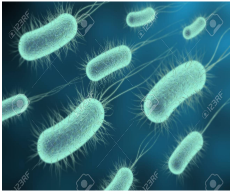
Active Matter

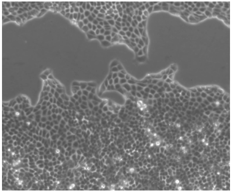
Drive at the microscopic level → *Strongly out of equilibrium* → *Fundamentally new physics*



Swimming bacteria

Bird Flocks

[COBBS Lab, Rome]



Crawling cells

[Poujade, PNAS 2007]

- Biological relevance
- Explore new dynamical phenomenology

Active Matter

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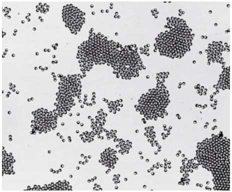
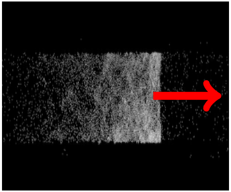
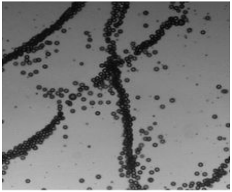


Clusters without attractive interactions
[van der Linden, PRL 2019]



Solitonic waves
[Bricard , Nature 2013]



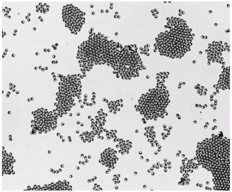
Filaments
[Thutupalli, PNAS 2018]

- Biological relevance
- Active Soft Materials

- Explore new dynamical phenomenology
- Build generic framework for Active Matter

Active Matter

Drive at the microscopic level → *Strongly out of equilibrium* → *Fundamentally new physics*



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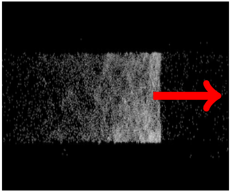

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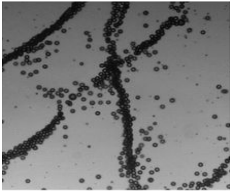


Filaments
[Thutupalli, PNAS 2018]

- Biological relevance
- Active Soft Materials

- Explore new dynamical phenomenology
- Build generic framework for Active Matter

→ A whole spectrum of questions ranging from applied maths to experimental physics

- Cécile Cottin-Bizonne: Active matter in real life
- Clément Erignoux: Exact coarse-grained descriptions of (some) active systems
- Liesbeth Janssen: Glasses, from passive to active
- Yariv Kafri: Random inhomogeneous active systems
- Martin Evans: Exact results on active lattice gases
- Mike Cates: Time-reversal symmetry in active field theories
- Vincent Calvez: How applied maths meet spreading organisms

First-order fluctuation-induced phase transition to collective motion

D. Martin, G. Spera, H. Chaté, C. Nardini, A. Solon, J. Tailleur, F. van Wijland

Laboratoire MSC
CNRS - Université Paris Diderot

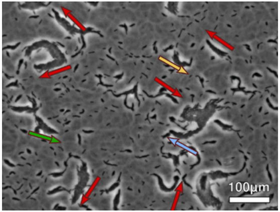
Random Inhomogeneous Systems

Collective motion

Birds

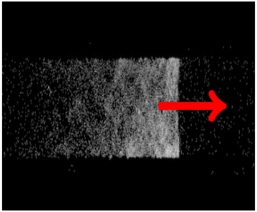
Fish

Sheep



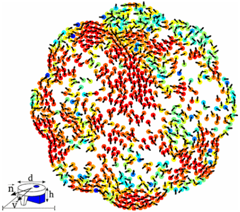
Myxobacteria

[Peruani *et al.*, PRL 2012]



Quincke rollers

[Bricard *et al.*, Nature 2013]



Vibrated disks

[Deseigne *et al.*, PRL 2010]

Minimal ingredients: Self-propulsion + Alignment

The Vicsek model [Vicsek et al. PRL 75, 1226 (1995)]

