Stretching of a fractal polymer near a disc reveals KPZ-like statistics

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Summary

1. Fluctuations of elongated 2D fractal paths above an impermeable disc and 1D Anderson localization

2. Fluctuations of (1+1)D path above a semi-impermeable disc and phase transition by dimensional reduction

3. Biased tracer diffusion in channel through curved bottleneck

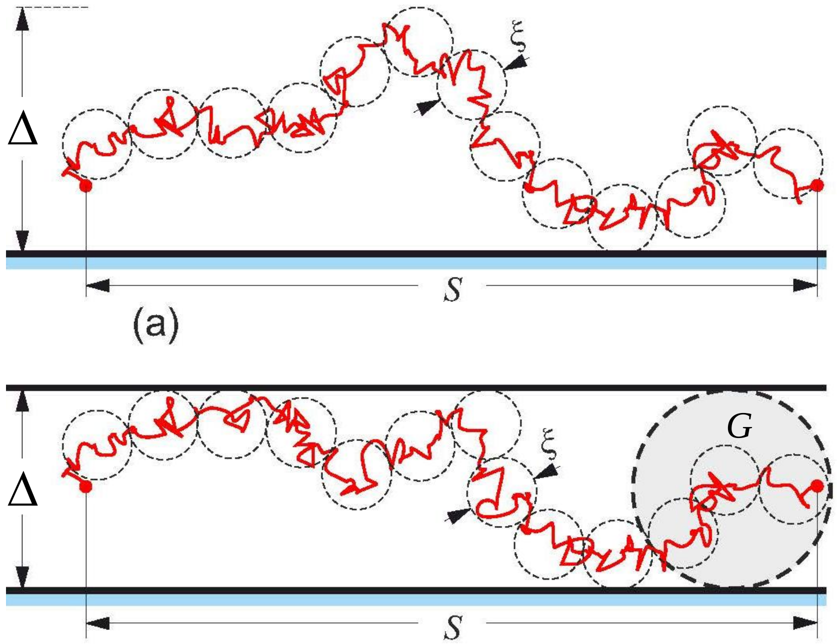
4. Mean-field approach to random matrices: finite-size scaling of Brownian bridges on supetrees

1. Fluctuations of elongated 2D fractal paths above an impermeable disc and 1D Anderson localization

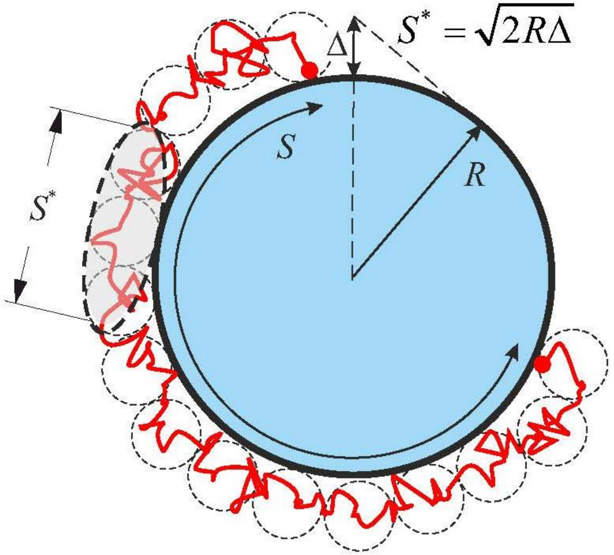
(Scaling approach “à la P.J. de Gennes”)

A. Grosberg, K. Polovnikov, S.N. (2022), in preparation
A. Gorsky, S.N., A. Valov, J. High. Energ. Phys. (2021)

Fractal polymer ($R_0 = bN^{\nu}$) stretching in flat tube or above impermeable disc



curvature is important on length S*



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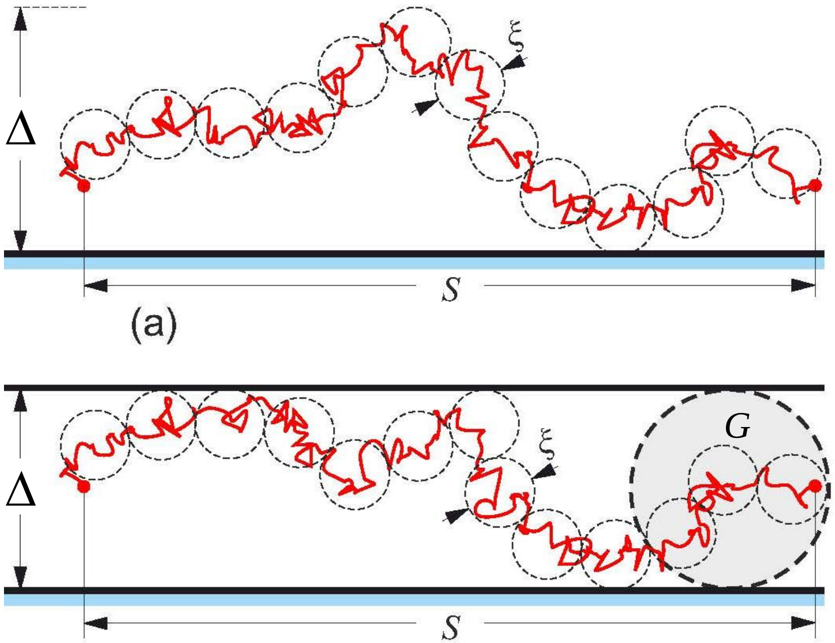

Free energy in a tube

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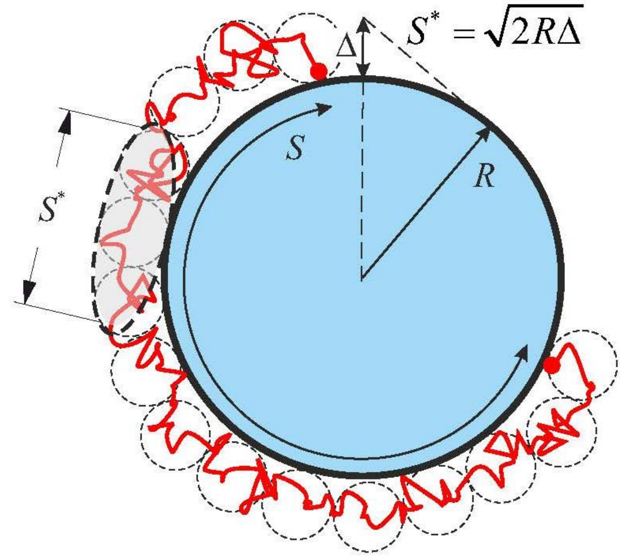

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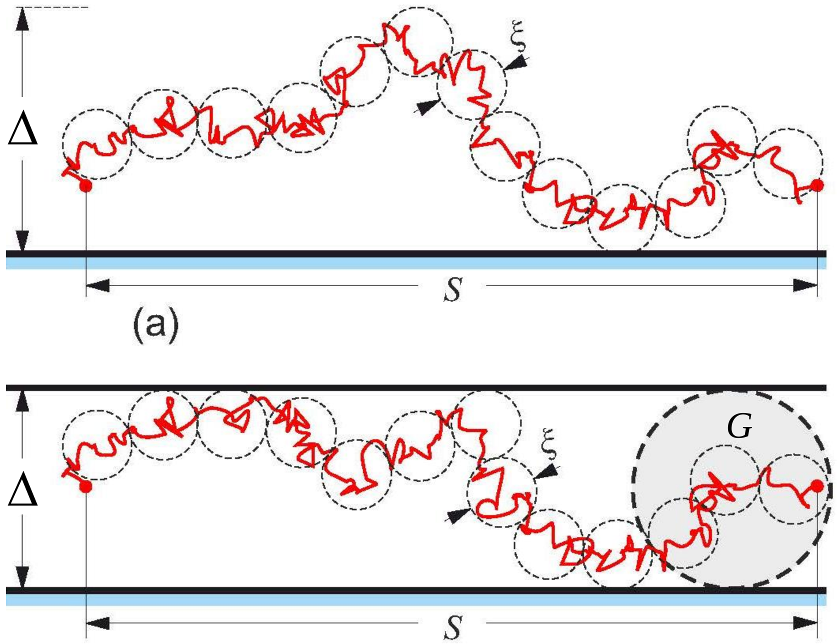

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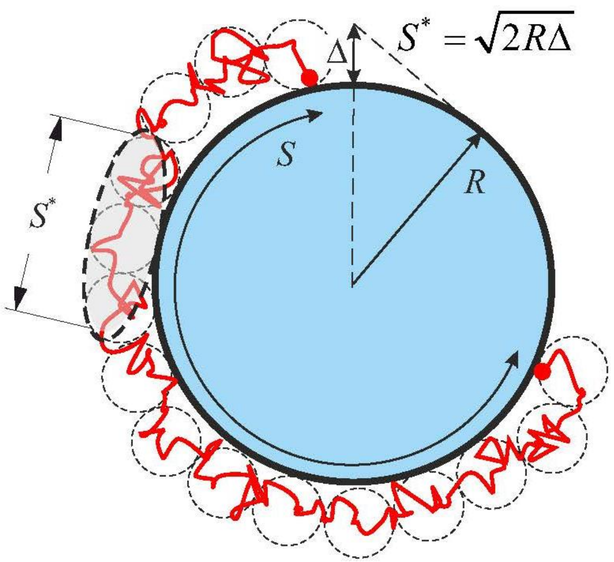

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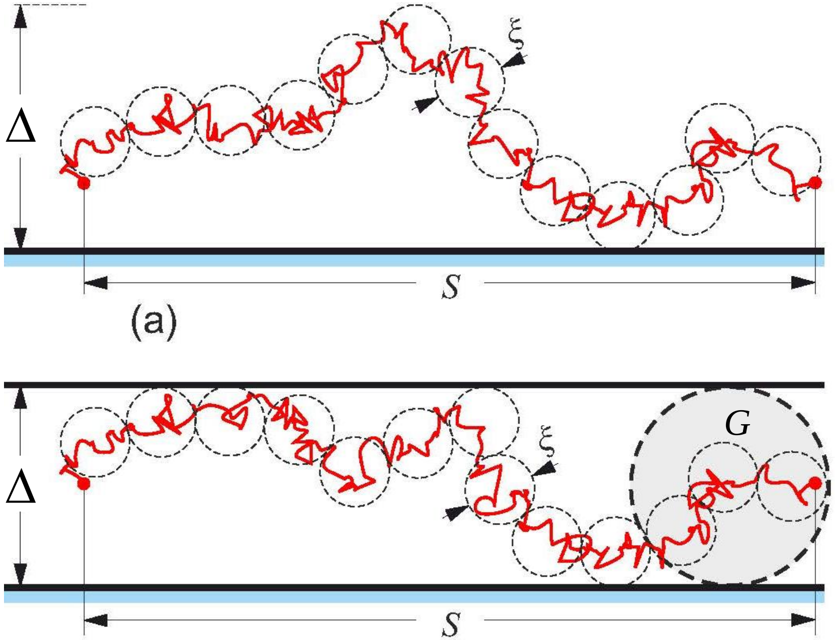

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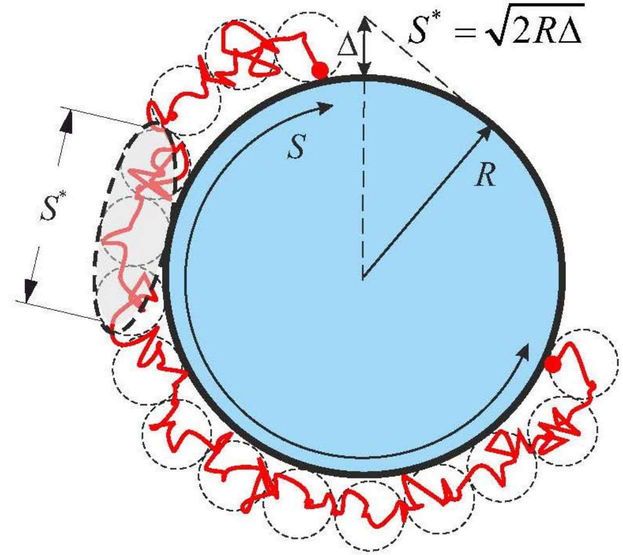

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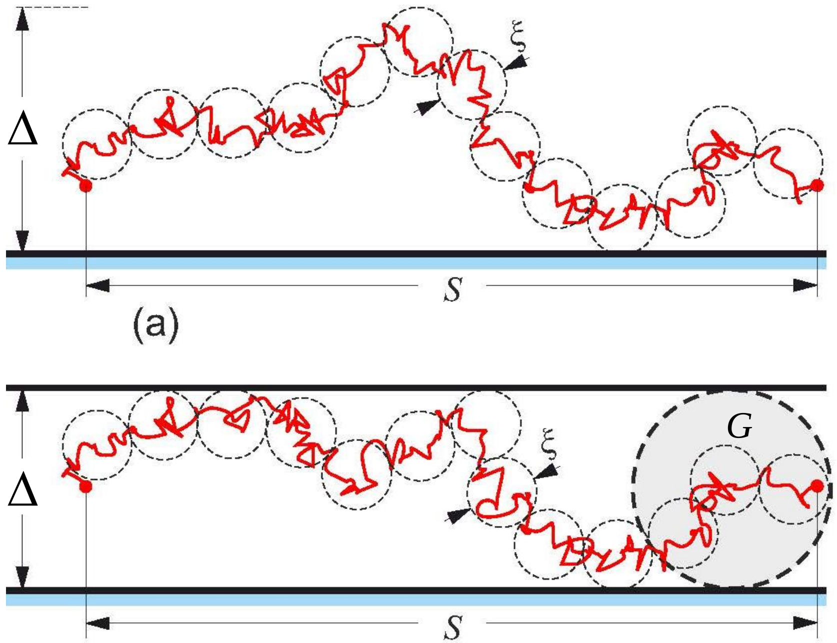

