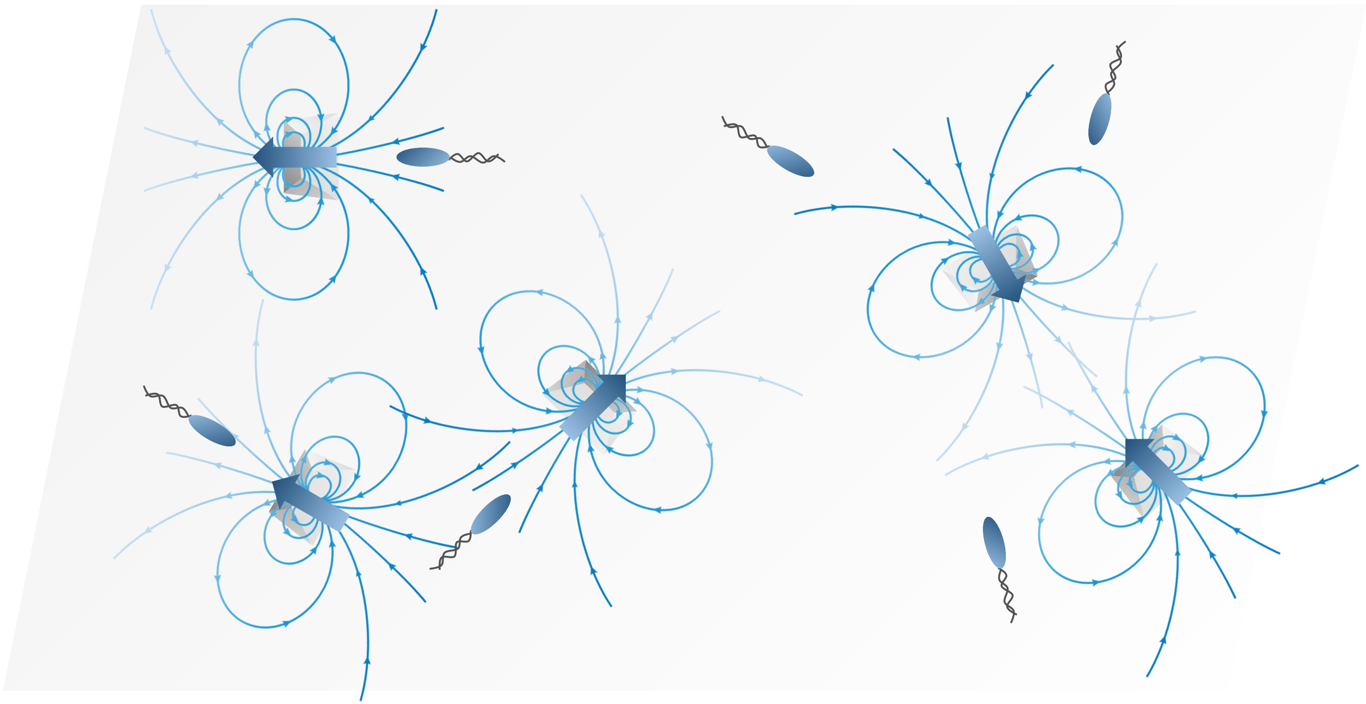
Particles on quenched disorder

$$\dot{\mathbf{r}}_i(t) = \boldsymbol{\eta}_i(t) + v\mathbf{u}_i - \mu\nabla V$$

$$\langle \boldsymbol{\eta}(t)\rangle = 0, \quad \langle \eta_i(t)\eta_j(t')\rangle = 2D\delta_{ij}\delta(t-t')$$

V - short range correlated and bounded

Each speck is a current source (modulating density)



Use result for single speck

$$\langle \rho(\mathbf{r}) \rangle = \rho_0 + \frac{\beta_{\text{eff}}}{S_d} \frac{(\mathbf{r} - \mathbf{r}') \cdot \mathbf{p}}{|\mathbf{r} - \mathbf{r}'|^d} + \mathcal{O}\left(|\mathbf{r} - \mathbf{r}'|^{-d}\right)$$

Take dipole field to be randomly distributed (dilute system)

sloppy have to take distributions….