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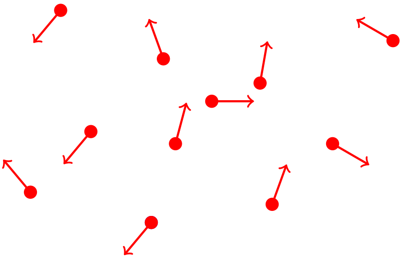
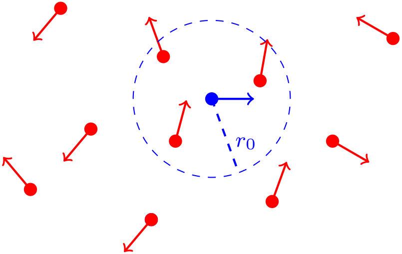


- N self-propelled particles in 2D continuous space
- Local alignment rule

The Vicsek model [Vicsek et al. PRL 75, 1226 (1995)]



- N self-propelled particles in 2D continuous space
- Local alignment rule with metric rules iVj iff $|r_i - r_j| < r_0$

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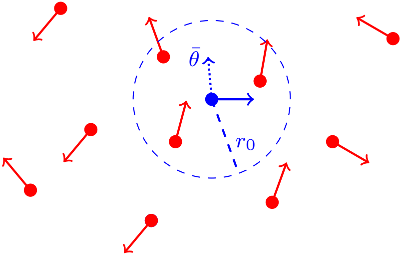

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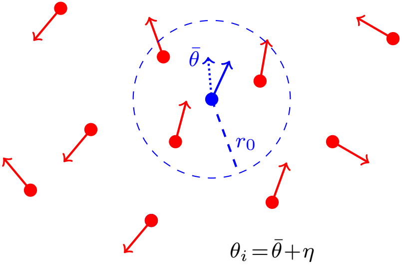

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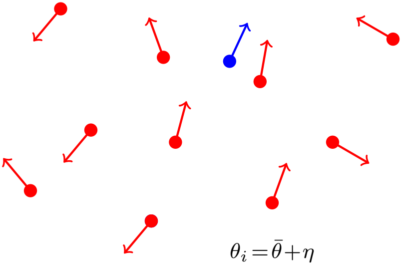

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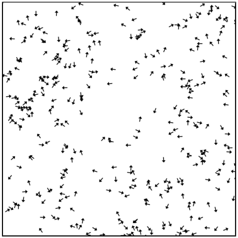
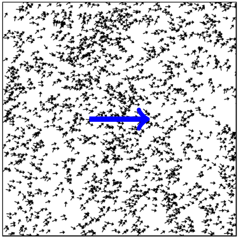
Flocking transition



Disordered

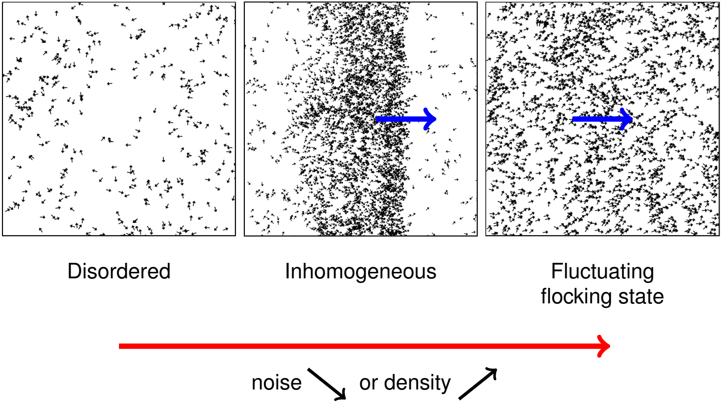


Fluctuating
flocking state

noise ↘ or density ↗

- Non-equilibrium transition to long-range order in $d = 2$ [Toner-Tu, PRL (1995)]

Flocking transition

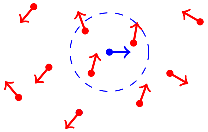


- Non-equilibrium transition to long-range order in $d = 2$ [Toner-Tu, PRL (1995)]
- First-order transition [Grégoire, Chaté, PRL (2004)] (after a long debate)