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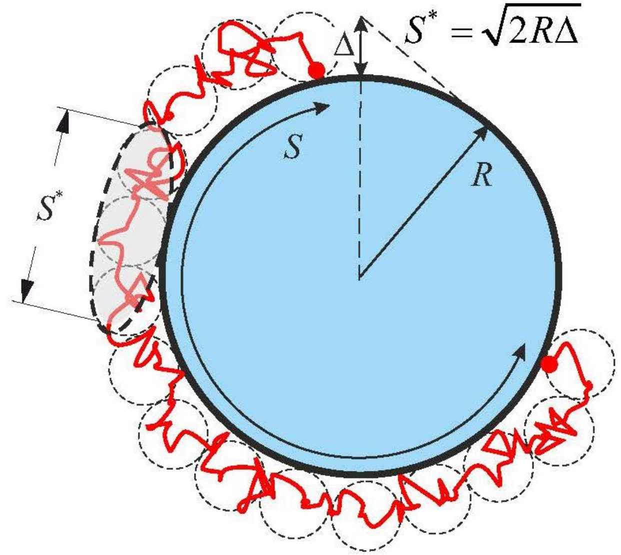

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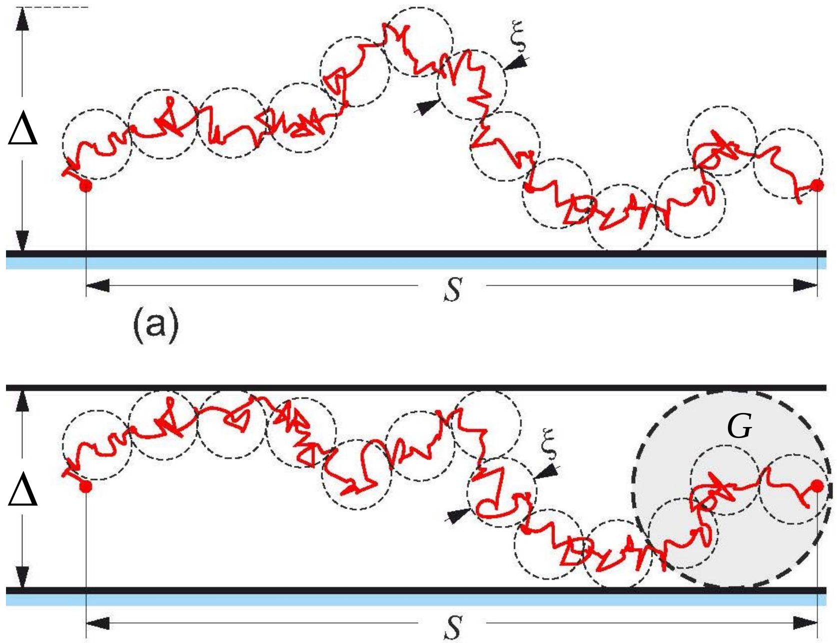

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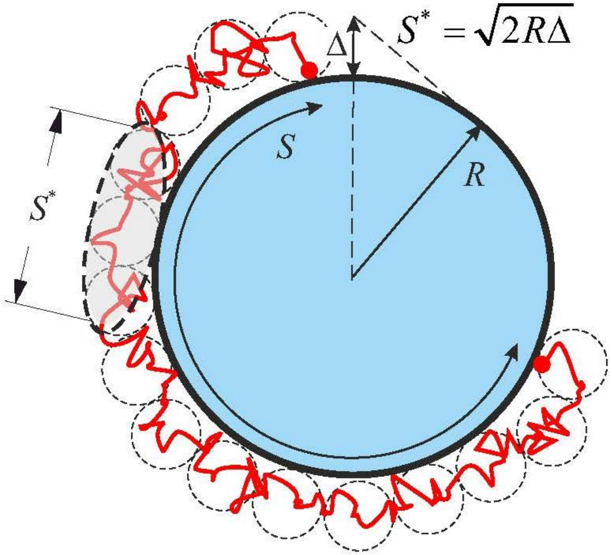

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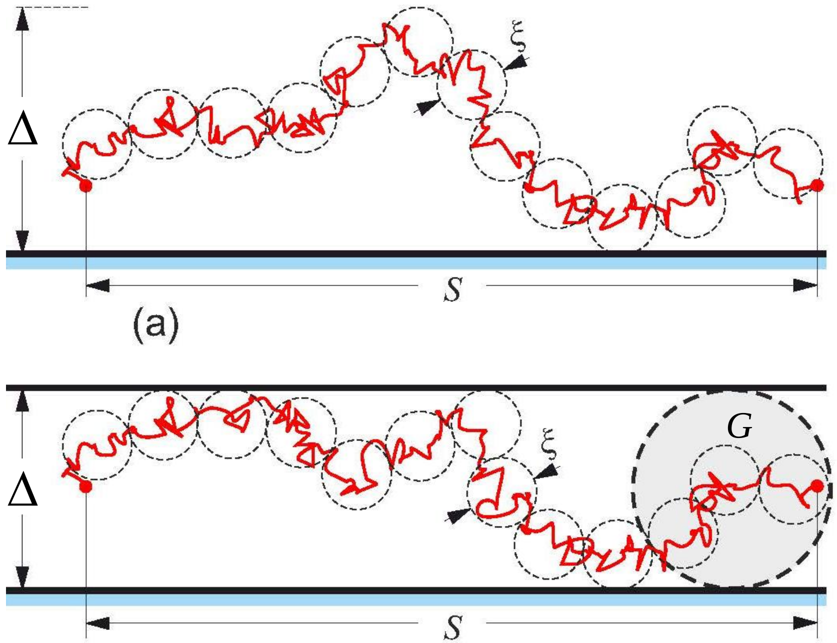

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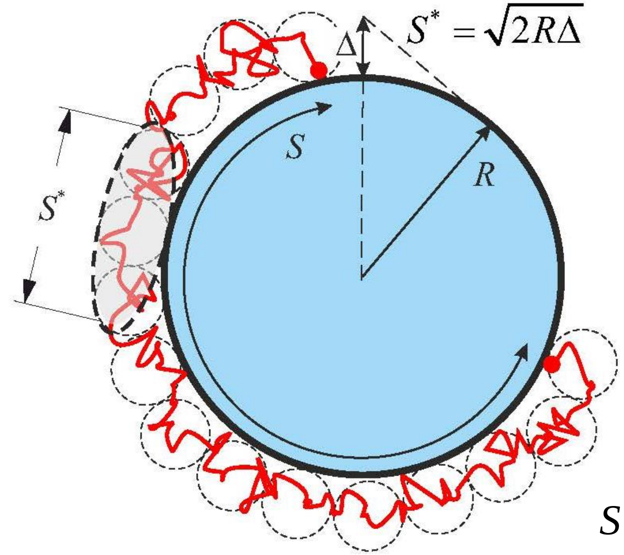

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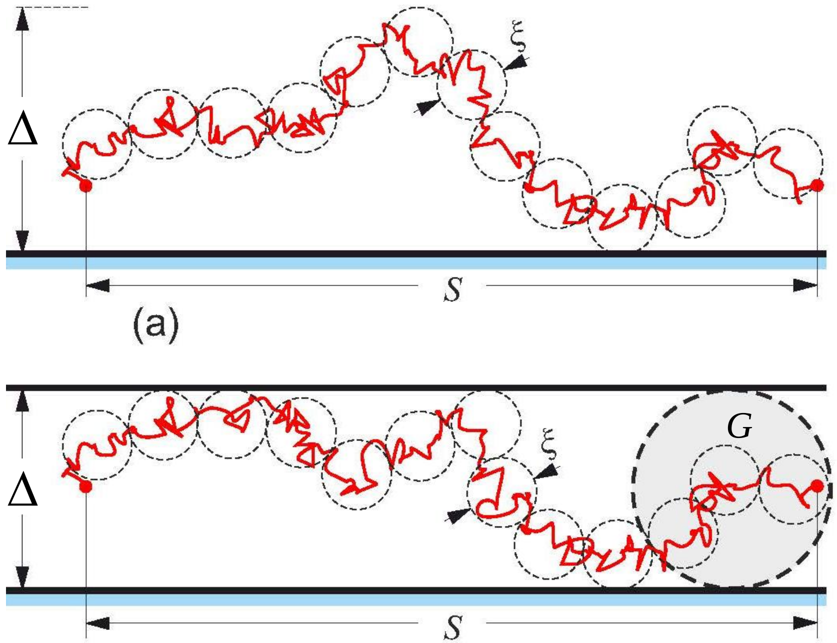

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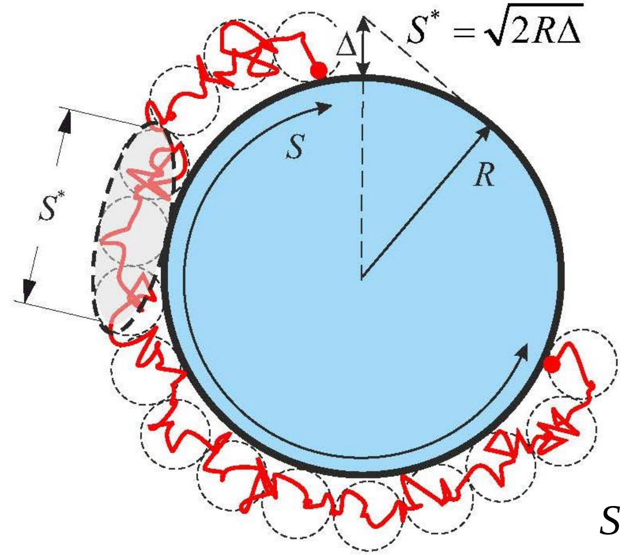

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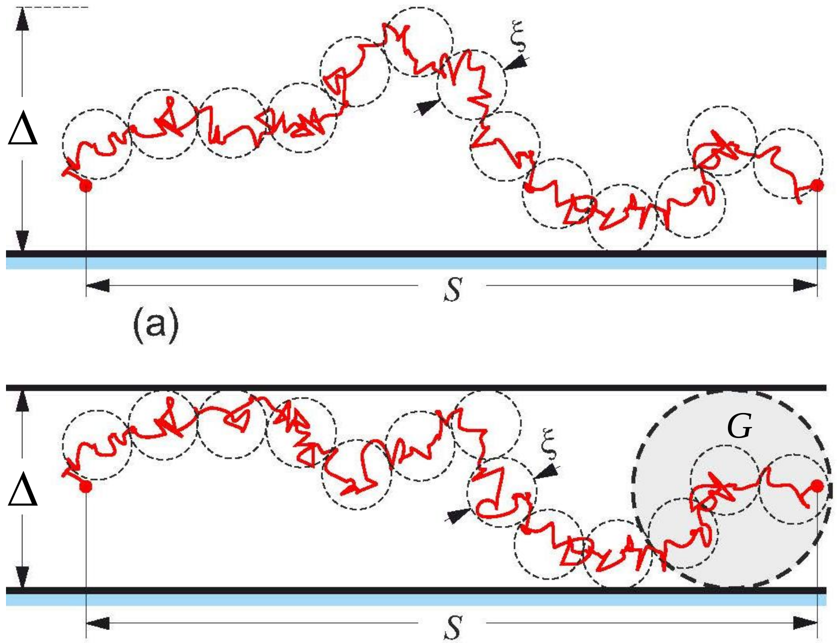

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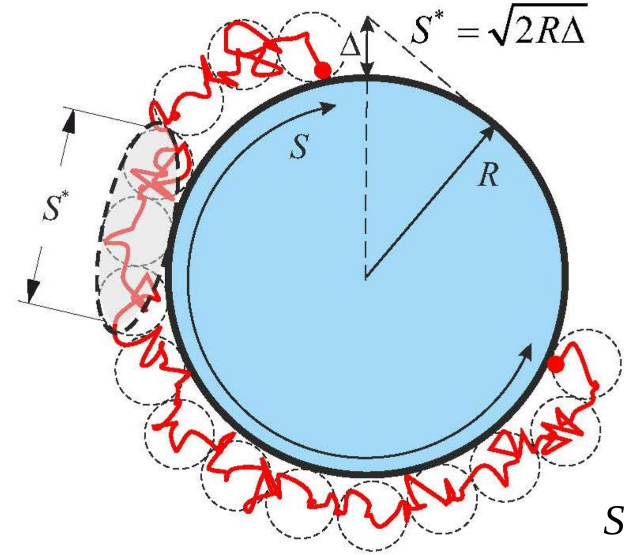

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at $\Delta \ll R$ one has:

$$\frac{\Delta}{bN^{\nu}} = \left(\frac{R}{bN^{\nu}}\right)^{\frac{1}{3}}\left(\frac{bN^{\nu}}{S}\right)^{\frac{2\nu}{3(1-\nu)}}$$

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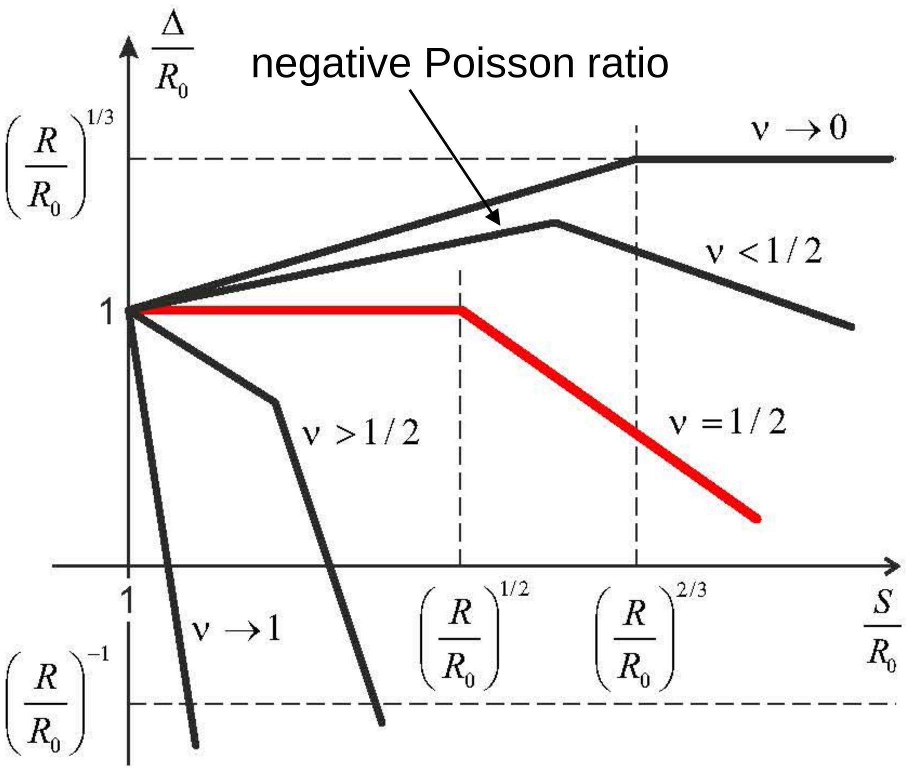