disorder average ↗

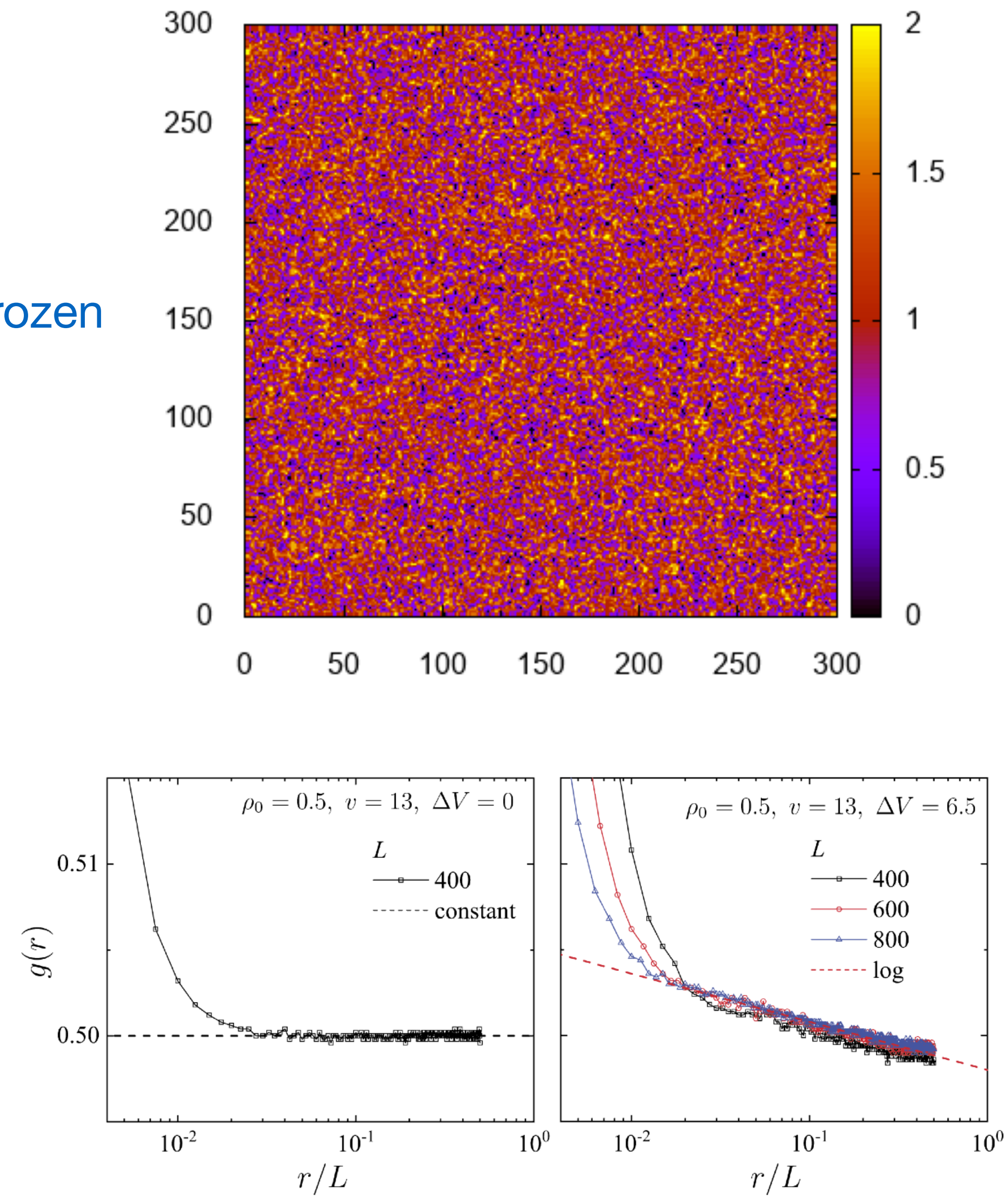
$$\overline{P_i(\mathbf{r}) P_j(\mathbf{r}')} = \chi^2 \delta_{ij} \delta(\mathbf{r} - \mathbf{r}')$$

Get

$$\overline{S}(\mathbf{q}) = \frac{\beta_{\text{eff}}^2 \chi^2}{q^2}$$

In dilute limit system is generically scale invariant!