The two scenarios for the transition to collective motion

1st order transition

- Metric Vicsek model



⟶ analytical (MF) & numerical results

The two scenarios for the transition to collective motion

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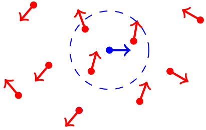
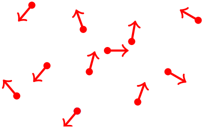


⟶ analytical (MF) & numerical results

Continuous transition

- “Topological” Vicsek model

⟶ Voronoi neighbours



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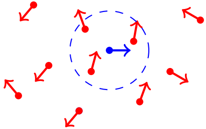

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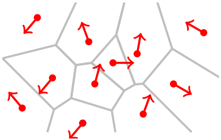

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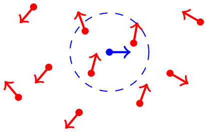

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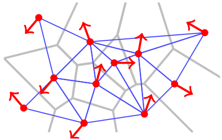

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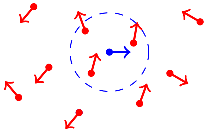

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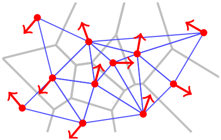


→ k-nearest neighbours

→ numerical & analytical (MF) results

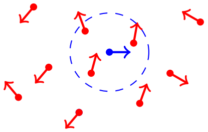
- Some metric model

→ Scaling limits

The two scenarios for the transition to collective motion

1st order transition

- Metric Vicsek model



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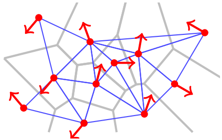

→ k-nearest neighbours

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1 Characterize the “1st-order transition”

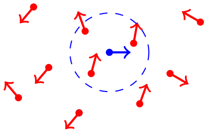
2 Discuss the underlying mechanism

3 Show/argue that “1st-order scenario” is generic

The two scenarios for the transition to collective motion

1st order transition

- Metric Vicsek model



⟶ analytical (MF) & numerical results

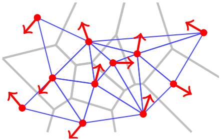
- Active Ising Model

⟶ numerical results

Continuous transition

- “Topological” Vicsek model

⟶ Voronoi neighbours



⟶ k-nearest neighbours

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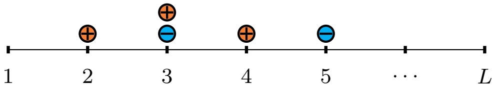
- Active Ising Model

⟶ mean-field computations

1. Characterize the “1st-order transition”
2. Discuss the underlying mechanism
3. Show/argue that “1st-order scenario” is generic

Active Ising model [Solon, Tailleur, PRL 2013]



- Density $\rho_i = n_i^+ + n_i^-$ Magnetisation $m_i = n_i^+ - n_i^-$

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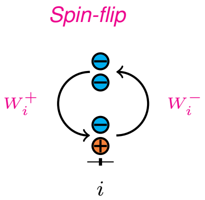
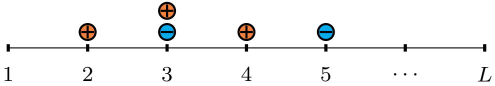




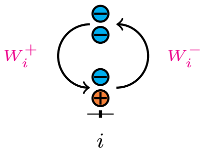
- Density $\rho_i = n_i^+ + n_i^-$ Magnetisation $m_i = n_i^+ - n_i^-$

- Local alignment ⟶ $W_i^{\pm} = \exp(\pm\beta \frac{m_i}{\rho_i})$

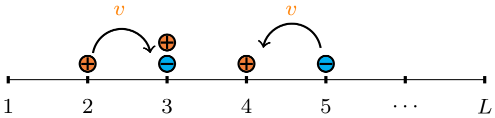
  ⟶ Fully connected Ising models on each site

Active Ising model [Solon, Tailleur, PRL 2013]