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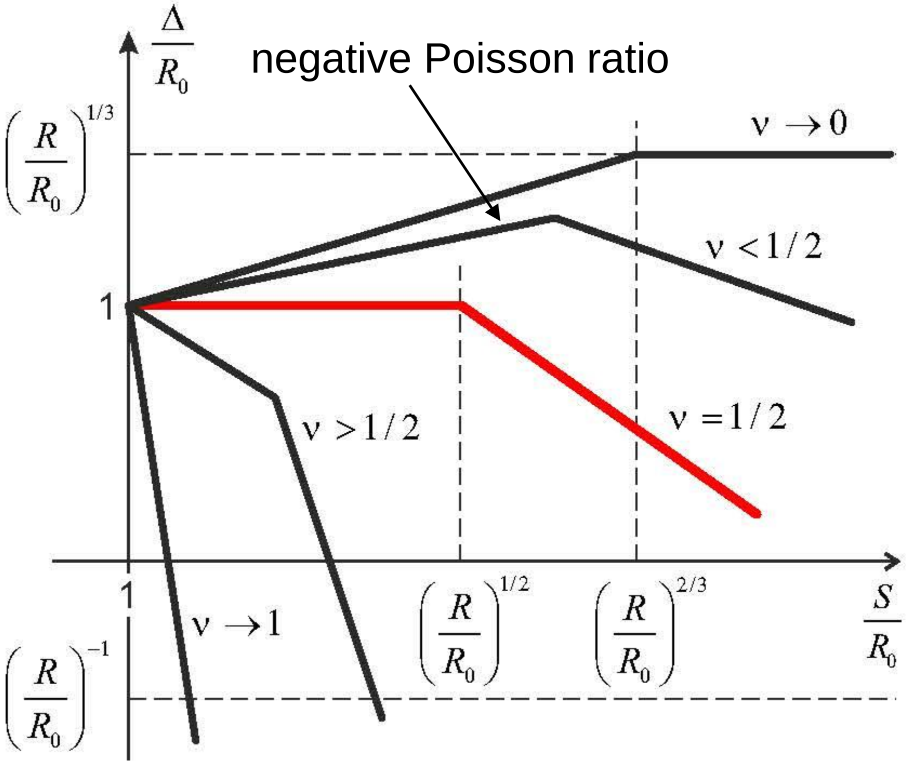

We have found scaling dependence

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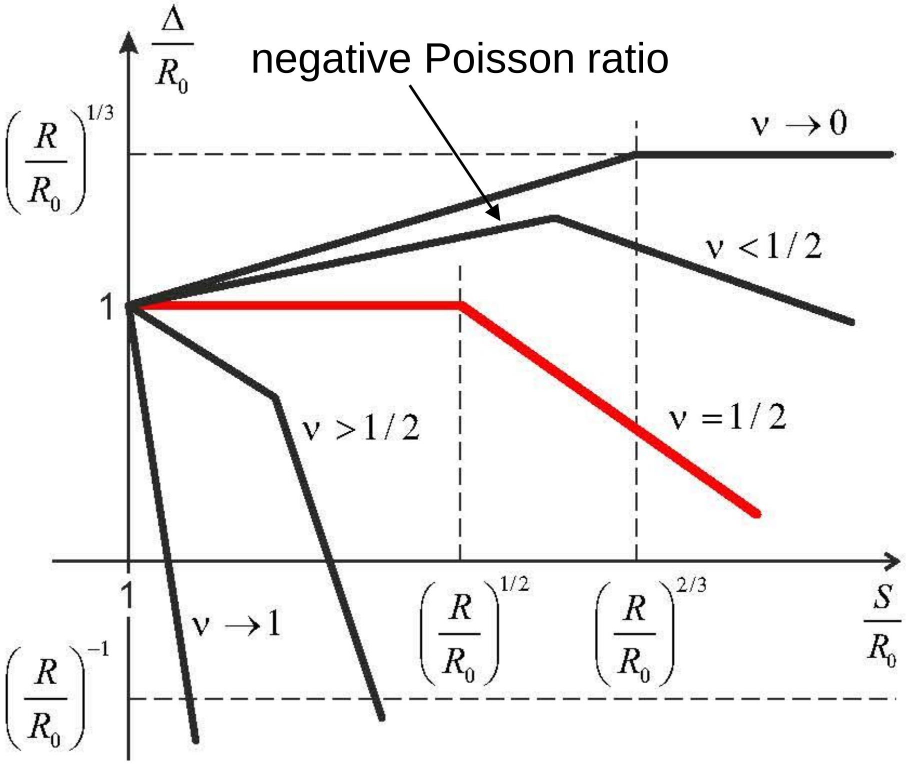

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Set $S = N^{\gamma} b \gg N^{\nu} b$ $\quad (\nu < \gamma < 1)$

The boundary of a disc is effectively flat

for a fractal polymer at $R > R^*$, where

$$\frac{R^*}{bN^{\nu}} = N^{\frac{(3-2\nu)(\gamma-\nu)}{2(1-\nu)}}$$

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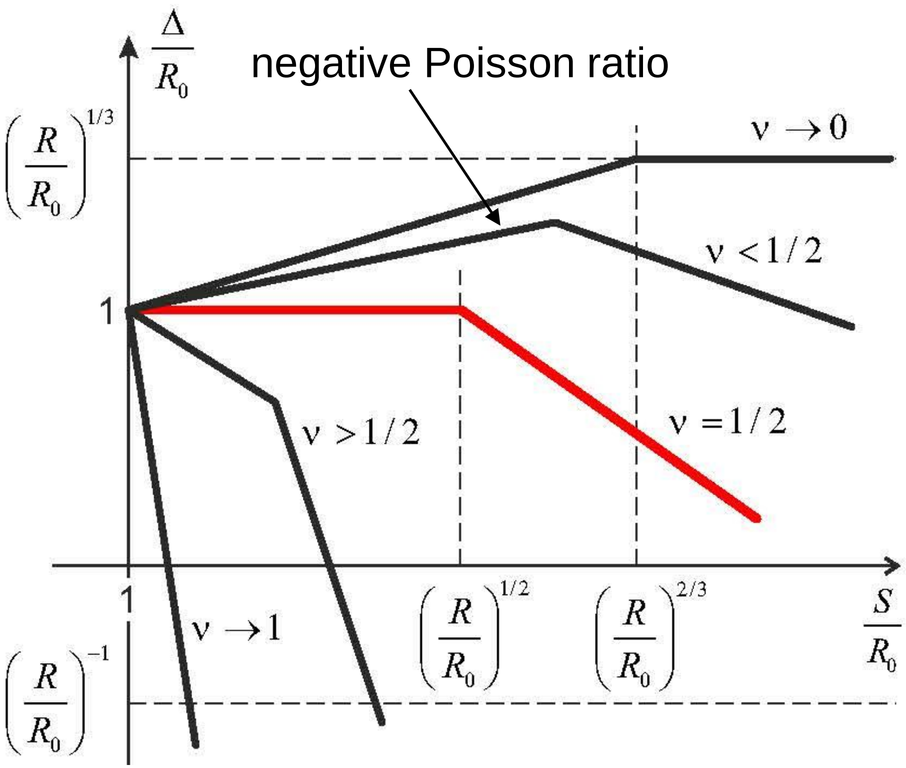

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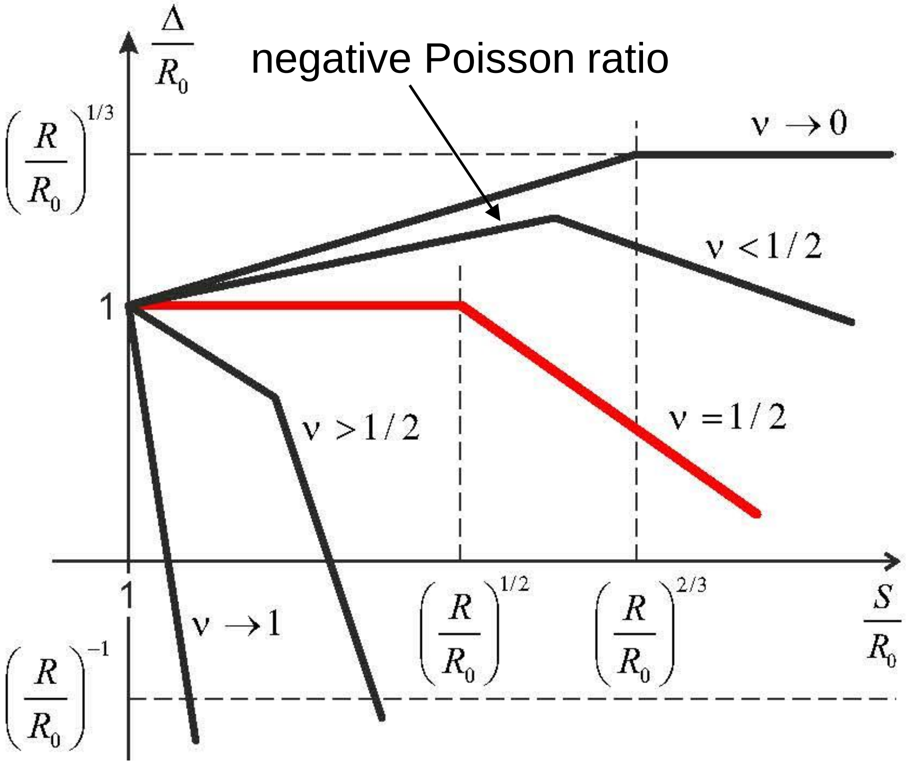

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Distance $R^* \sim N^z$ at $\gamma \to 1$ becomes independent on fractal dimension, ν, providing for z the universal value of 1D KPZ **dynamic exponent**, $z = 3/2$.

Optimal fluctuation for ultimately stretched paths above the disc

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Balancing stretching above the disc and confinement in a slit, we have (as above):

$$\frac{F_{\text{circ}}}{k_B T} \propto \left(\frac{S}{bN^{\nu}} \frac{R+\Delta}{R} \right)^{1/(1-\nu)} + \frac{b^2 N^{2\nu}}{\Delta^2} \left(\frac{bN^{\nu}}{S} \right)^{(2\nu-1)/(1-\nu)} \Bigg|_{\substack{\nu=1/2 \\ \gamma\to 1}} \propto \frac{\Delta}{b} + \frac{b^2}{\Delta^2} N$$

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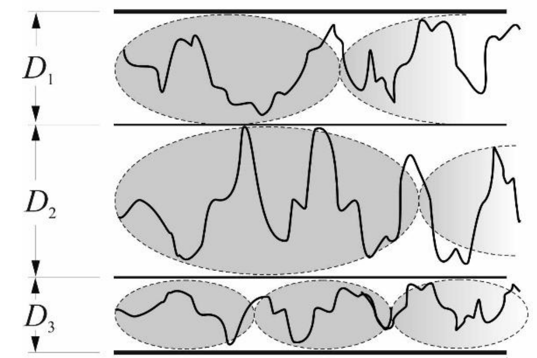

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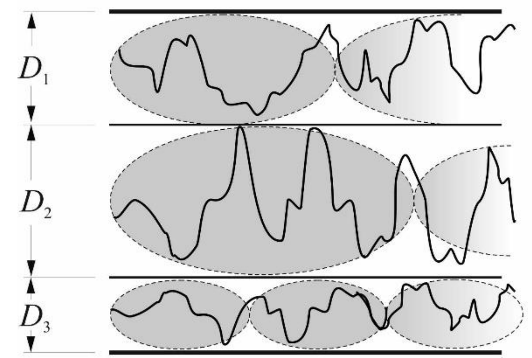

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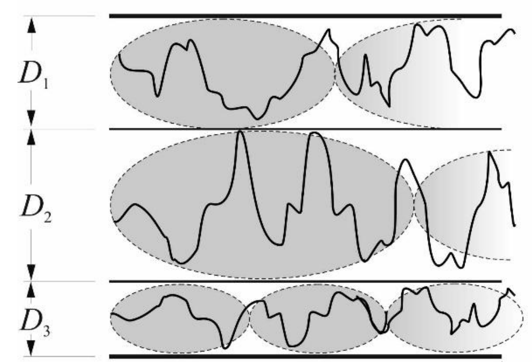

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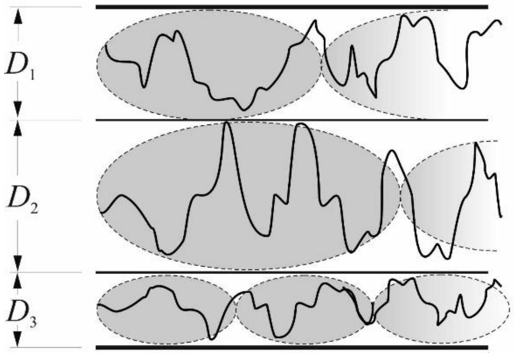

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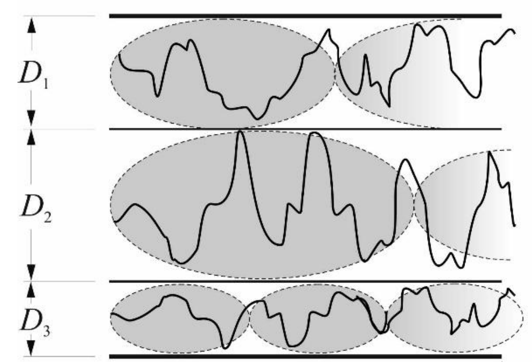

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The inverse Laplace transform gives the Lifshitz tail of 1D Anderson localization

$$P(s) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} P(N) e^{sN} \sim e^{-\beta/\sqrt{s}}$$