note, structures frozen in space



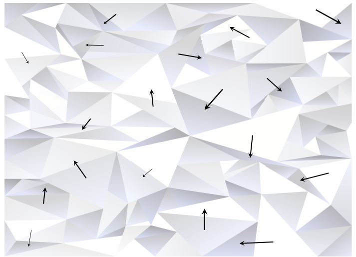
clean

disordered

- **Interacting systems**

Approach
- simplest linear field theory
- check when self-consistent

Field theoretic treatment



Linear theory with random forcing

$$\frac{\partial}{\partial t}\phi(\mathbf{r},t) = -\nabla\cdot\mathbf{j}(\mathbf{r},t),$$
$$\mathbf{j}(\mathbf{r},t) = -\nabla\mu[\phi]$$

linear - $$\mu[\phi(\mathbf{r},t)] = u\phi(\mathbf{r},t) + K\nabla^2\phi(\mathbf{r},t)$$

uncorrelated in space $$\overline{f_i(\mathbf{r})f_j(\mathbf{r}')} = \sigma^2\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')$$

Use to study density and currents

For single particle,
B. Derrida, J. Stat. Phys. (1983)
D. S. Fisher, PRA (1984)
J.-P. Bouchaud, A. Comtet, A. Georges, and P. Le Doussal, Annals of Physics (1990)

Density

$$\overline{S(\mathbf{q})} = \frac{\sigma^2}{q^2(u+Kq^2)^2} + \frac{D}{(u+Kq^2)} \quad \xrightarrow[\text{small } q]{} \frac{1}{q^2}$$

Leading order behavior same as heuristic picture

To understand powerlaw use a ***Helmholtz decomposition***

$$\frac{\partial}{\partial t}\phi(\mathbf{r},t) = -\nabla\cdot\mathbf{j}(\mathbf{r},t),$$

$$\mathbf{j}(\mathbf{r},t) = -\nabla\mu[\phi] + \mathbf{f}(\mathbf{r}) + \boldsymbol{\eta}(\mathbf{r},t),$$

$$\mathbf{f}(\mathbf{r}) = -\nabla U(\mathbf{r}) + \boldsymbol{\xi}(\mathbf{r})$$

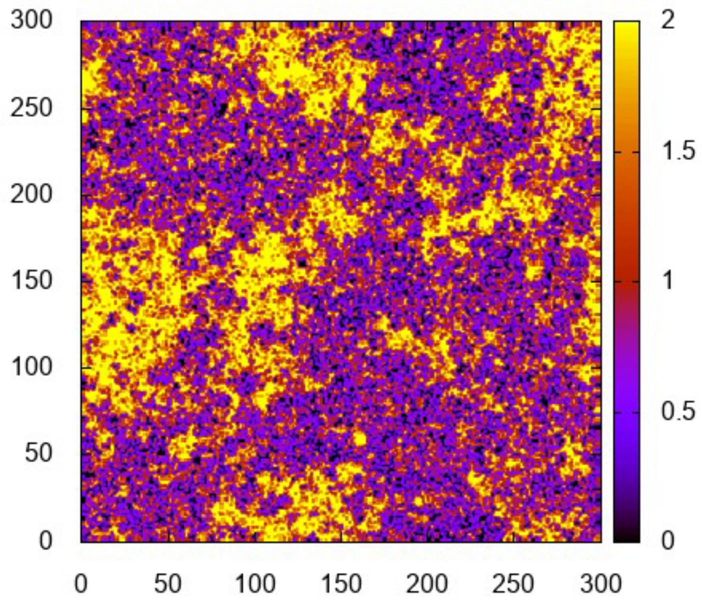
$\nabla\cdot\boldsymbol{\xi}(\mathbf{r}) = 0$ does not enter density fluctuations

$$\mathbf{f}(\mathbf{r}) = -\nabla U(\mathbf{r}) + \boldsymbol{\xi}(\mathbf{r})$$

Find that effect potential obeys:

$$\overline{U(\mathbf{q})U(\mathbf{q}')} = \frac{\sigma^2}{q^2}\delta^d_{\mathbf{q},-\mathbf{q}'}$$

Effective potential self-affine with deep wells.