Spin-flip



Biased hopping



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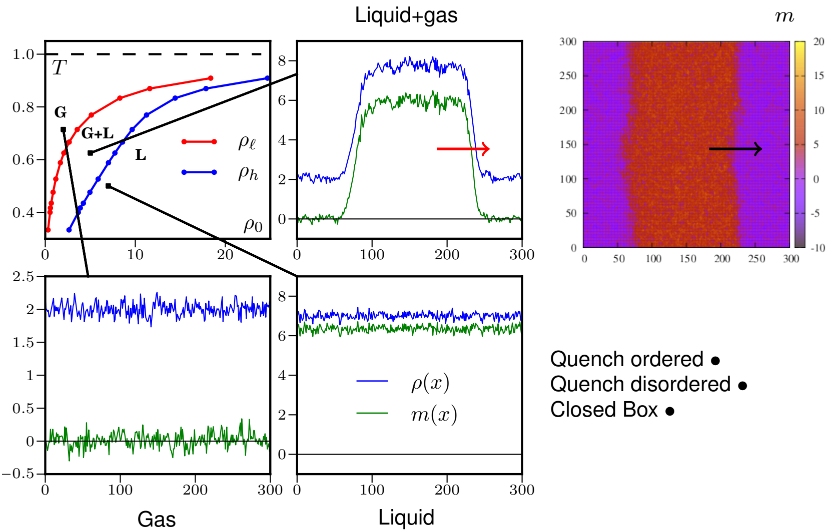
- Local alignment ⟶ $W_i^\pm = \exp(\pm\beta \frac{m_i}{\rho_i})$

  ⟶ Fully connected Ising models on each site

- Self-propulsion ⟶ Hoping along $\hat{e}_x$ biased by the spins

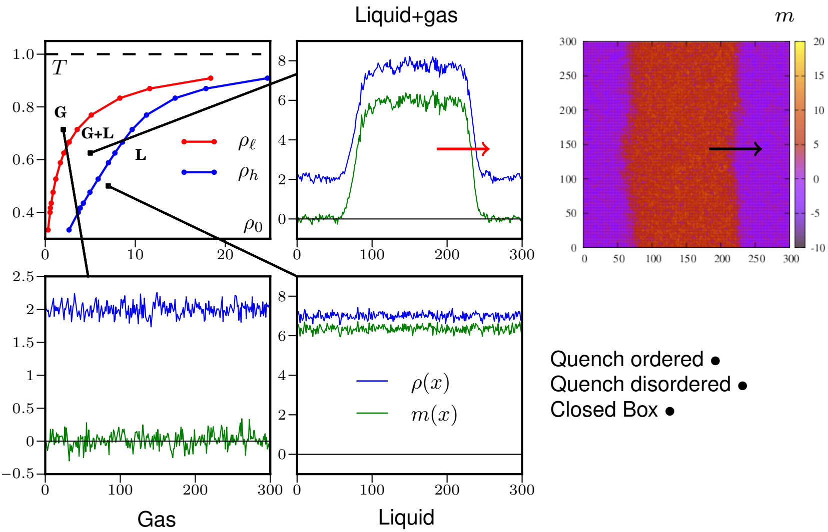
  (Symmetric diffusion along $d-1$ other directions)

Phase diagram in 2d $(\rho, T \equiv \beta^{-1})$



Quench ordered •
Quench disordered •
Closed Box •

Phase diagram in 2d $(\rho, T \equiv \beta^{-1})$

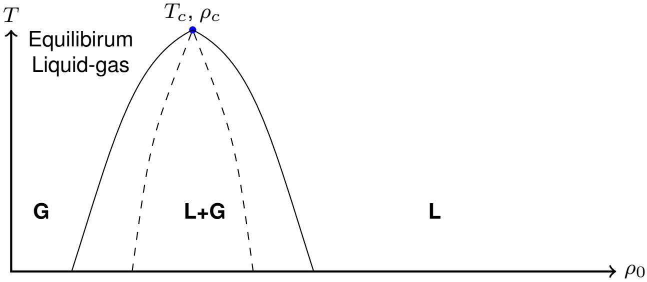


Quench ordered •
Quench disordered •
Closed Box •

A liquid-gas phase transition: nucleation, hysteresis, lever rule, finite-size scaling