2. Fluctuations of (1+1)D path above semi-impermeable disc and phase transition by dimensional reduction

A. Gorsky, S.N., A. Valov, JETP (2022)

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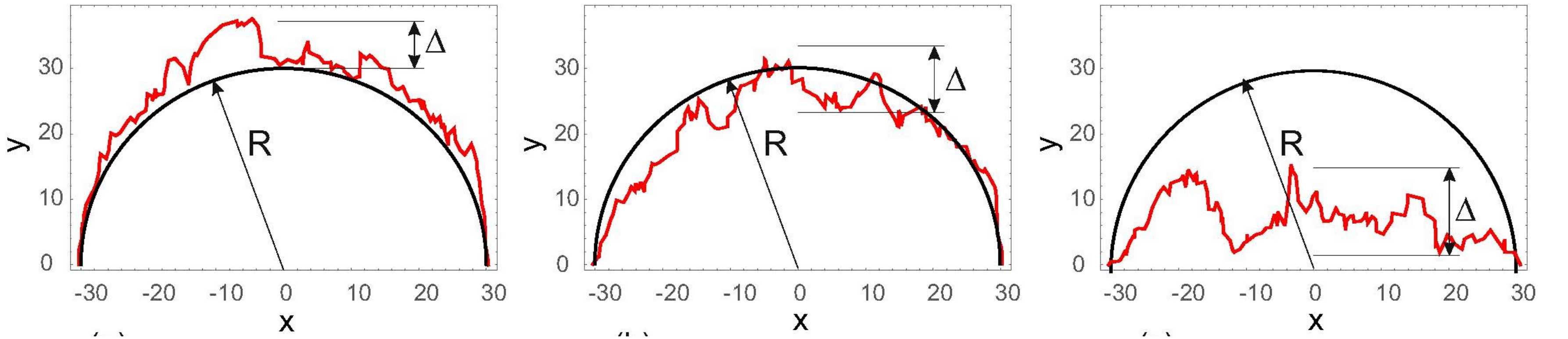

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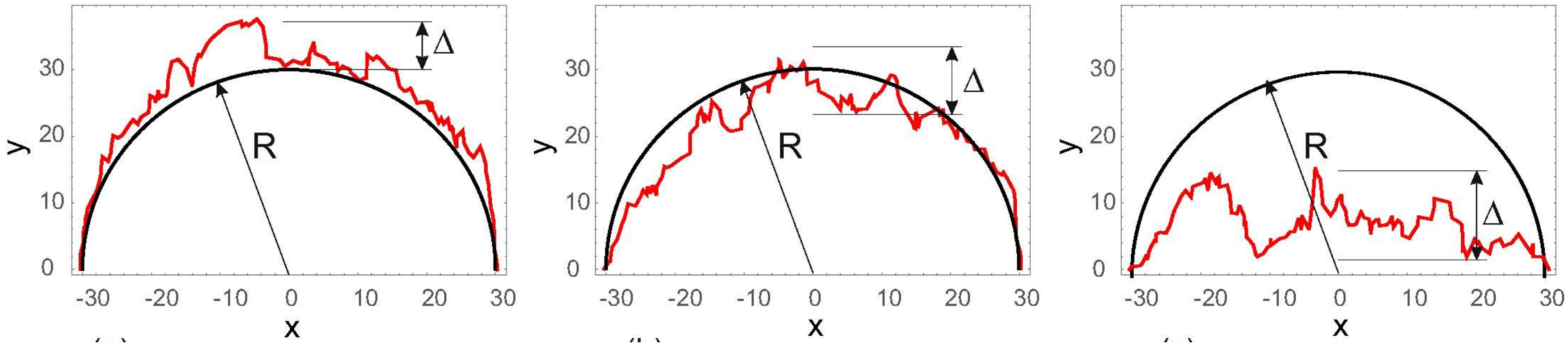

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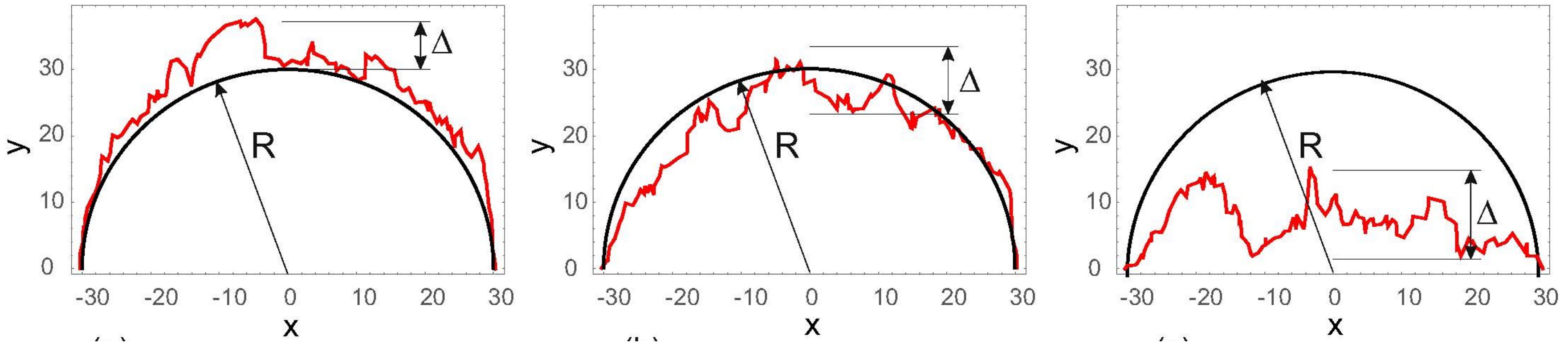

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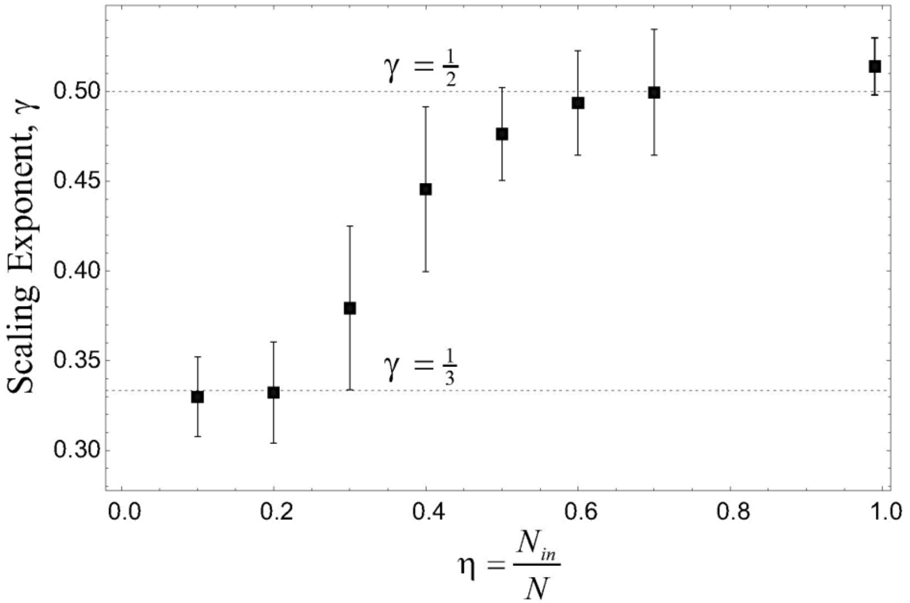
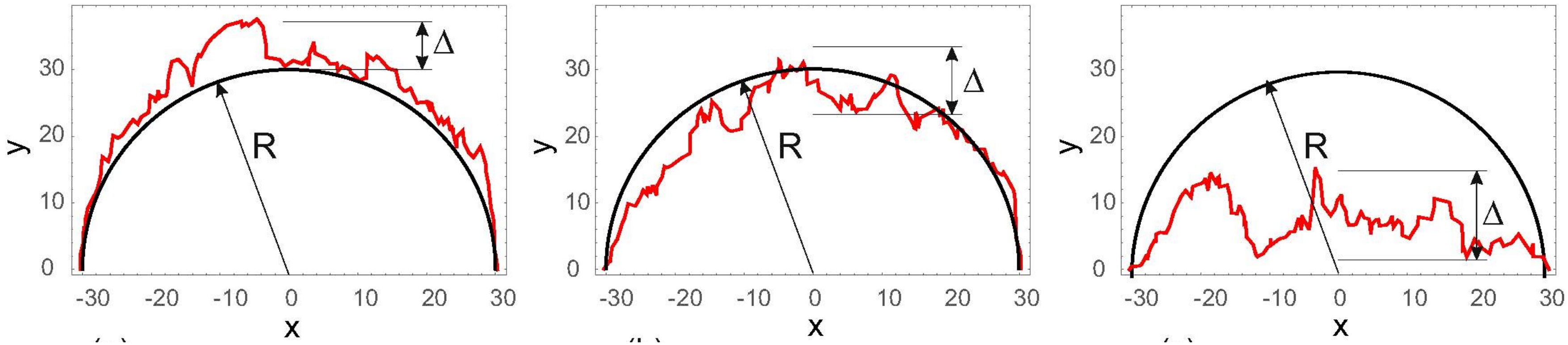


Probing the 1/3 → 1/2 transition by dimensional reduction (UMAP)



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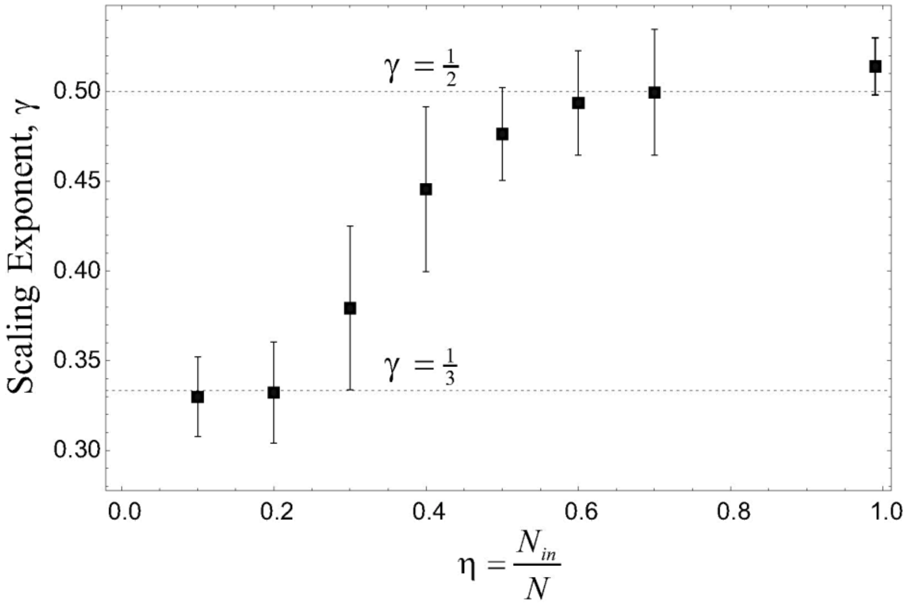
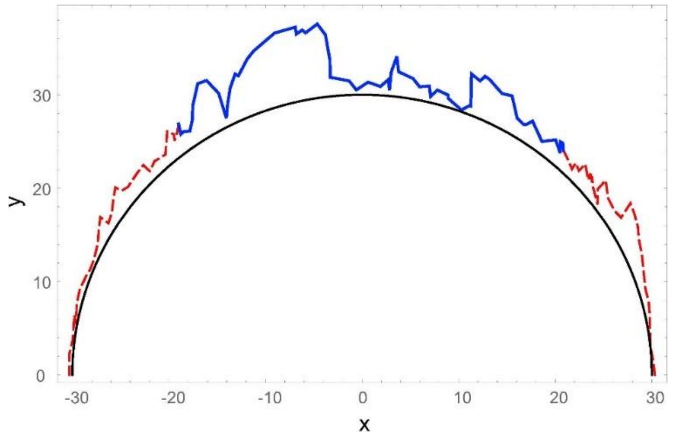


Path preparation for UMAP



UMAP (Uniform Manifold Approximation and Projection)

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We have generated 1000 vectors $\boldsymbol{X}$. Each vector $\boldsymbol{X}$ consists of 120 coordinates of all chain monomers at all values of η. Then by UMAP the dimension of $\boldsymbol{X}$ is reduced to 2

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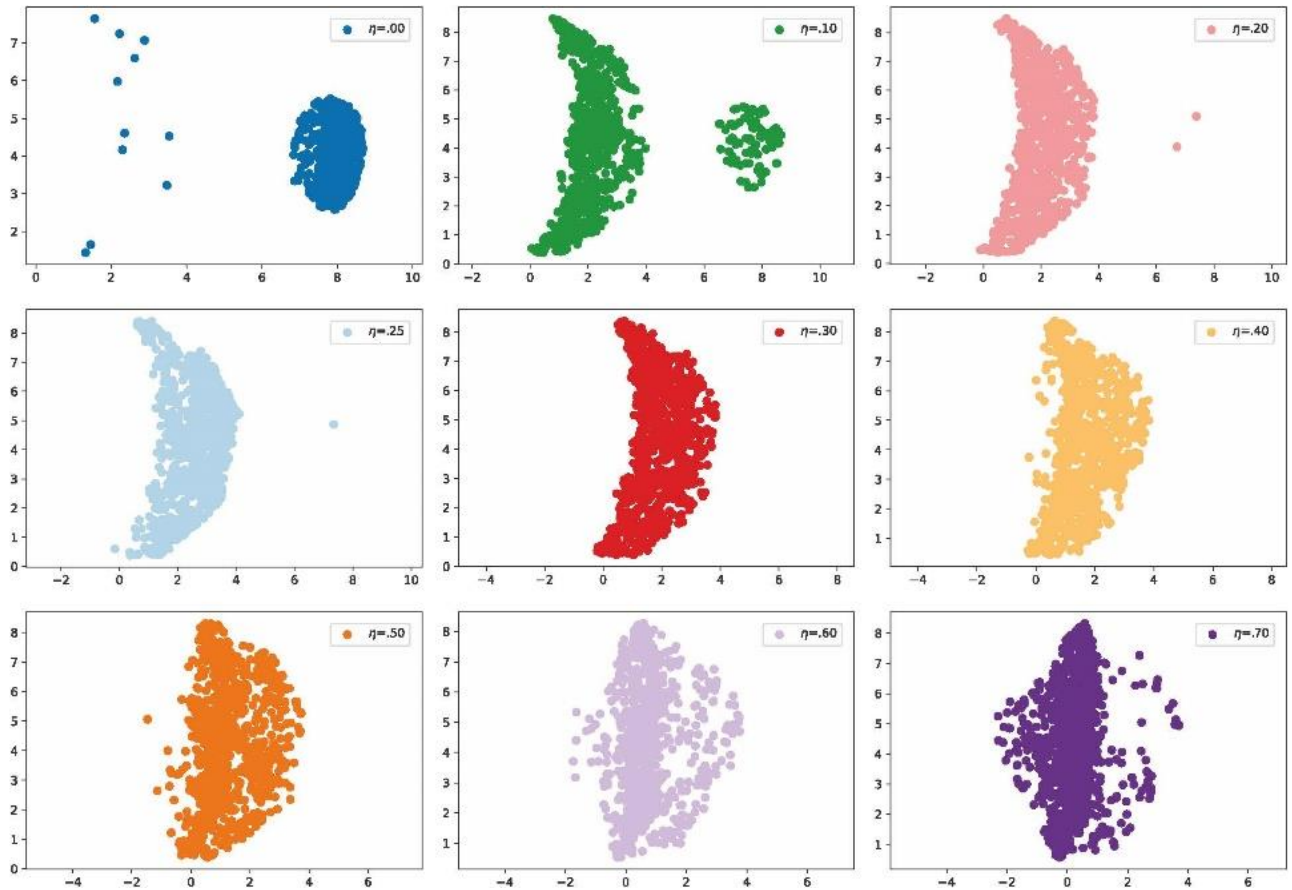