Next *check when linear theory is self-consistent*

$$\overline{\langle \delta\rho^2(\ell) \rangle} \ll \rho_b^2$$

ℓ - scale of box we are looking in

Find (a - uv cutoff)

$$\frac{\overline{\langle \delta\rho^2(\ell)\rangle}}{\rho_b^2} = \begin{cases} \dfrac{\sigma^2 \ln(\ell/a)}{\pi u^2 \rho_b^2} & \text{for} \quad d = 2 \\ \dfrac{\sigma^2 a^{2-d}}{(d-2) S_d u^2 \rho_b^2} & \text{for} \quad d > 2\ . \end{cases}$$

eventually fails (for $d = 2$)

always ok for weak (for $d > 2$)

Implies

- In 2d beyond a length scale ℓ^* the behavior is expected to break down (in numerics never see this)

$$\ell^* \equiv a \exp(\pi u^2 \rho_b^2 / \sigma^2)$$

- In d>2 for weak disorder theory self consistent. *For strong disorder this suggests a new phase*.

Currents

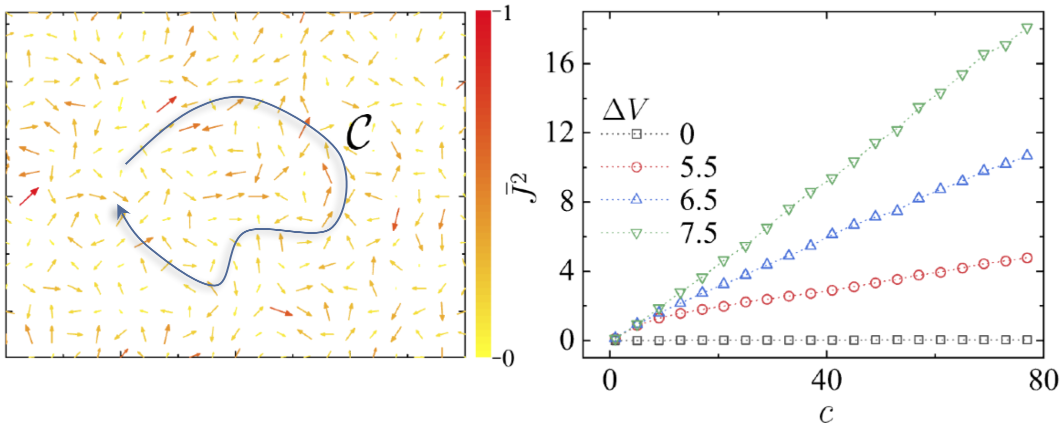
Equation of motion

$$\mathbf{j}(\mathbf{r},t) = -\nabla\mu\,[\phi] + \mathbf{f}(\mathbf{r}) + \boldsymbol{\eta}(\mathbf{r},t)$$

Circulation of current

$$J(\mathcal{C}) \equiv \oint_{\mathcal{C}} \mathrm{d}\mathbf{l}\cdot\mathbf{j}(\mathbf{r})$$

$$\overline{J^2}(c) \propto c$$



So far -

- Circulating currents
- Generic disorder induced long range correlations
- Effective potential self-affine Gaussian surface

