1999

Entropic Repulsion and Fluctuations of Random Surfaces

Jean-Dominique Deuschel (Berlin): "Entropic repulsion and wetting transition for massless fields"

We consider a lattice free field, that is an SOS model with an effective gradient interaction. We investigate the effect of a hard wall condition (entropic repulsion) and weak pinning on the interface. Without pinning, in a box of size N, the spins are repelled at height $\sqrt{\log N}$ for $d \geq 3$, $\log N$ for $d = 2$ and $\sqrt{N}$ for $d = 1$. When repulsion and pinning compete, the field undergoes a wetting transition in dimension one, that is repulsion for weak pinning and localization for strong pinning. This is in contrast to the transient dimensions $d \geq 3$ where pinning always dominates. The delicate two dimensional case is still open.

This is a joint work with Erwin Bolthausen and Ofer Zeitouni.

Giambattista Giacomin (Milano): "Large Deviations for massless fields"

We will introduce and discuss the continuum spin models which go under the name of massless fields. These can be interpreted as simplified models for a random interfaces. We will then state a LD principle and we will link the LD rate functional with the surface tension of the model. The emphasis will be on some new tools introduced by various authors to deal with fields of massless type.

Servet Martinez (Santiago): "One-dimensional interfaces: Hamiltonians on random walk trajectories"

We consider Gibbs measures on the set of paths of nearest neighbor random walks on $\mathbb{Z}_+$. The basic measure is the uniform measure on the set of paths of the simple random walk on $\mathbb{Z}_+$ and the Hamiltonian awards each visit to site $x \in \mathbb{Z}_+$ by an amount $\alpha_x \in \mathbb{R}$, $x \in \mathbb{Z}_+$. We give conditions on (α_x) that guarantee the existence of the (infinite volume) Gibbs measure. When comparing the measures in $\mathbb{Z}_+$ with the corresponding measures in $\mathbb{Z}$, the so called entropic repulsion appears as a counting effect.

Johannes Sjoestrand (Palaiseau): "Integrals and Schrödinger operators in high dimension"

This is mainly a survey talk about some methods and results which may be useful for the problems more in the center of the "journée". We will discuss certain estimates for Schrödinger operators in high dimension both using the maximum principle and L^2-methods, and explain how they can be applied to the study of correlations for integrals in high dimension (continuous spin systems), via Witten Laplacians. In particular we discuss more recent work on the asymptotics of correlations, when these decay exponentially.

Sources: Mostly joint works in various combinations, including the speaker, B.Helffer (to a large part), W.M.Wang, V.Bach, Th.Jecko.

Yvan Velenik (Berlin): "Localization of a 2D massless gradient field"

We consider a massless gradient model in 2 dimensions, which can be seen as describing a 2D interface in a 3D medium. As is well known, the presence of a continuous symmetry prevents the existence of the (infinite volume) Gibbs state (the interface has unbounded fluctuations). The aim of this talk is to explain how an arbitrarily weak breaking of the continuous symmetry (by a so-called pinning potential) is sufficient to recover a well-defined infinite-volume state (i.e. to localize the interface). In fact, as will be explained, even more is true, since the localized field necessarily exhibits exponential decay of correlations.

Milos Zahradnik (Prague): "Translation noninvariant Gibbs states and entropic repulsion in low temperature lattice models with "multiple well" potentials"

We explain a recent method of the author (developed in a paper submitted to AMS Dobrushin Memorial volume) and apply it to the problem of interfaces.

Our method is suited to the study of the Gibbs states of low temperature lattice models with a continuous spin and "multiple well" potentials. With the help of a suitable "gaussian" transformation of the exponential $\exp(-U)$ of the potential U, the partition functions of the original model (on a torus, say) are converted to partition functions of some discrete spin model (of "abstract Pirogov Sinai" type). A typical example we are able to control is the model with a Hamiltonian $H(x) = \sum_{|i-j|=1}(x_i - x_j)^2 + \sum_i U(x_i)$ where $U(x_i) = \cos(x_i)$ and i is a point of a given three dimensional lattice. Also small perturbations of such a model can be treated by the method.

The Gibbs states of the Dobrushin type and the entropic repulsion can be also rigorously studied in this example, with the help of some additional technical constructions I will outline: Using the above "gaussian transformation" and expanding the gaussian partition functions which thus appear, the problem of Dobrushin states in the above example is transcripted to an analogous problem in a suitable discrete spin model with $\mathbb{Z}$ valued spins. The latter models can be studied by a general technique recently developed for the "stratified" Ising type models.