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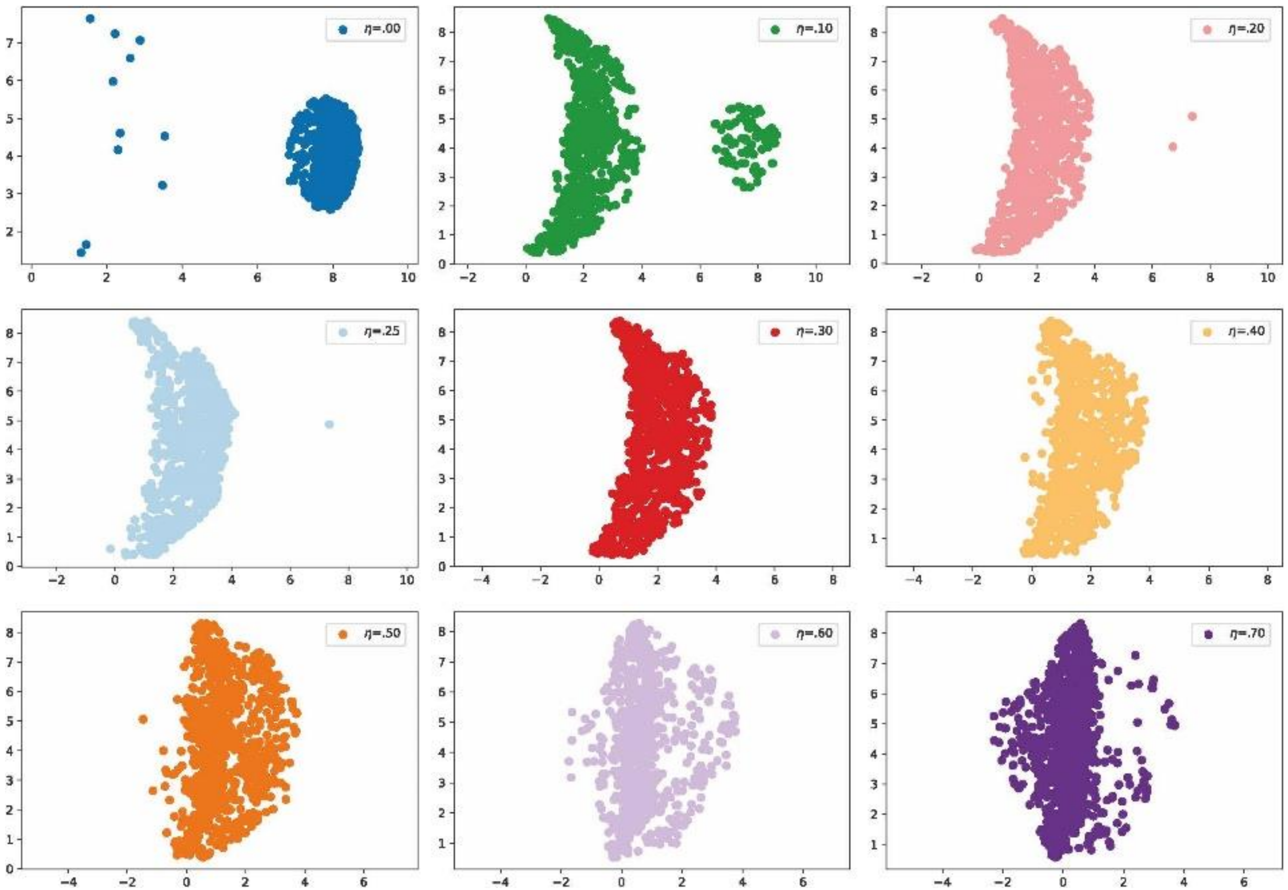

The coordinates in low-dimensional space are the solutions to optimization problem for informational functional: Kullback-Leibler divergence for SNE, cross-entropy for UMAP

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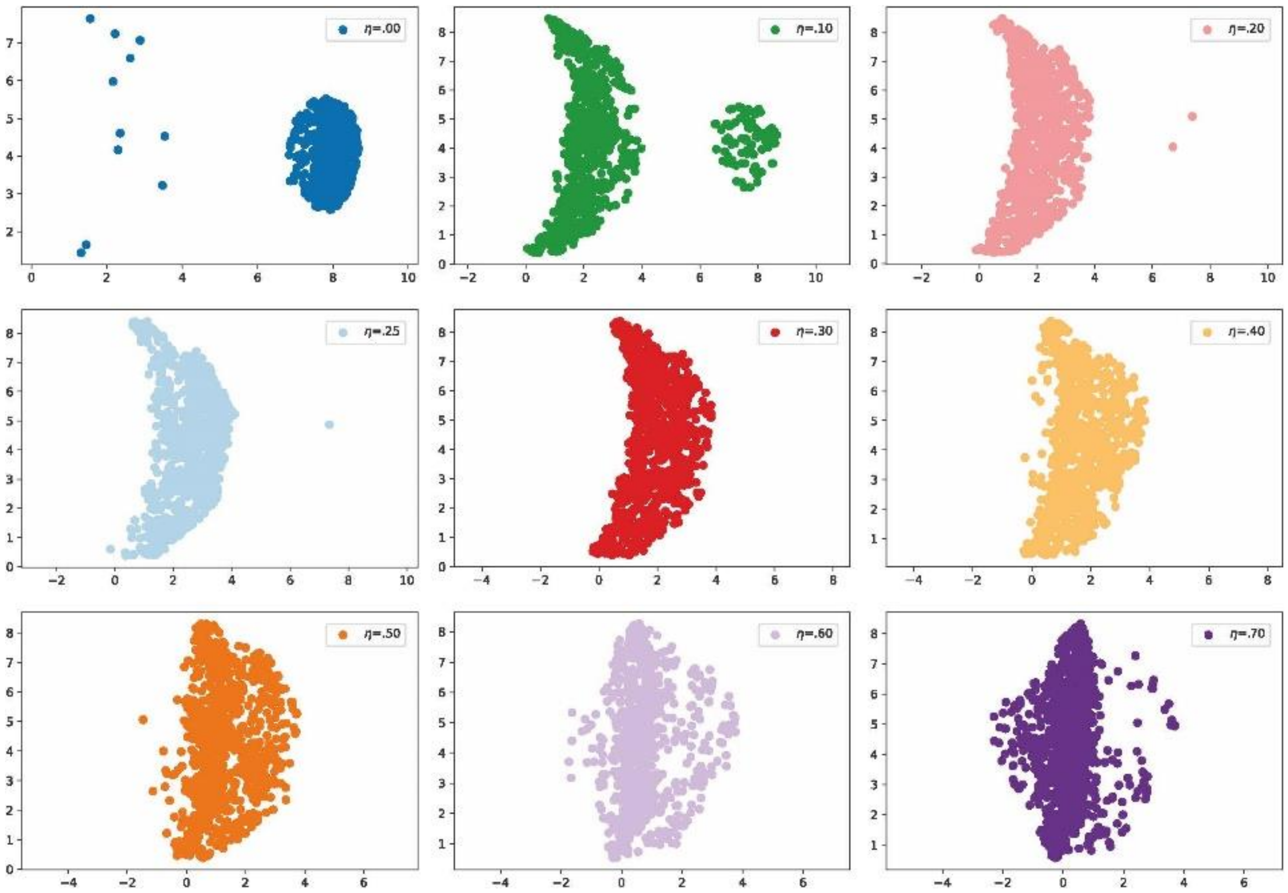

Baik-Ben Arous-Péché (BBP) transition?

BBP transition occurs when part of path gets localized on an extended defect at some critical coupling between path and defect

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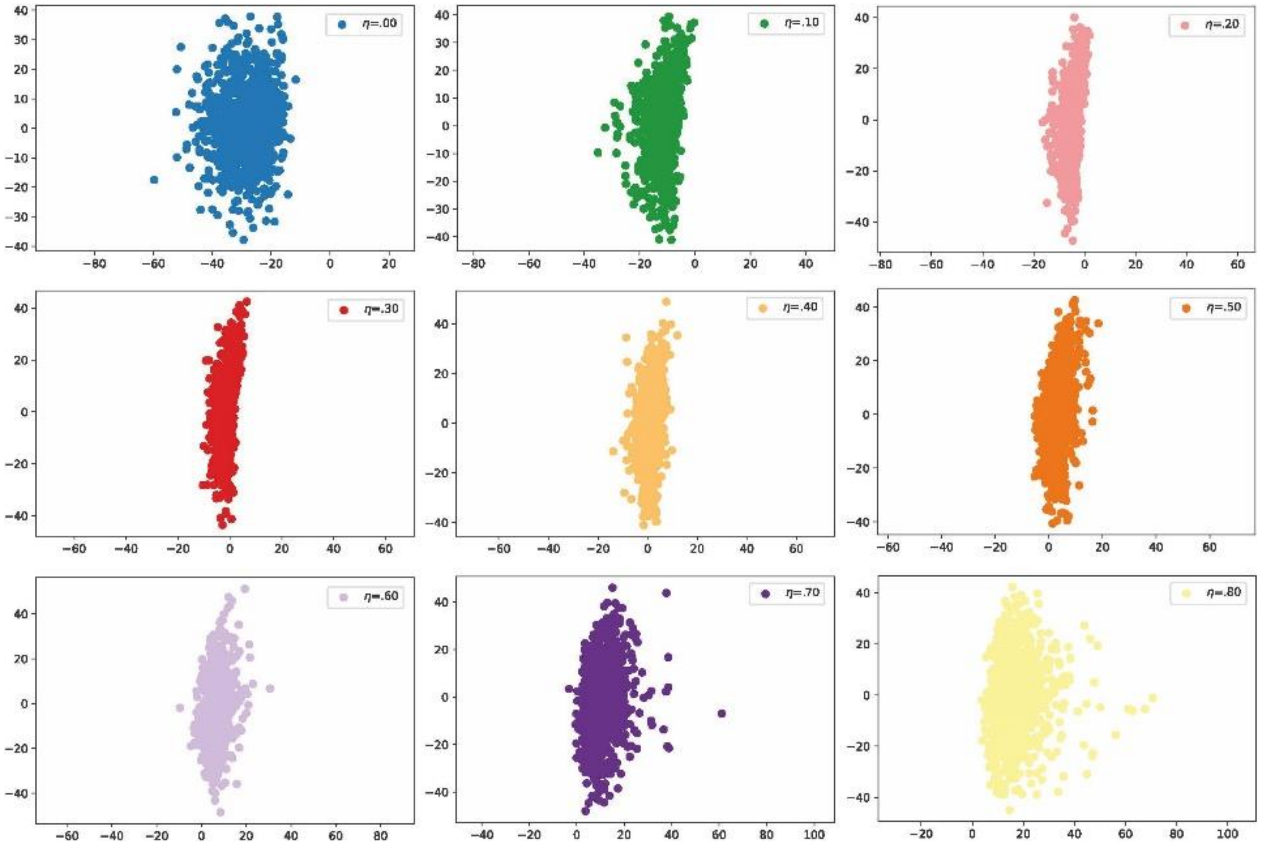

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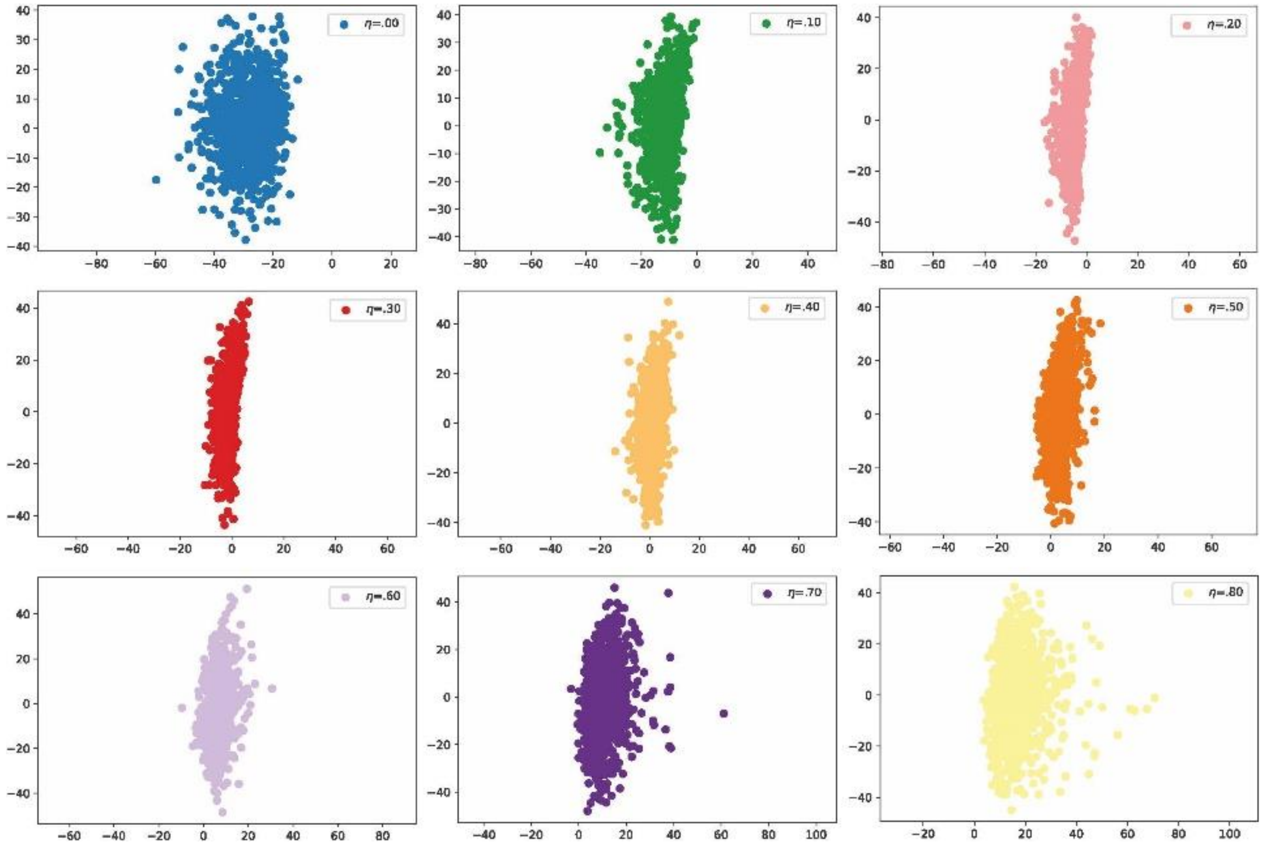