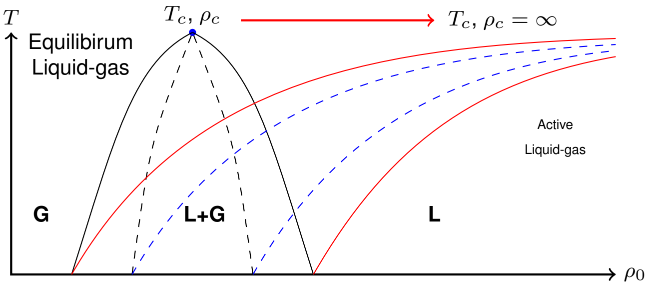
1st take-home Message

- Flocking transition ⟶ Liquid-gas transition in canonical ensemble



1st take-home Message

- Flocking transition ⟶ Liquid-gas transition in canonical ensemble
- Symmetry of the liquid phase ⟶ $\rho_c = \infty$



Can we understand (analytically) the nature of the transition ?

Phenomenological hydrodynamic description in 1d

- Density $\rho_i = \sum_i (n_i^+ + n_i^-)$ and magnetisation $m_i = \sum_i (n_i^+ - n_i^-)$
- Mean-field approximation $\langle f(\rho_i, m_i) \rangle \simeq f(\langle \rho_i \rangle, \langle m_i \rangle)$ + gradient expansion

$$\dot{\rho}(x,t) =$$

$$\dot{m}(x,t) =$$

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$$\dot{\rho}(x,t) = D\partial_{xx}\rho$$

$$\dot{m}(x,t) = D\partial_{xx}m$$

- Random hopping ⟶ Diffusion

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$$\dot{\rho}(x,t) = D\partial_{xx}\rho - v\partial_x m$$
$$\dot{m}(x,t) = D\partial_{xx}m - v\partial_x \rho$$

- Random hopping ⟶ Diffusion
- Self-propulsion ⟶ advective terms •

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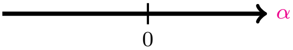
$$\dot{\rho}(x,t) = D\partial_{xx}\rho - v\partial_x m$$
$$\dot{m}(x,t) = D\partial_{xx}m - v\partial_x\rho + 2\rho\sinh(\beta\frac{m}{\rho}) - 2m\cosh(\beta\frac{m}{\rho})$$

- Random hopping ⟶ Diffusion
- Self-propulsion ⟶ advective terms •
- Alignment ⟶ Mean-field theory with local intensive magnetisation $\hat{m}(x) = \frac{m(x)}{\rho(x)}$

The transition as β is varied

- Expansion for small $\frac{m}{\rho}$ with $\alpha \equiv 1 - \beta$ and $\Gamma \equiv \frac{2\beta(1-\beta/3)}{\rho^2}$

$$\dot{\rho}(x,t) = D\Delta\rho - v\partial_x m$$
$$\dot{m}(x,t) = D\Delta m - v\partial_x \rho - \alpha m - \Gamma m^3$$



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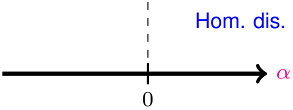

- $\alpha > 0 \longrightarrow$ Homogeneous disordered system ($\rho(x) = \rho_0, m(x) = 0$) linearly stable

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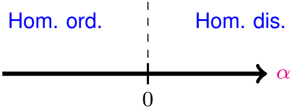

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- $\alpha < 0$ ⟶ Homogeneous ordered solution $\rho(x) = \rho_0$, $m(x) = \sqrt{\alpha/\Gamma(\rho_0)}$ linearly stable

- Mean-field treatment predicts a continuous transition ☹

- Bands solution do not exist [Caussin *et al.*PRL 2014]

The transition as β is varied

- Expansion for small $\frac{m}{\rho}$ with $\alpha \equiv 1 - \beta$ and $\Gamma \equiv \frac{2\beta(1-\beta/3)}{\rho^2}$