With potential surface can check when phase separation stable against disorder

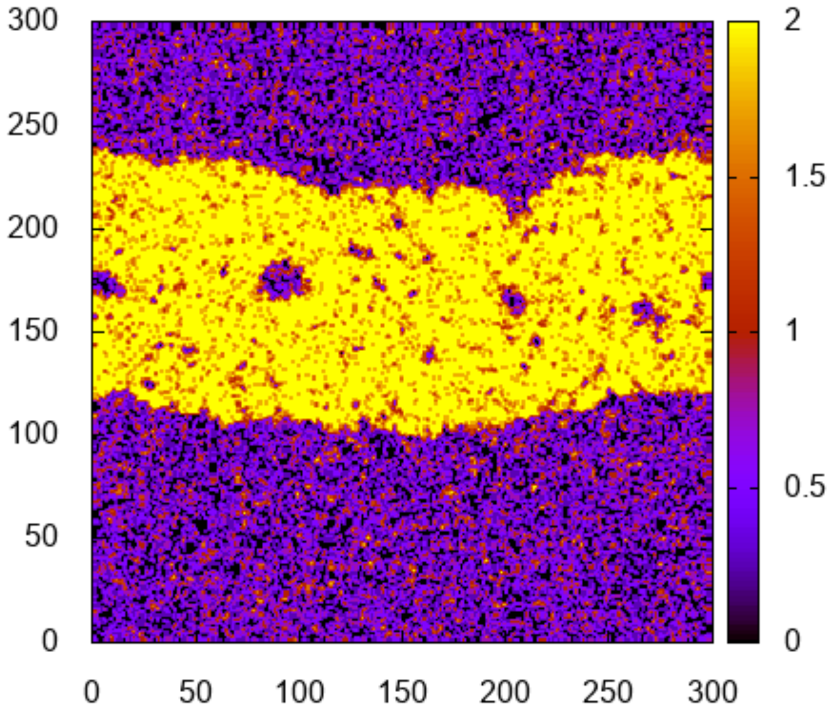
Lower critical dimension

Use standard Imry-Ma argument with

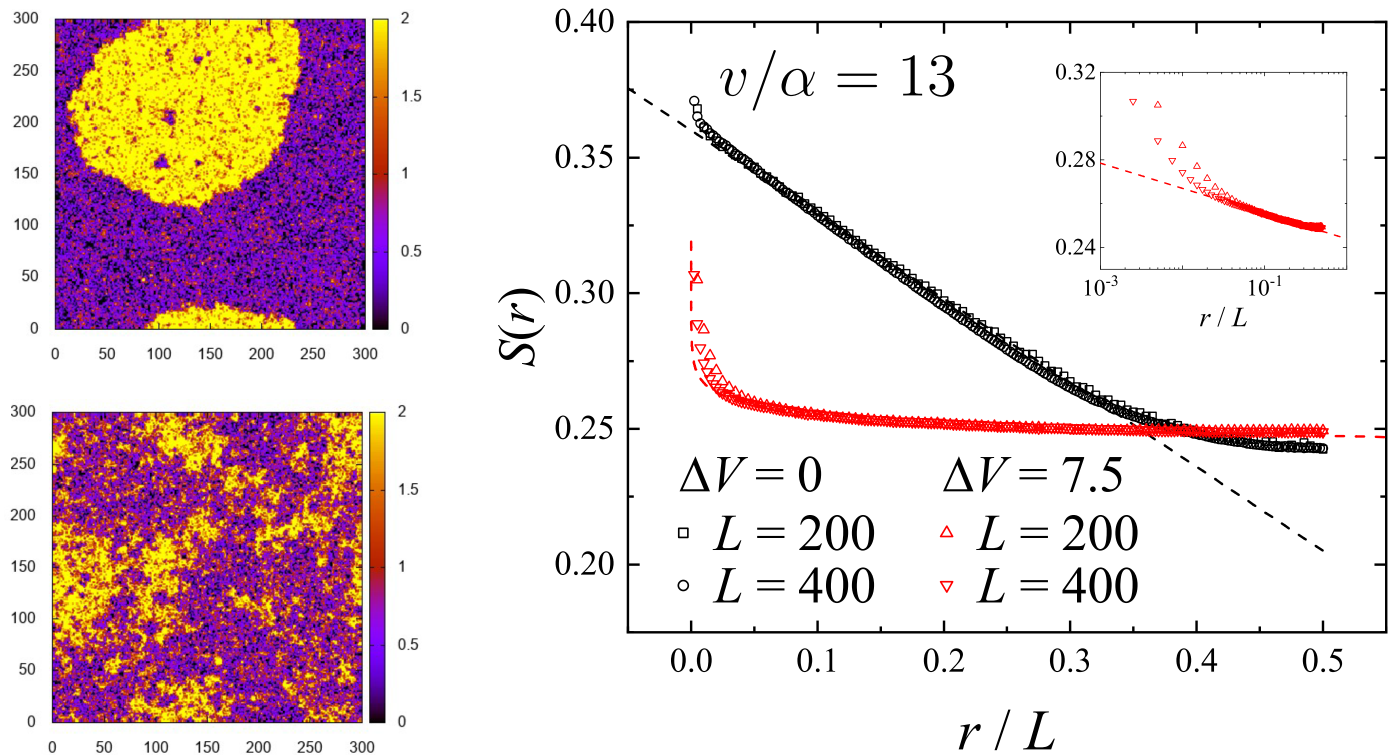
$$\overline{U(\mathbf{q})U(\mathbf{q}')} = \frac{\sigma^2}{q^2}\delta^d_{\mathbf{q},-\mathbf{q}'}$$

Obtain - no MIPS below

$$d_c = 4$$



Movie - disorder growing in time



Agrees rather well (in 2d)

Comment:

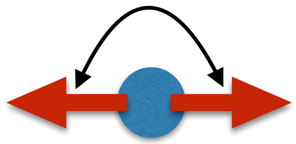
Recent work (Toner, Guttenberg, Tu PRL 2018) showed that the Vicsek model with disorder has quasi-long range order in d=2 and long range in d>2 - less sensitive to disorder when compared to equilibrium $d_c = 4$

See also Duan et. al. arxiv:2010.02356

In our case, a discrete symmetry is more sensitive to disorder compared to the equilibrium case $d_c = 2$

- **One-dimensional systems**

1D RUN-AND-TUMBLE PARTICLES



first non-interacting

- Changing stochastically between velocities $\pm v$ with rate $\frac{\alpha}{2}$

- Fokker-Planck equation for the probability densities $\mathcal{P}_{\pm}(x,t)$

$$\partial_t \mathcal{P}_+(x,t) = -\,\partial_x \left[v(x)\mathcal{P}_+(x,t) - \mu\left(\partial_x V\right)\mathcal{P}_+(x,t)\right] - \frac{\alpha(x)}{2}\left[\mathcal{P}_+(x,t) - \mathcal{P}_-(x,t)\right]$$
$$\partial_t \mathcal{P}_-(x,t) = -\,\partial_x \left[-v(x)\mathcal{P}_-(x,t) - \mu\left(\partial_x V\right)\mathcal{P}_-(x,t)\right] - \frac{\alpha(x)}{2}\left[\mathcal{P}_-(x,t) - \mathcal{P}_+(x,t)\right]$$

- With no disorder, constant speed and tumbling rate on long-time scales — diffuse with an effective diffusion coefficient

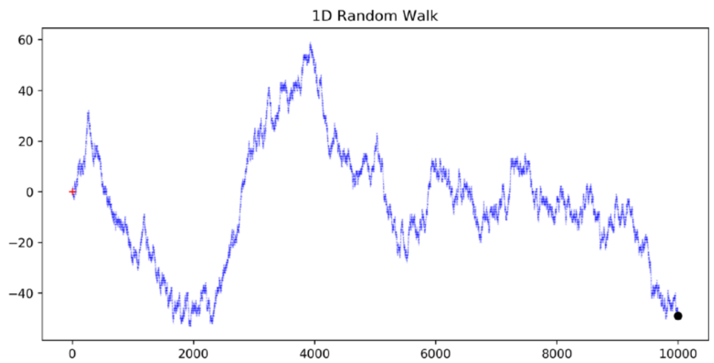
$$\mathcal{D}_{\text{eff}} = \frac{v^2}{\alpha}$$

Steady-state distribution known -
Can show exactly that random forcing energy landscape

$$\rho_V^{(s)}(x) = \frac{1}{1 - \left(\frac{\mu}{v}\right)^2 (\partial_x V)^2} \exp\left(-\frac{\alpha\mu}{v^2} \int^x dx' \, \frac{(\partial_{x'} V)}{1 - \left(\frac{\mu}{v}\right)^2 (\partial_{x'} V)^2} \right)$$

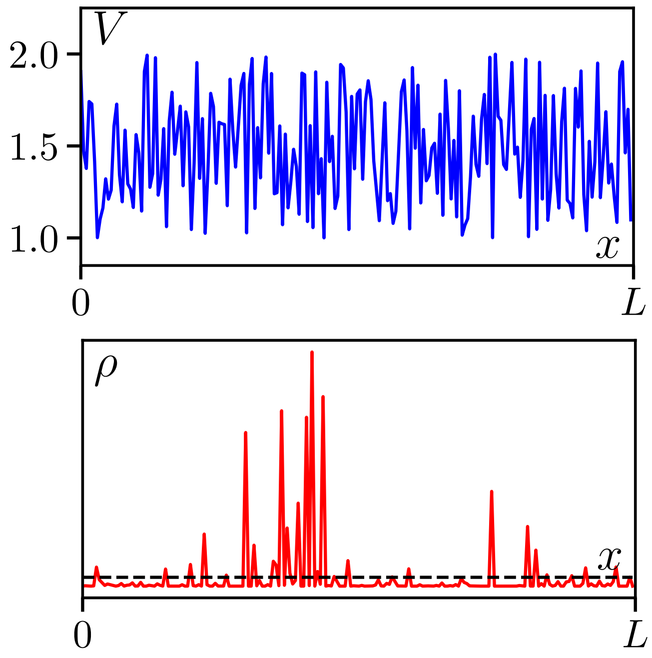
non-local `effective potential'

asymmetric potential - integral does not vanish
looks on large length scales like a tilt



Effects of potential disorder?