Related references:

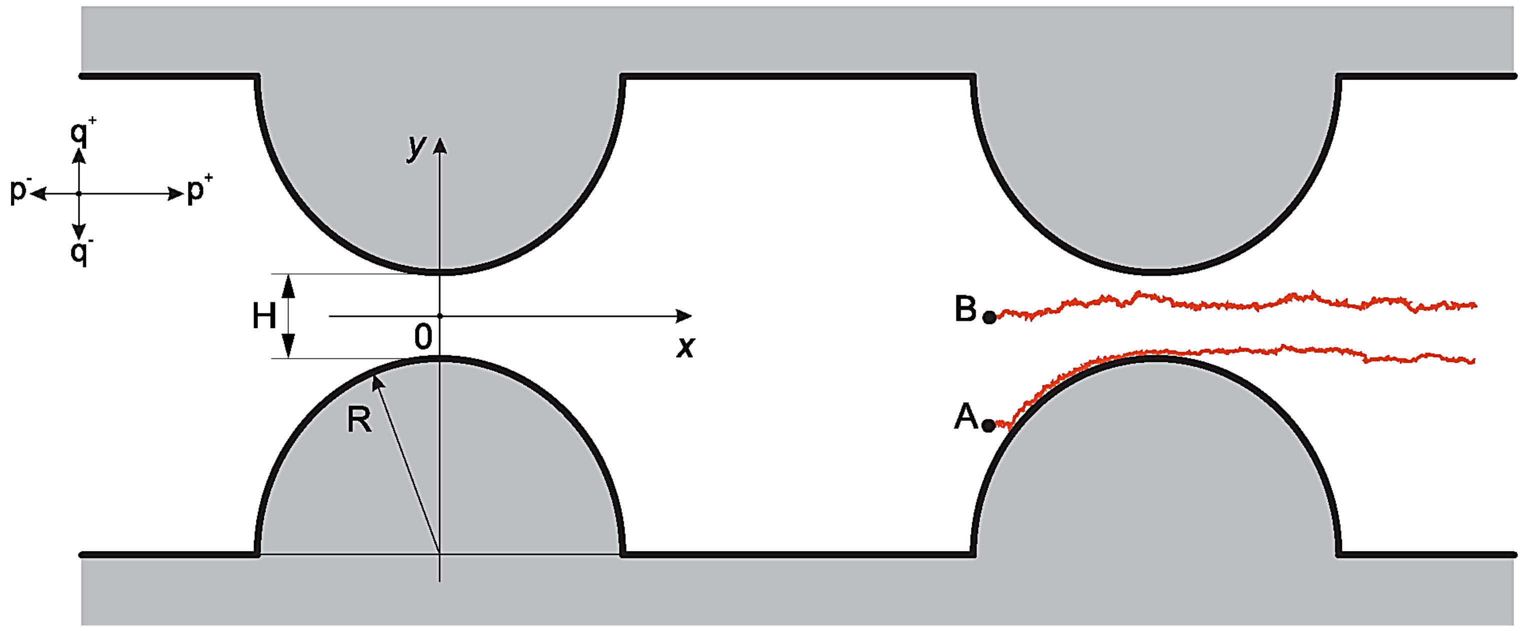
UMAP: L. McInnes, J. Healy, and J. Melville, UMAP, arXiv:1802.03426 (2018)

BBP: J. Baik, G. Ben-Arous, and S. Péché, Ann. Probab. (2005),
A. Krajenbrink, P. Le Doussal, N. O'Connell, PRE (2020),
G. Barraquand, P. Le Doussal, PRE (2021)

3. Biased tracer diffusion in channel through curved bottleneck

S.N., K. Polovnikov, S. Shlosman, A. Vladimirov, A. Valov, Phys. Rev. E (2019);
A. Valov, V. Avetisov, S.N., G. Oshanin, Polym. Chem. Chem. Phys. (2021)

Diffusion with a drift in non-uniform channels



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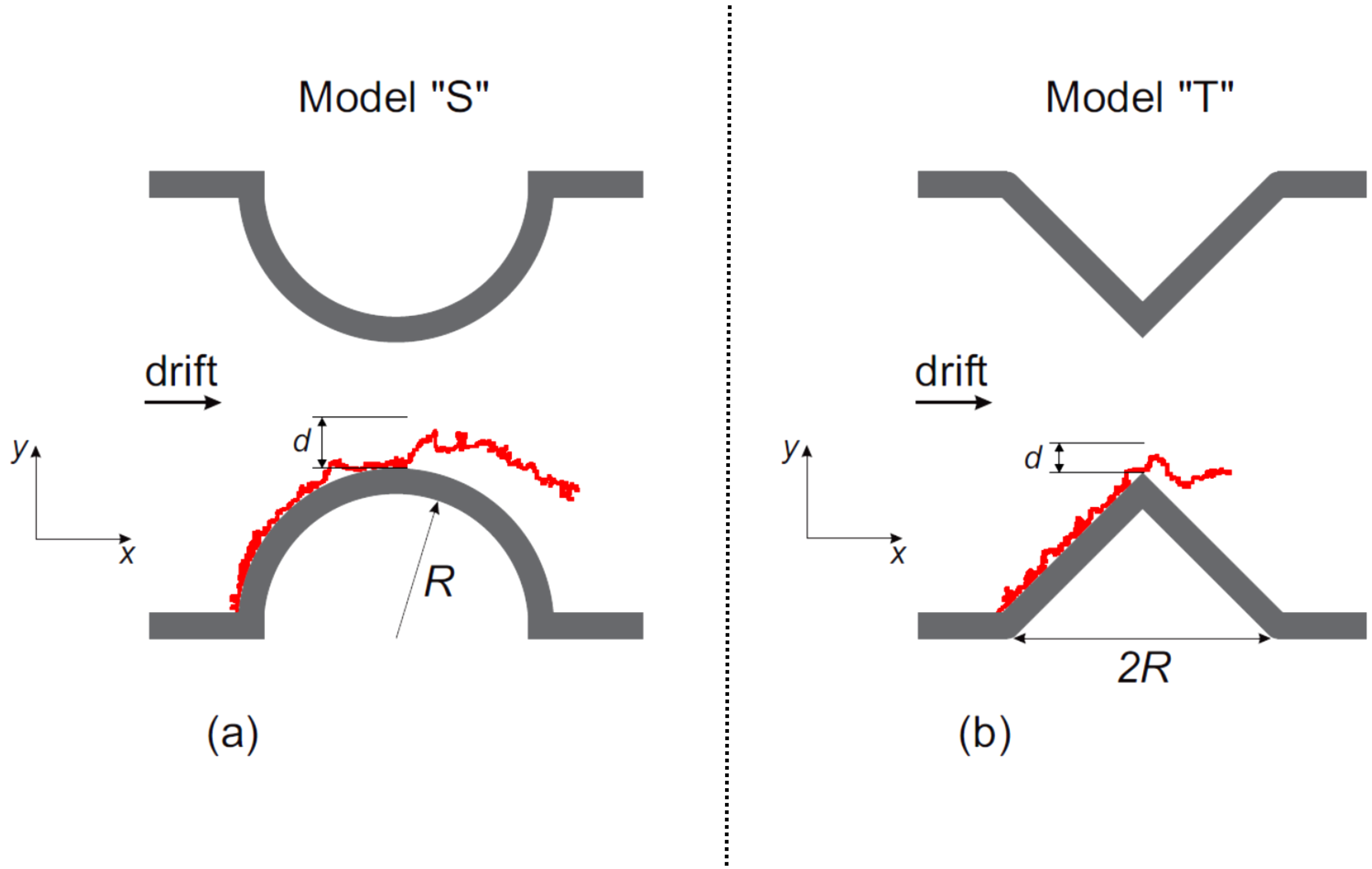
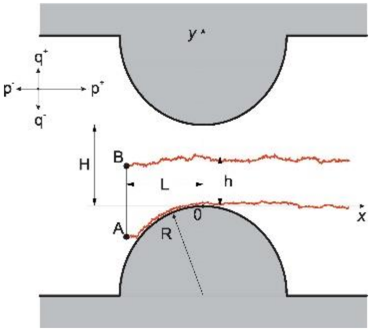


Typical trajectories of a tracer particle starting away from a funnel and passing through it.



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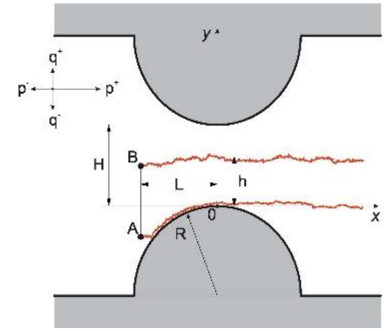
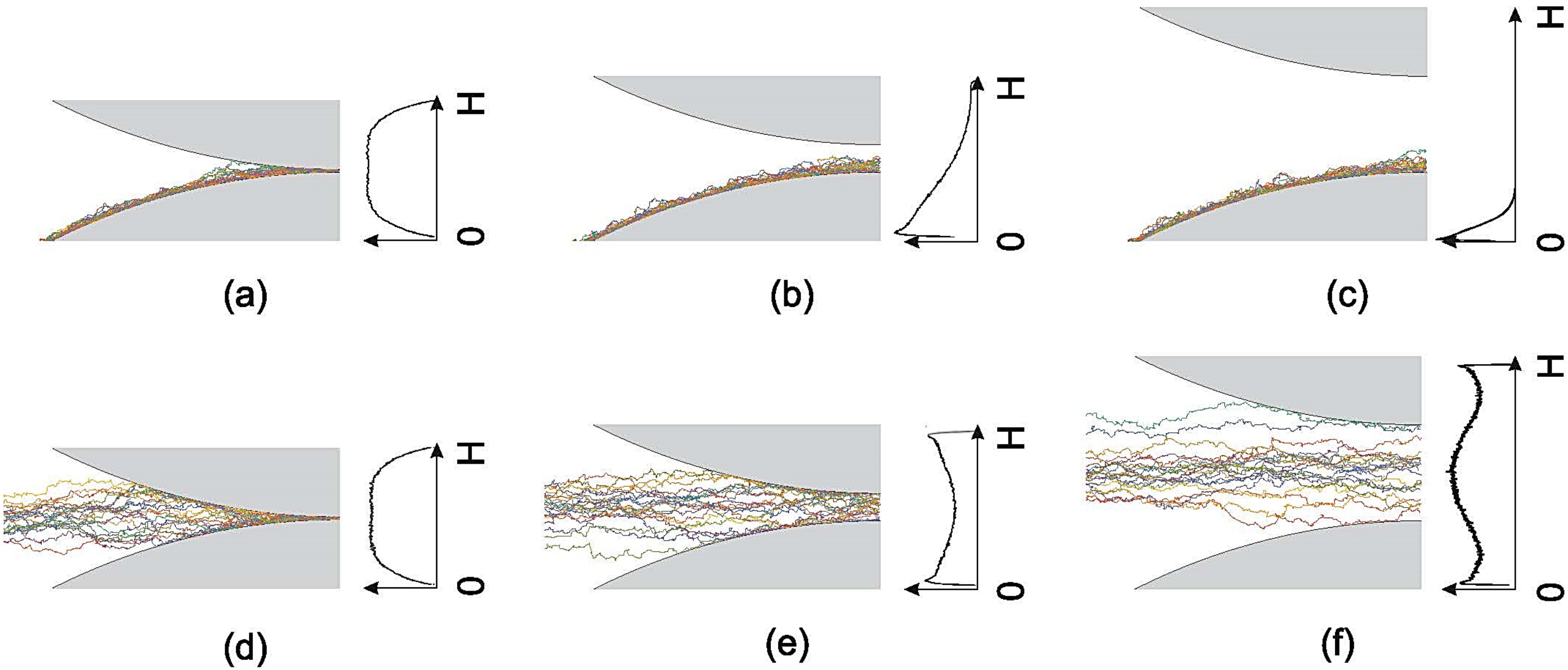
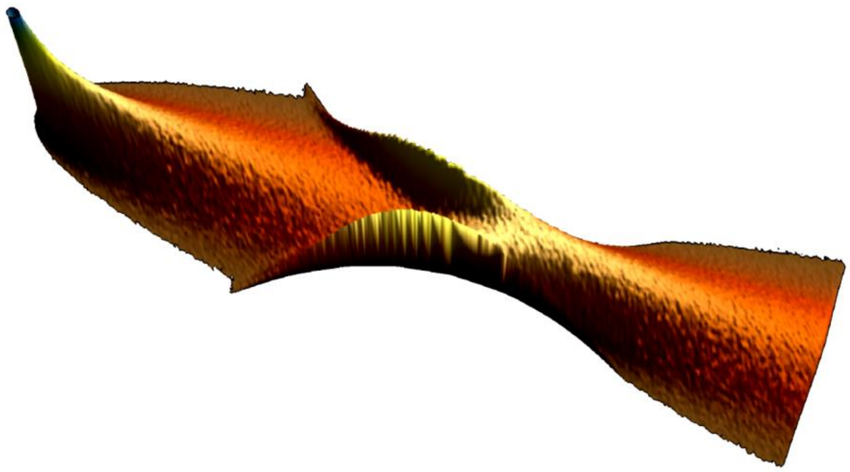
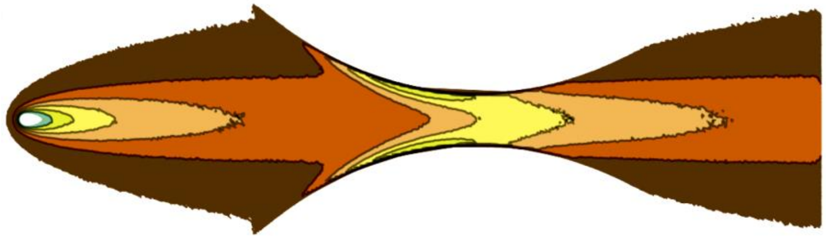
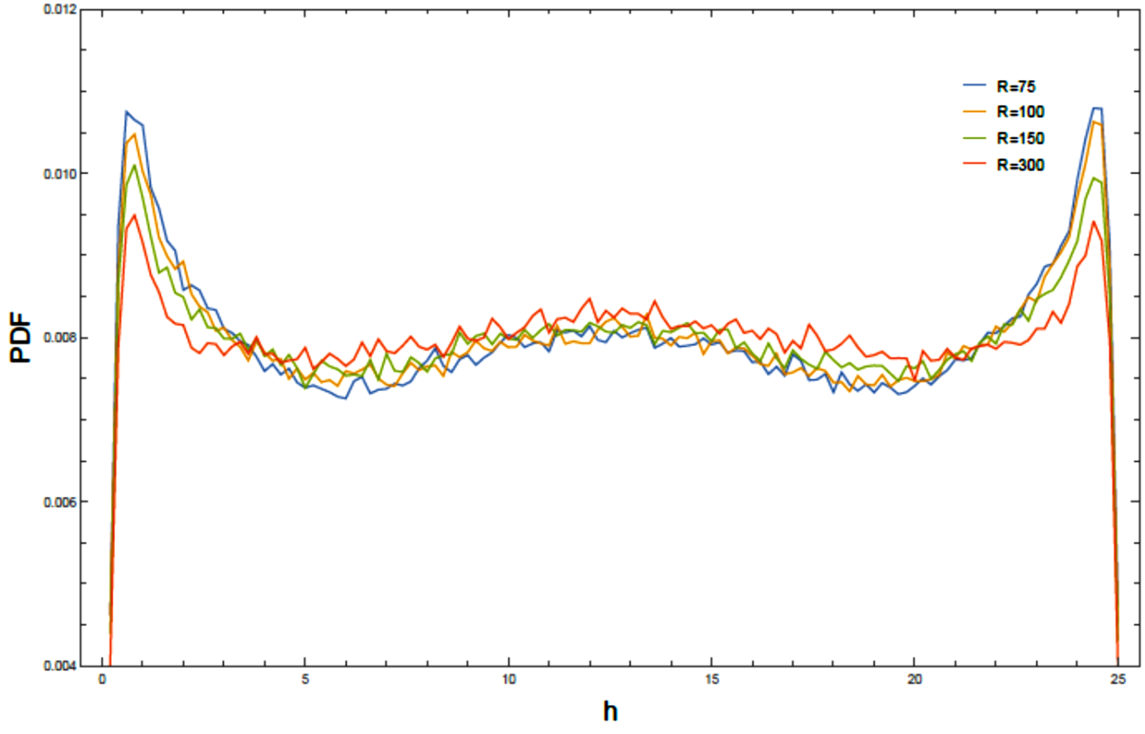