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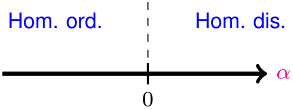

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- Bands solution do not exist [Caussin *et al.*PRL 2014]
- What is missing? Density-dependency of $\alpha(\rho)$ [Gregoire *et al.*, J. Phys. A 2009]

The transition as β is varied

- Expansion for small $\frac{m}{\rho}$ with $\alpha \equiv 1 - \beta$ and $\Gamma \equiv \frac{2\beta(1-\beta/3)}{\rho^2}$

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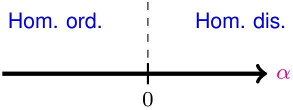

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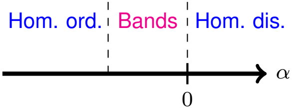

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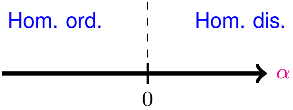

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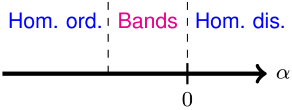

- The dependence of the 'critical temperature' α with ρ_0 leads to a liquid-gas scenario

Refined Mean-Field Model (RMFM)

- AIM: MF is WRONG (phase diagram, order of transition, no inhomogeneous profile)

$$\dot{\rho} = D\Delta\rho - v\partial_x m$$

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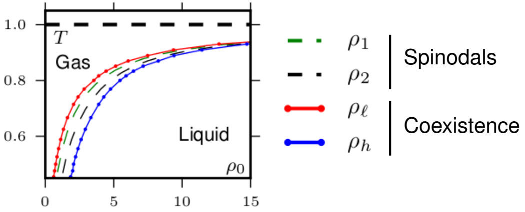

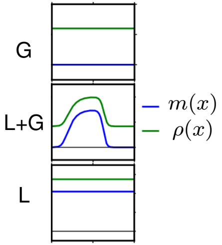

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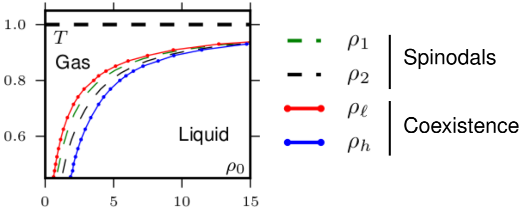

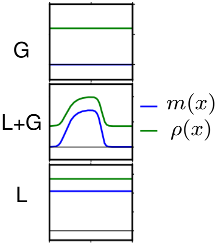

- Same phenomenology as microscopic model
- Q1: Can we indeed show that fluctuations make $\alpha(\rho)$
- Q2: How generic is this?

Quasi-linear renormalization

- We add noise on the mean-field hydrodynamics of AIM

$$\partial_t \rho = D\partial_{xx}\rho - v\partial_x m \tag{1}$$
$$\partial_t m = D\partial_{xx}m - v\partial_x\rho - \mathcal{F}(\rho,m) + \sqrt{2\sigma\rho}\,\eta \tag{2}$$

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- Need to compute $\langle\delta\rho^2\rangle$, $\langle\delta m^2\rangle$, $\langle\delta\rho\delta m\rangle$ as functions of ρ_0 and m_0, to leading order in σ.

- Using a Gaussian approximation on the dynamics of $\delta\rho$ and δm:

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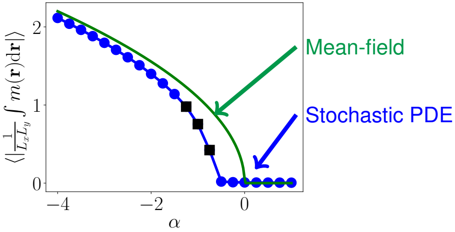