(steady-state measure)



Dynamics

Using mean-first passage time

$$\Rightarrow \quad \overline{\langle x^2(t) \rangle} \underset{t \to \infty}{\propto} \ln^4(t)$$

At the exponential level equivalent to Sinai diffusion

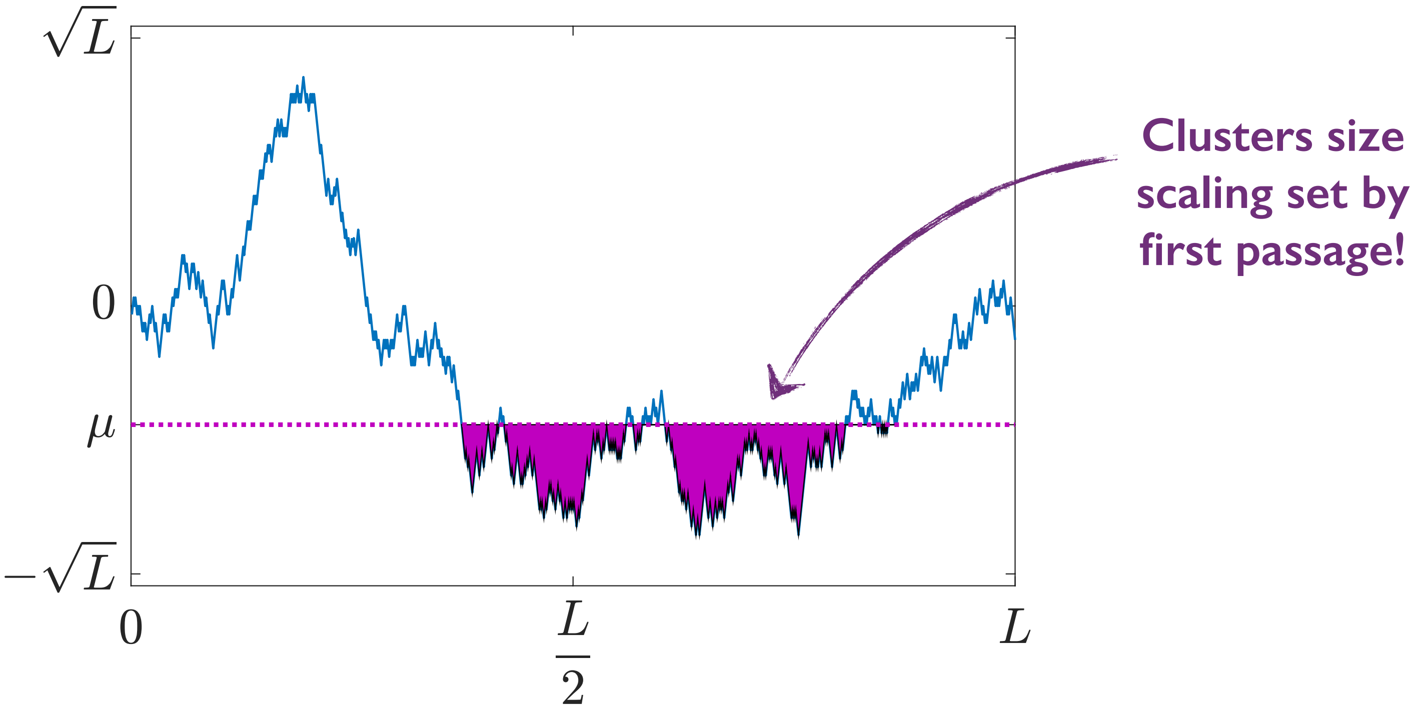
Review: JP Bouchaud, A Comtet, A Georges, P Le Doussal
Annals of Physics 201 (2), 285-341

Many interacting (hard core) - **strong disorder** behavior $\ell \gg \frac{D^2}{\sigma^2}$

