Interfaces and equilibrium shapes in three dimensions

Thierry Bodineau (Paris): "The Wulff construction in three and more dimensions"

We prove the Wulff construction in three and more dimensions for an Ising model with nearest neighbor interactions.

Raphael Cerf (Orsay): "Large deviations for three dimensional supercritical percolation"

We consider Bernoulli bond percolation on the three dimensional lattice in the supercritical regime. We prove a large deviation principle for the rescaled configuration, from which we derive a formulation of the Wulff construction.

Dima Ioffe (Haifa): "Large Deviations and Concentration Properties for $\nabla\phi$ Interface Models"

We consider the massless field with zero boundary conditions outside $D_N \equiv D \cap (\mathbb{Z}^d/N)$ $(N \in \mathbb{Z}^+)$, D a suitable subset of $\mathbb{R}^d$, i.e. the continuous spin Gibbs measure P_N on $\mathbb{R}^{\mathbb{Z}^d/N}$ with Hamiltonian given by $H = \sum_{x,y:\|x-y\|=1} V(\phi(x) - \phi(y))$ and $\phi(x) = 0$ for $x \notin D_N$. The interaction V is taken to be strictly convex and with bounded second derivative. This is a standard effective model for a $(d+1)$-dimensional interface: ϕ represents the height of the interface over the *base* D_N. Due to the choice of scaling of the base, we scale the height with the same factor by setting $\xi_N = \phi/N$. We first establish a Large Deviation principle for the sequence of random surfaces $\{\xi_N\}_{N\in\mathbb{Z}^+}$ and we show that the rate function can be written, for $u \in H_0^1(D)$, as $\int_D \sigma(\nabla u(x))\mathrm{d}x$, where σ is the surface tension of the model. We use this result to study the concentration properties of P_N under a volume constraint, i.e. the constraint that $(1/N^d)\sum_{x\in D_N} \xi_N(x)$ stays in a neighborhood of a fixed volume $V > 0$, and the hard–wall constraint, i.e. $\xi_N(x) \geq 0$ for all x. This is therefore a model for a droplet of volume V lying above a hard wall. Our main result is the concentration of the rescaled heights ξ_N around the solution of a variational problem involving the surface tension, as predicted by the phenomenological theory of phase boundaries. The proofs are close in spirit to hydrodynamic limit type of arguments and they have therefore both probabilistic and analytic aspects. Essential analytic tools are L^p estimates for elliptic equations and the theory of Young measures. On the side of probability tools, a central role is played by the Hellfer–Sjöstrand PDE representation for continuous spin systems which we rewrite in terms of *random walk in random environment* and by recent results of Funaki and Spohn on the structure of gradient fields. (Joint work with J-D.Deuschel and G.Giacomin)

Richard Kenyon (Orsay): "Forme d'équilibre dans le modèle des dimères"

Pour une grande région R du plan, pavable par dominos (rectangles 1X2), nous donnons la forme asymptotique, quand Aire(R)$\to\infty$, d'un pavage typique de R par des dominos. Travail en commun avec H. Cohn, J. Propp.

Jacques Magnen (Palaiseau): "A Wulff Shape from Constructive Field Theory"

We consider a sessile droplet as given by a height function $h(x)$, subject to a Hamiltonian of the form $|\nabla h|^2 + \lambda P(\nabla h)$, where P is a polynomial and λ is small. The corresponding Gibbs measure is conditioned on the value of the droplet volume $\mathrm{V} = \int_\Lambda dx\, h(x)$, where $\Lambda \subset R^2$ is a bounded domain of the plane.

The droplet shape, in a scaling limit $\mathrm{V} \approx |\Lambda|^{3/2} \to \infty$, is then a Wulff shape, with logarithmic fluctuations. The proof is based on a phase space cluster expansion, which includes a renormalization of a varying slope chemical potential as well as a renormalization of the drop shape. The renormalization group process consists in integrating stepwise on fluctuations of lower and lower frequencies, leaving at the end a no fluctuation problem, whose solution depends on the precise form of the domain Λ. From this latter solution one reconstructs back the actual drop.

Salvador Miracle-Sole (Marseille): "Statistical mechanics of equilibrium shapes"

According to the Wulff construction the shape of the equilibrium crystal is determined by the surface tension considered as a function of the interface orientation. We present some rigorous results concerning this function, in the case of a lattice gas, and apply them to study the shape of the equilibrium crystal and, in particular, the shape of the facets of this crystal.