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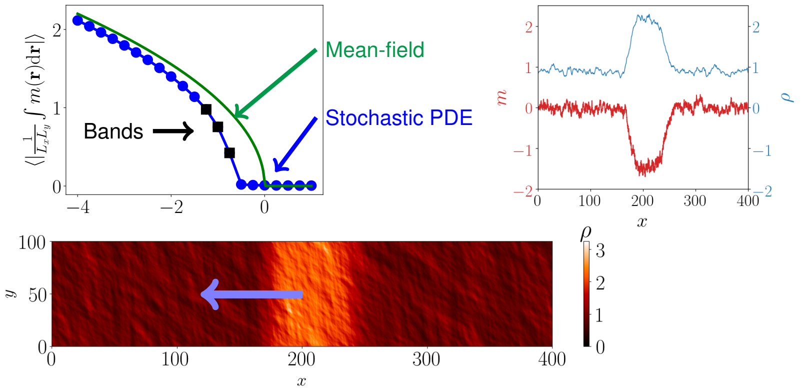

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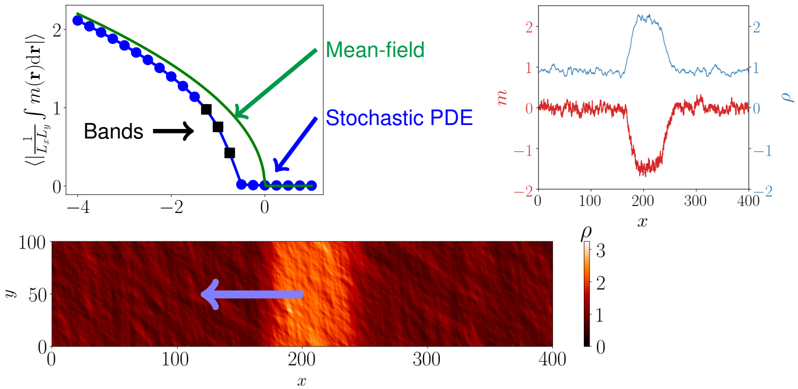

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- Fluctuation-induced phase-separation scenario is generic in metric models
- Scaling limits in which $\alpha \in \mathbb{R}$ are singular limits

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PNAS

Interaction ruling animal collective behavior depends on topological rather than metric distance: Evidence from a field study

M. Ballerini[*,†], N. Cabibbo[‡,§], R. Candelier[‡,¶], A. Cavagna[*,‖,**], E. Cisbani[†], I. Giardina[*,‖], V. Lecomte[††,‡‡], A. Orlandi[*], G. Parisi[*,‡,§,**], A. Procaccini[*,‡], and M. Viale[‡,§§], and V. Zdravkovic[*]

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Voronoi neigbours

PHYSICAL REVIEW X **6,** 021011 (2016)

Motility-Driven Glass and Jamming Transitions in Biological Tissues

Dapeng Bi,[1,3] Xingbo Yang,[1,4] M. Cristina Marchetti,[1,2] and M. Lisa Manning[1,2]

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[4]*Department of Physics, Northwestern University, Evanston, Illinois 60208, USA*

(Received 25 October 2015; revised manuscript received 24 February 2016; published 21 April 2016)

Cell motion inside dense tissues governs many biological processes, including embryonic development and cancer metastasis, and recent experiments suggest that these tissues exhibit collective glassy behavior. To make quantitative predictions about glass transitions in tissues, we study a self-propelled Voronoi model that simultaneously captures polarized cell motility and multibody cell-cell interactions in a confluent

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- Idea: particles at $\mathbf{r}$ align with a field $\bar{m}(\mathbf{r})$ resulting from topological construction
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- E.g: k nearest neighbours.
- Position-dependent interaction range $y(x)$ such that

$$\int_{x-y(x)}^{x+y(x)} dz \rho(z) = k$$

- $\bar{m}(x)$ is averaged over $[x - y(x), x + y(x)]$:

$$\bar{m}(x) = \frac{1}{k} \int_{x-y(x)}^{x+y(x)} dz m(z)$$

- Landau term: $\mathcal{F} = 2m \cosh(\beta \bar{m}) - 2\rho \sinh(\beta \bar{m}) \simeq 2m - 2\rho\beta\bar{m} - \frac{\rho\beta^3}{3}\bar{m}^3 + \beta^2 m \bar{m}^2$
- Mean-field hydrodynamics

$$\begin{aligned}
\partial_t \rho &= D\partial_{xx}\rho - v\partial_x m \\
\partial_t m &= D\partial_{xx} m - v\partial_x \rho - \mathcal{F}(\rho, m, \bar{m})
\end{aligned}$$

- Predicts a continuous transition, in agreement with the literature

Stability against fluctuations?