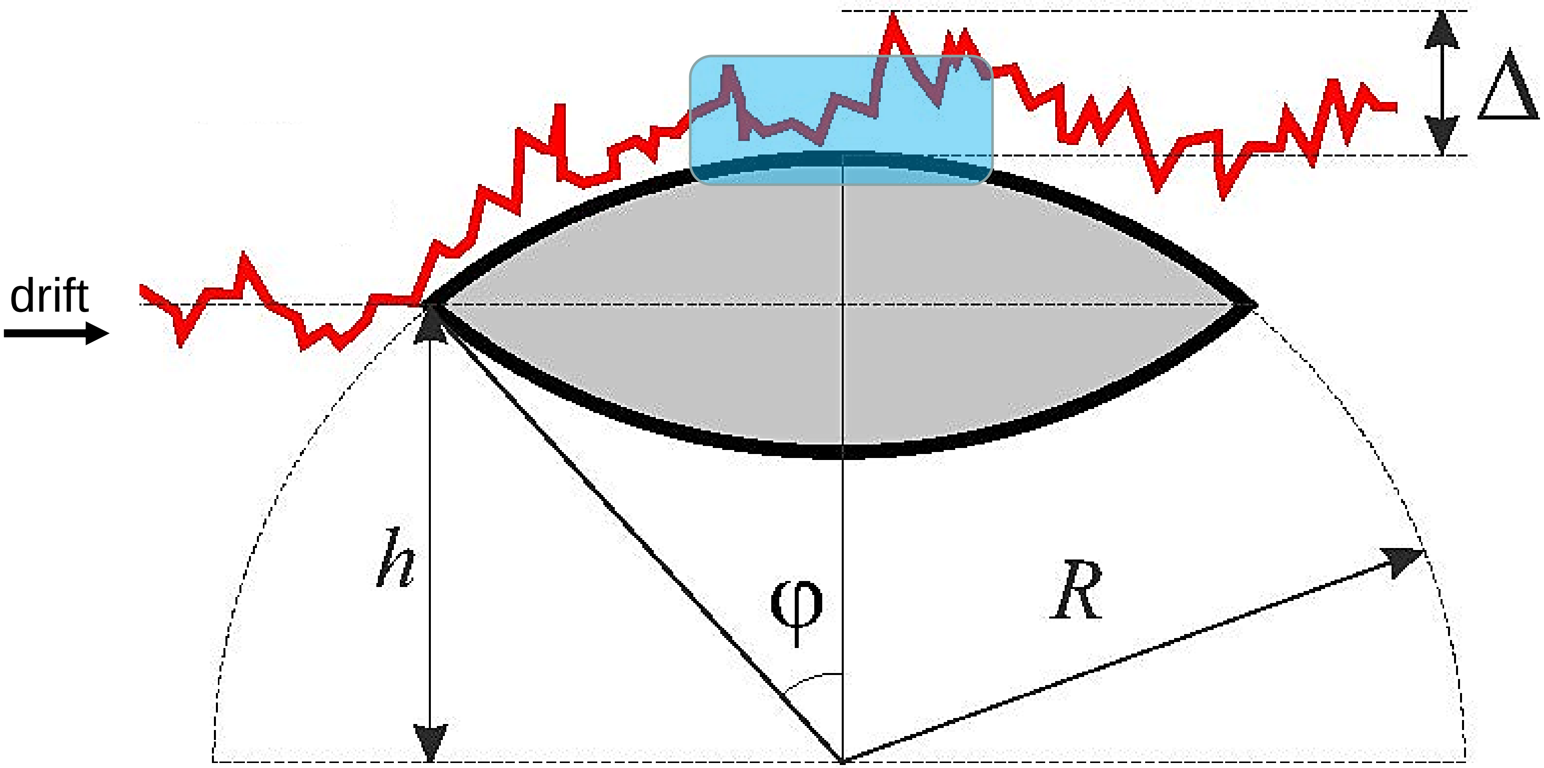


Probability density of paths in a channel's section

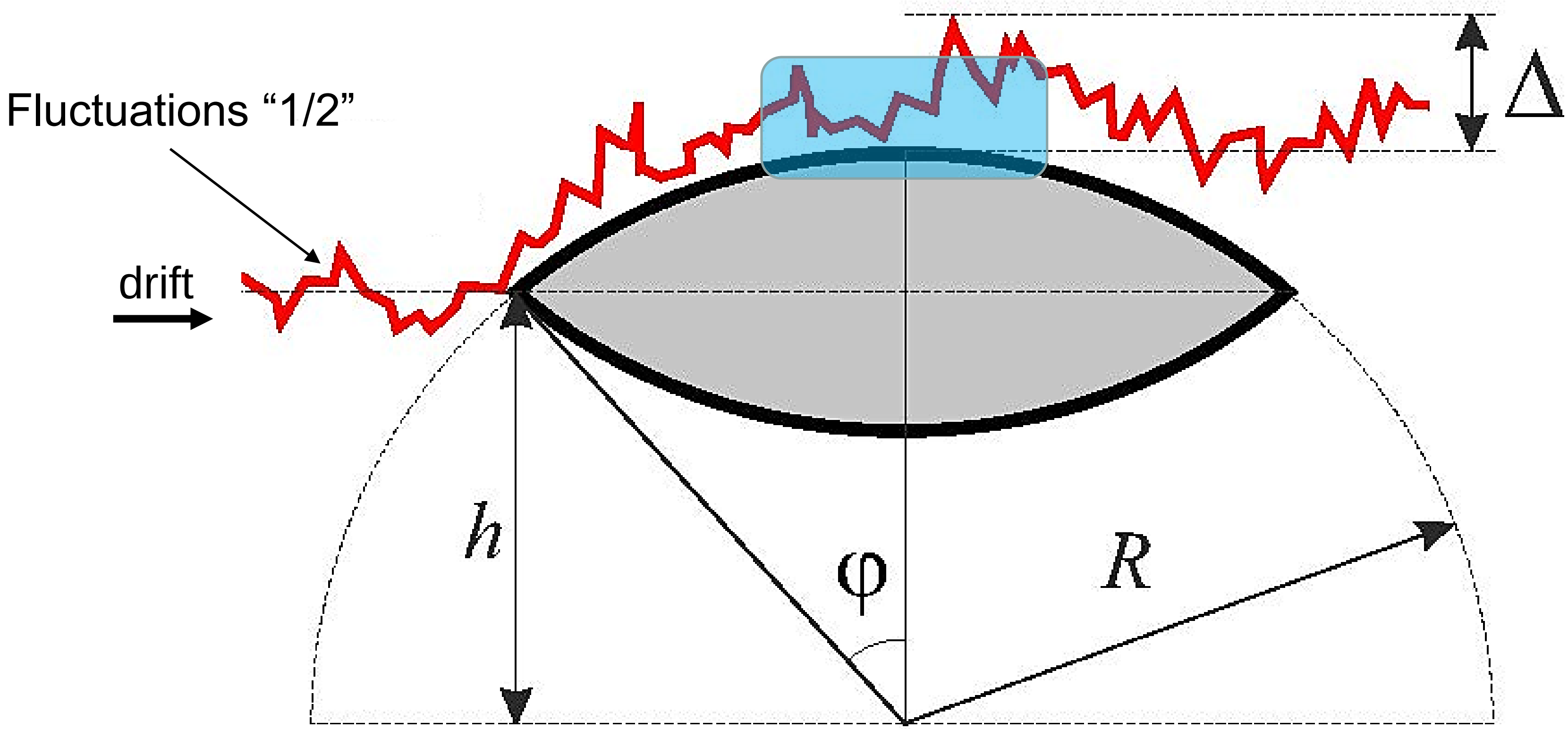


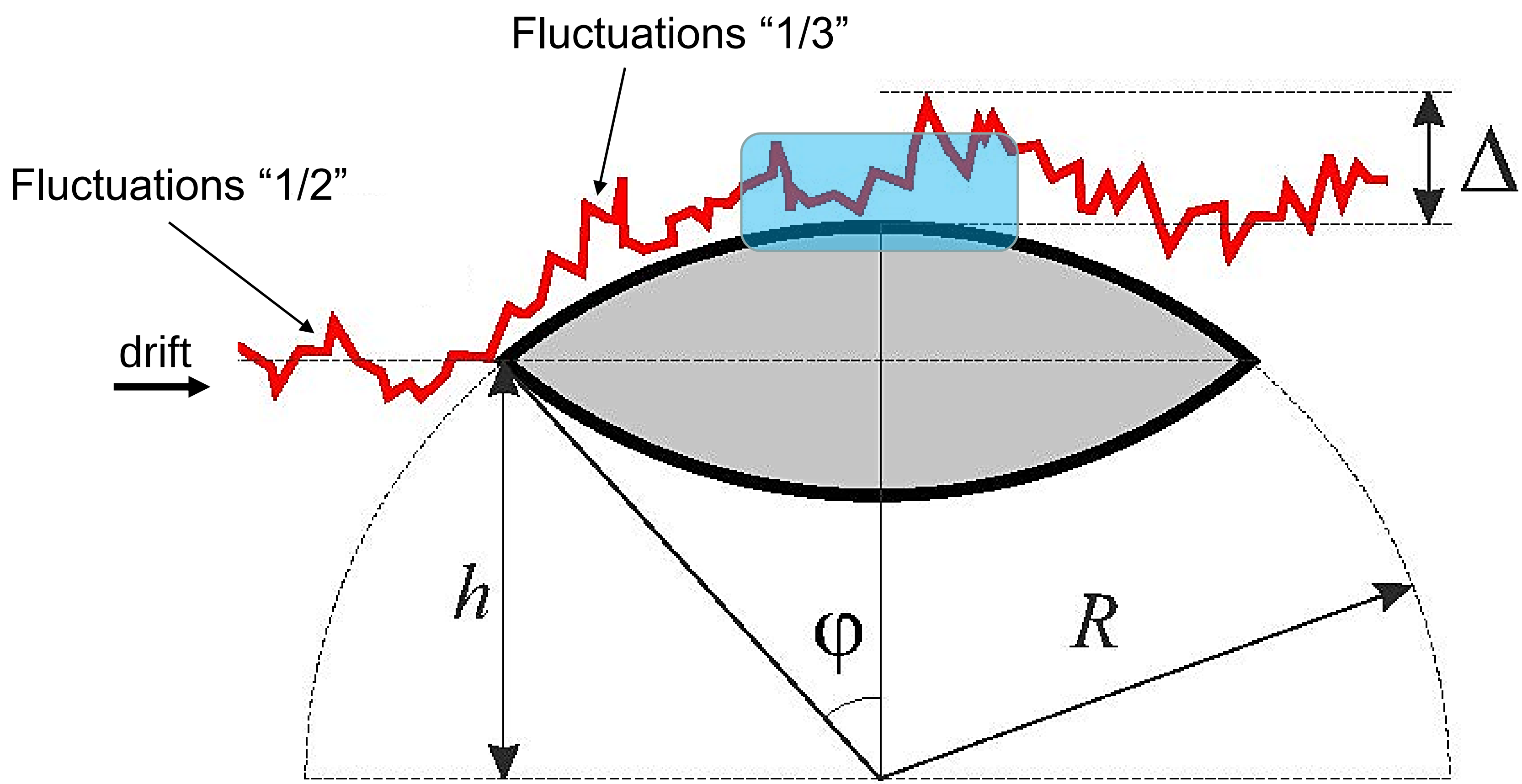




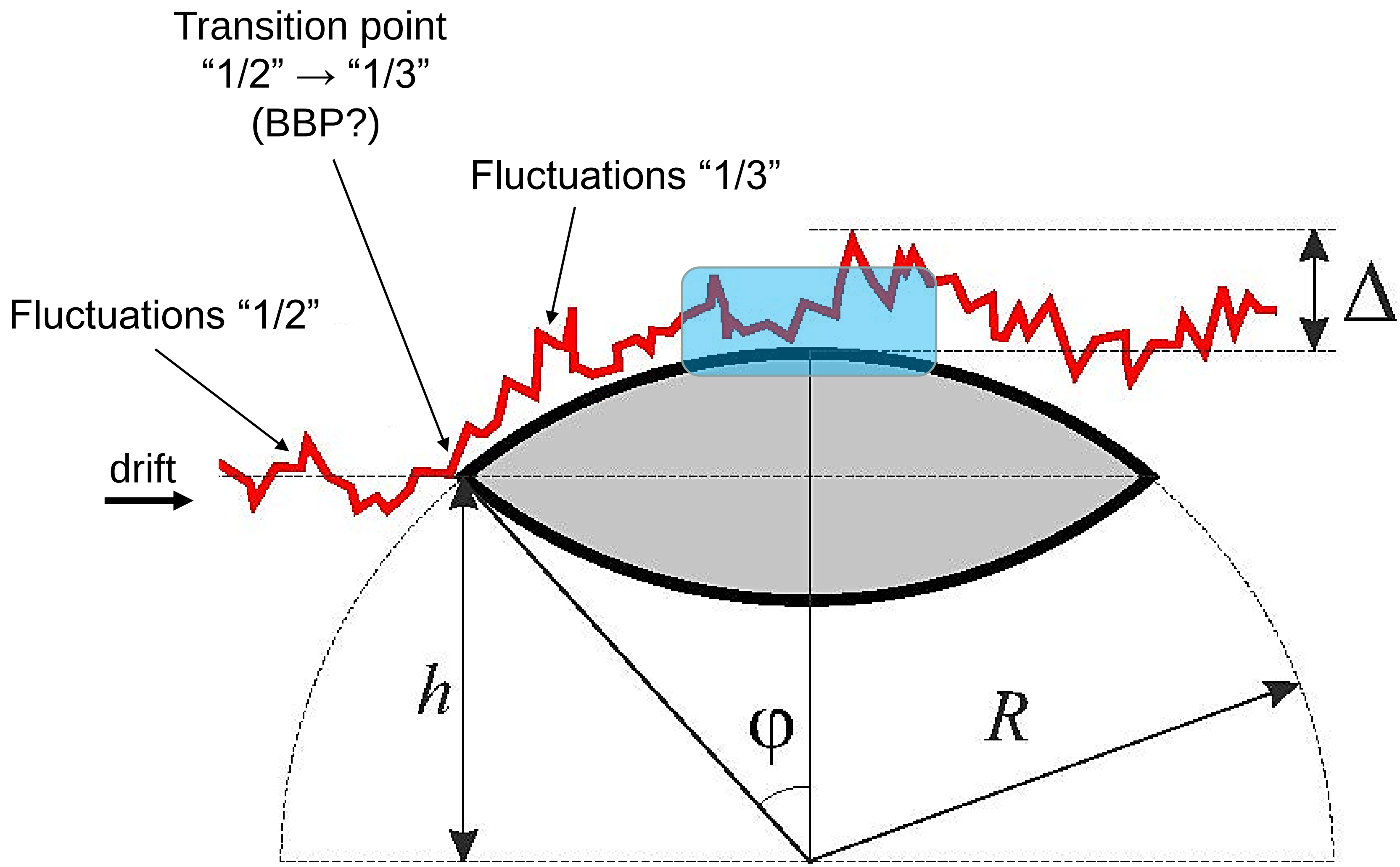


drift
Δ
h
φ
R

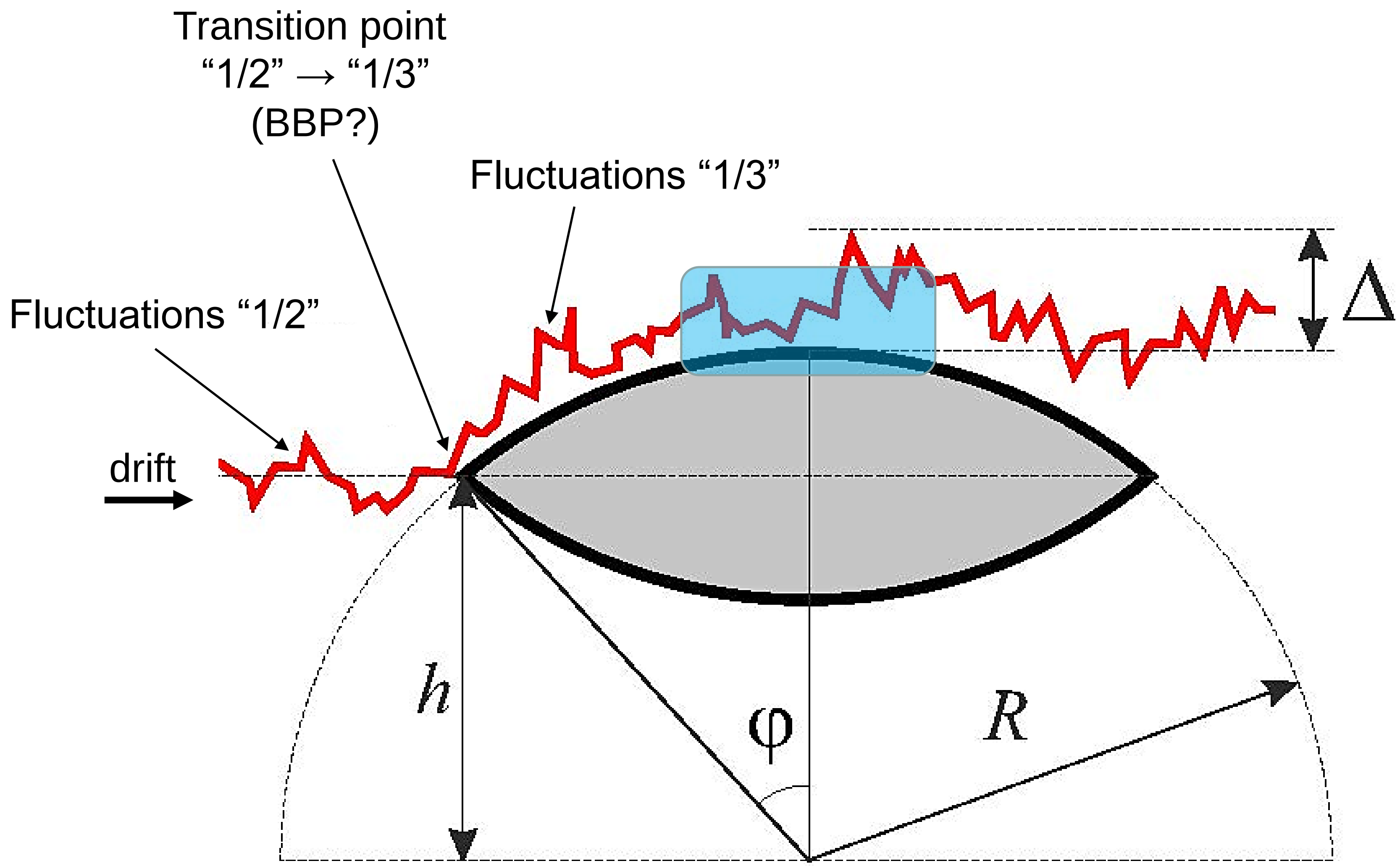

Fluctuations "1/2"
drift
Δ
h
φ
R



Fluctuations “1/3”
Fluctuations “1/2”
drift
Δ
h
φ
R



Transition point
“1/2” → “1/3”
(BBP?)
Fluctuations “1/3”
Fluctuations “1/2”
drift
Δ
h
φ
R



If h is such that the path hits the semicircle is a blue region of length $R^{2/3}$, there is no "1/3" → "1/2" transition in scaling behavior for fluctuations.

4. Mean-field approach to random matrices: finite-size scaling of Brownian bridges on supetrees

A. Gorsky, S.N, A. Valov, J. High Energ. Phys. (2018);
A. Valov, A. Gorsky, S.N., Physics of Particles and Nuclei (2021)

Joint eigenvalue distribution for matrix ensemble (β = 1, 2, 4)

$$P_\beta(\lambda_1, ..., \lambda_N) \propto \prod_{i \neq j}^{N} |\lambda_i - \lambda_j|^\beta \; e^{-c \sum_{i=1}^{N} \lambda_i^2}$$

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I. Dumitriu and A. Edelman have shown that same distribution appears in ensembles of symmetric **three-diagonal** random matrices with independent (but non-identically) distributed elements [I. Dumitriu and A. Edelman, J. Math. Phys. (2002)]