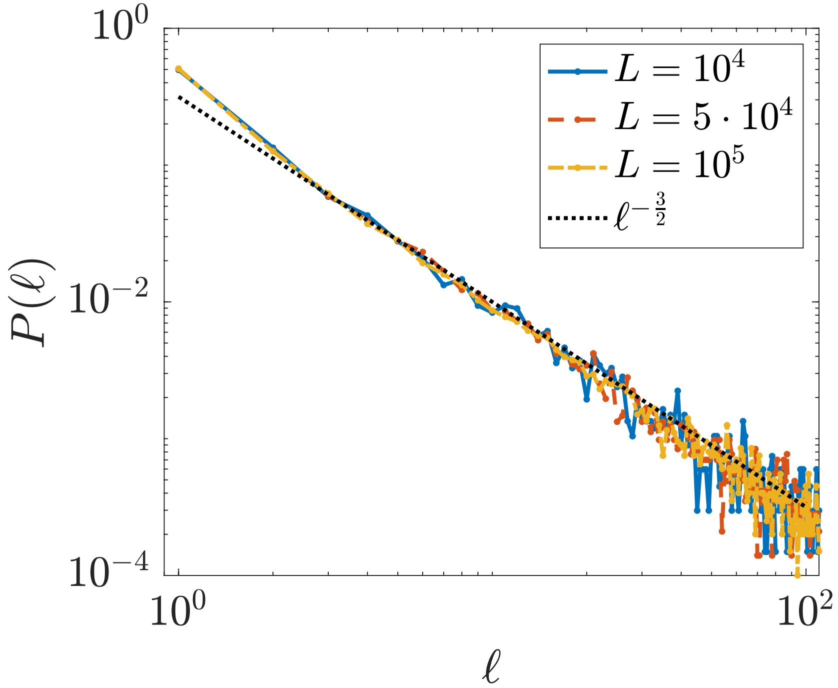
No disorder - finite clusters in one-dimension

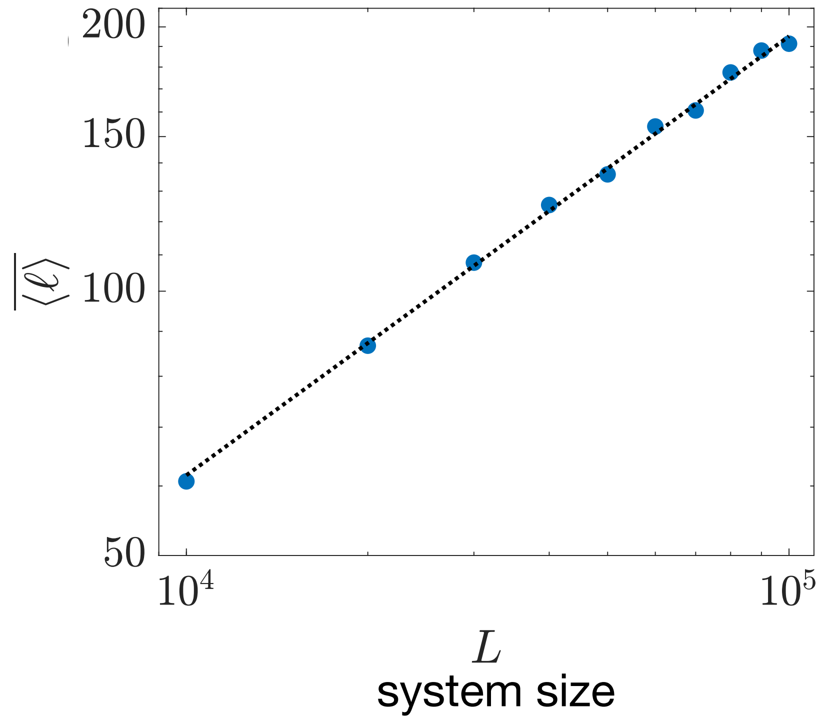
ME Cates and J Tailleur. Annual Review of Condensed Matter Physics, 6(1):219–244, 2015.
R. Soto and R. Golestanian. Phys. Rev. E **89**, 012706, 2014.

Using analogy to random forcing - Fermion on random forcing landscape



Cluster size distribution and mean cluster size





$$P(\ell) \sim 1/\ell^{3/2}$$

$$\langle \ell \rangle \sim \sqrt{L}$$

in one dimension disorder enhances clustering !

Summary -

- Disorder induces circulating currents
- Disorder induces generic long range correlations
- Holds *for any* dilute active system
- Lower critical dimension for MIPS is $d_c = 4$