Random Matrices
23 January 2002, Neuville sur Oise
Moderator: Vincent Rivasseau

Evgueni Bogomolny (CNRS et Université Paris-Sud Orsay, France):
Spectral Statistics of Singular Billiards

Rank-one perturbation of diagonal random matrices of the form

$$M_{mn} = e_n \delta_{mn} + V_n^* V_m$$

where e_n are i.i.d. random variables and V_n is a fixed vector appear naturally in different situations. The simplest physical realization of such model is an integrable billiard with a short-range impurity inside. It is demonstrated that all spectral correlation functions of such models can be computed analytically. We present explicit calculations of the two level correlation function and the nearest-neighbor distribution and prove that the spectral statistics of these models belong to intermediate type characterised by level repulsion at short distances (as for standard random matrix ensembles) and exponential decrease of the nearest-neighbor distribution at large distances (as for the Poisson statistics). Different generalizations of these results are shortly discussed.

Margherita Disertori (Institute for Advanced Study, Princeton, USA):
Density of states for random band matrices
By applying the supersymmetric approach we prove smoothness of the averaged density of states for a three dimensional random band matrix ensemble, in the limit of infinite volume and fixed band width. We also prove that the resulting expression for the density of states coincides with the Wigner semicircle law with a precision $1/W^2$, for W large but finite.

Bertrand Duplantier (CEA, Saclay France):
Double Conformal Multifractality
The double multifractal spectrum $f(\alpha, \alpha')$ of the potential near any simple conformally invariant scaling curve in the plane is given. It describes the Hausdorff dimension of the set of points along the curve where the potential H varies jointly as r^α on one side, and as $r^{\alpha'}$ on the other. The universal result is labelled by the central charge c describing the associated conformal field theory. It includes the cases of the self-avoiding walk, the frontier of a planar Brownian motion, the external perimeter of a percolation cluster, which are all the same universal scaling curve, as well as the external perimeter of a critical Potts cluster. Use is made of two-dimensional quantum gravity.

Yan V. Fyodorov (Brunel University, UK):
On statistics of characteristic polynomials of random Hermitian matrices
We reconsider the problem of calculating a general spectral correlation function containing arbitary number of products and ratios of characteristic polynomials for a random matrix taken from the Gaussian Unitary Ensemble (GUE). Deviating from the standard "super-symmetry" approach we circumvent the use of the Hubbard-Stratonovich transformation

and integrate out Grassmann variables at early stage. The method is shown to be capable of calculation when reinforced with a generalisation of Duistermaat-Heckman localisation theorem to integrals over non-compact homogeneous Kähler manifolds. In the limit of large matrices the asymptotic expression for the correlation function reproduces the result anticipated earlier by Andreev and Simons. In the end we present similar results for characteristic polynomials of chiral GUE matrices.

Herbert Spohn (Technische Universität München, Germany):
Edge scaling for Dyson's Brownian motion model and the PNG growth model
We consider the Poisson model of point to point (directed) last passage percolation. As shown by Baik, Deift, Johansson asymptotically the length of the optimal path is given by the largest eigenvalue of a GUE random matrix. More detailed, we study the joint distribution of the length of the optimal paths for two distinct end points. Under appropriate scaling it is related to the last line of Dyson's Brownian motion model.

Paul Zinn-Justin (CNRS et Université Paris-Sud Orsay, France):
Counting tangles via matrix models
We shall show how matrix models and the combinatorics of planar diagrams allow to solve a certain class of enumeration problems for knots, links and tangles. Some particular cases allow to recover results known to mathematicians. Here the general problem of counting alternating tangles will be presented in a unified framework using quantum field-theoretic methods applied to a matrix model of colored links. The topological equivalences will be removed by means of a "renormalization" of the theory. The implementation of this method to obtain a computerized enumeration of tangles to high orders using a transfer matrix approach will also be sketched.

Martin Zirnbauer (Universität-Köln, Germany):
Dual pairs in random matrix theory
The traditional tool for computing the correlations of random matrix eigenvalues are orthogonal polynomials. In this talk an alternative method, rooted in the theory of dual pairs and symplectic geometry, is presented. By tensoring the oscillator representation of the metaplectic group with the spinor representation of the spin group, a representation of an orthosymplectic Lie supergroup OSp is obtained. Its character is the square root of a superdeterminant, and serves as a generating function for the eigenvalue correlations of unitary matrices. We then focus on the circular random matrix ensembles defined over the classical compact Lie groups. Their correlation functions are computed by considering any one of the dual pairs O(N) $\times$ OSp(2n—2n), U(N) $\times$ GL(n—n), or USp(2N) $\times$ OSp(2n—2n) inside the oscillator-spinor representation of OSp. On integrating over the first factor with Haar measure, and using Roger Howe's exposition of classical invariant theory, the correlation functions become characters of an irreducible highest-weight representation of the second factor. In the large-N limit, the latter can be evaluated by a supersymmetric generalization of the Duistermaat-Heckman theorem.