- Again complement with noise: $\partial_t m = [...] + \sqrt{2\rho\sigma}\eta$
- Pain & suffering $\longrightarrow$ Renormalized $\tilde{\alpha}(\rho)$ $\longrightarrow$ Fluctuation-induced $1^{\rm st}$-order transition

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$$\dot{\mathbf{r}}_i = s_i\, v_0\, \mathbf{u}_x + \sqrt{2D}\boldsymbol{\eta}_i \tag{6}$$

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$$W(s_i) = \Gamma e^{-\beta s_i \bar{m}_i}, \quad \text{where} \quad \bar{m}_i = \frac{1}{k}\sum_{j\in\mathcal{N}_i} s_j \tag{7}$$

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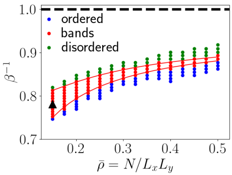

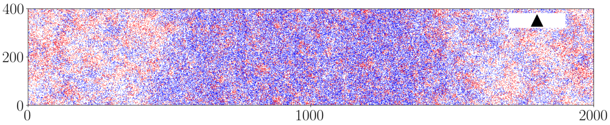

- In huge enough systems ⟶ 1st-order transition & bands

Summary

- Fluctuations generically make “critical temperature” depend on density
- A density-dependent critical temperature leads to phase-separation scenario
- Holds for metric as well as topological models

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- Fluctuations generically make “critical temperature” depend on density
- A density-dependent critical temperature leads to phase-separation scenario
- Holds for metric as well as topological models
  - Articles on the Active Ising model & the transition:
    [Solon, Tailleur, PRL 2013; PRE 2015; Solon, Tailleur, Chaté, PRL 2015]
  - Fluctuation-induced 1st-order scenario:
    [D. Martin *et al.* arXiv:2008.01397]

THANK YOU!

Active terms

$$\dot{\rho}(x,t) = D_r \partial_{xx} \rho$$
$$\dot{m}(x,t) = D_m \partial_{xx} m - \alpha m - \Gamma m^3$$

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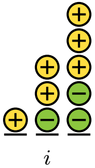

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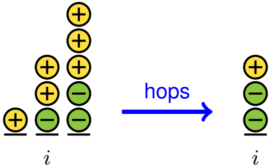

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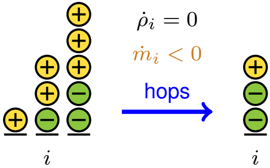
$\dot{\rho}_i = 0$

$\dot{m}_i < 0$



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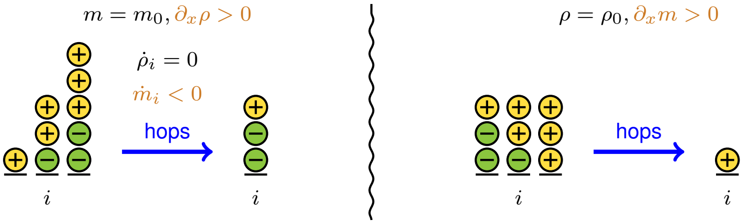

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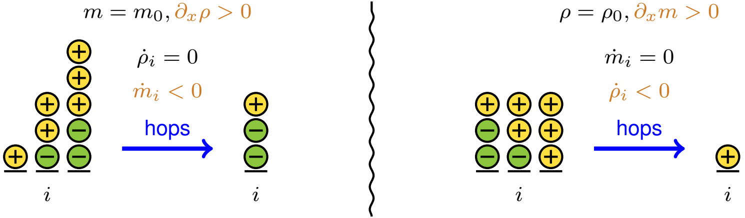

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