$$M = \begin{pmatrix} x_{11} & x_{12} & 0 & 0 & 0 & \dots \\ x_{21} & x_{22} & x_{23} & 0 & 0 & \\ 0 & x_{32} & x_{33} & x_{34} & 0 & \\ 0 & 0 & x_{43} & x_{44} & x_{45} & \\ 0 & 0 & 0 & x_{54} & x_{55} & \\ \vdots & & & & & \ddots \end{pmatrix}; \qquad M' = \begin{pmatrix} x_{11} & 1 & 0 & 0 & 0 & \dots \\ x_{21}^2 & x_{22} & 1 & 0 & 0 & \\ 0 & x_{32}^2 & x_{33} & 1 & 0 & \\ 0 & 0 & x_{43}^2 & x_{44} & 1 & \\ 0 & 0 & 0 & x_{54}^2 & x_{55} & \\ \vdots & & & & & \ddots \end{pmatrix}$$

$$\begin{cases} f(x|\mu,\sigma) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} & \text{for a normal, } \mathcal{N}(\mu,\sigma)\text{–distribution} \\ f(x|n) = \dfrac{x^{k-1} e^{-\frac{x^2}{2}}}{2^{\frac{k}{2}-1}\Gamma\left(\frac{k}{2}\right)}, \quad x \geq 0 & \text{for a } \chi\text{–distribution} \end{cases}$$

Mean-field approximation:

$$\begin{cases} \mu_1 = \mathbf{E}_{\chi(k)}(x) = \dfrac{\sqrt{2}\,\Gamma\left(\frac{k+1}{2}\right)}{\Gamma\left(\frac{k}{2}\right)} & \text{for subdiagonal matrix elements of } M \\ \mu_2 = \mathbf{E}_{\chi(k)}(x^2) = k & \text{for subdiagonal matrix elements of } M' \end{cases}$$

$$\langle M \rangle \approx \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & 0 & \cdots \\ \sqrt{1} & 0 & \sqrt{2} & 0 & 0 & \\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0 & \\ 0 & 0 & \sqrt{3} & 0 & \sqrt{4} & \\ 0 & 0 & 0 & \sqrt{4} & 0 & \\ \vdots & & & & & \ddots \end{pmatrix} \qquad \langle M' \rangle = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 1 & 0 & 0 & \\ 0 & 2 & 0 & 1 & 0 & \\ 0 & 0 & 3 & 0 & 1 & \\ 0 & 0 & 0 & 4 & 0 & \\ \vdots & & & & & \ddots \end{pmatrix}$$

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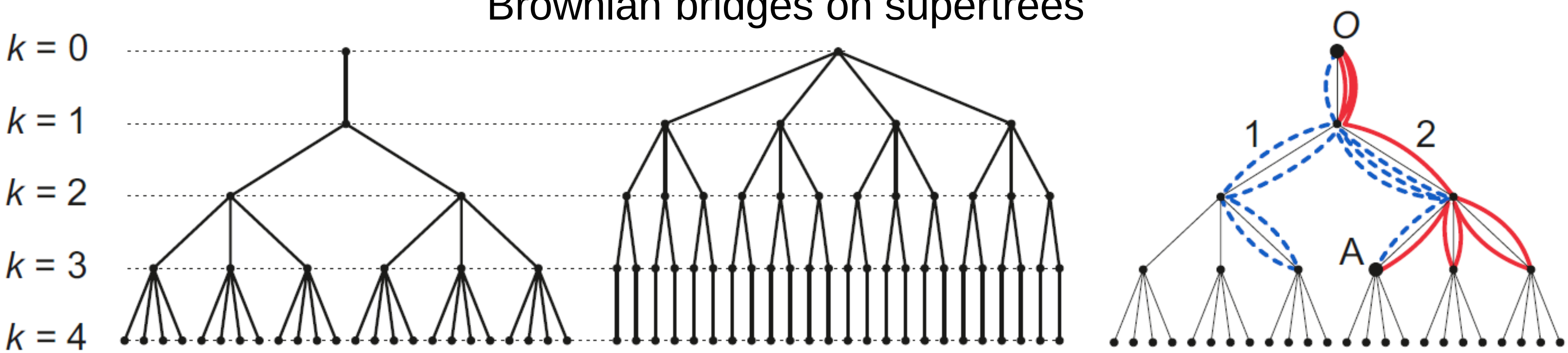
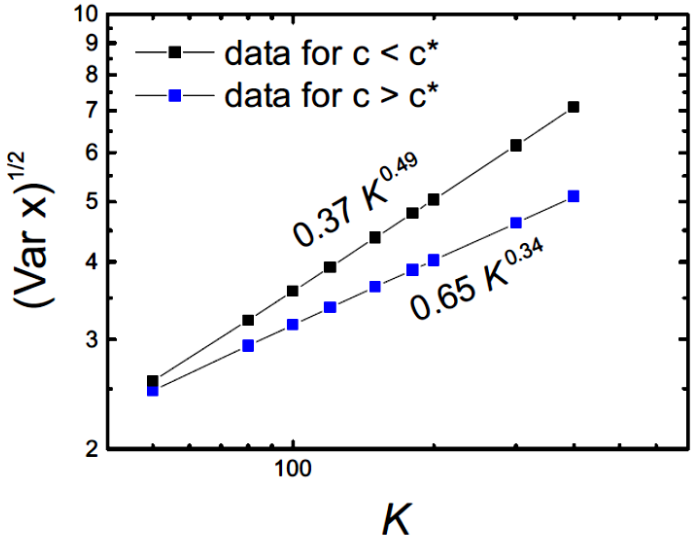
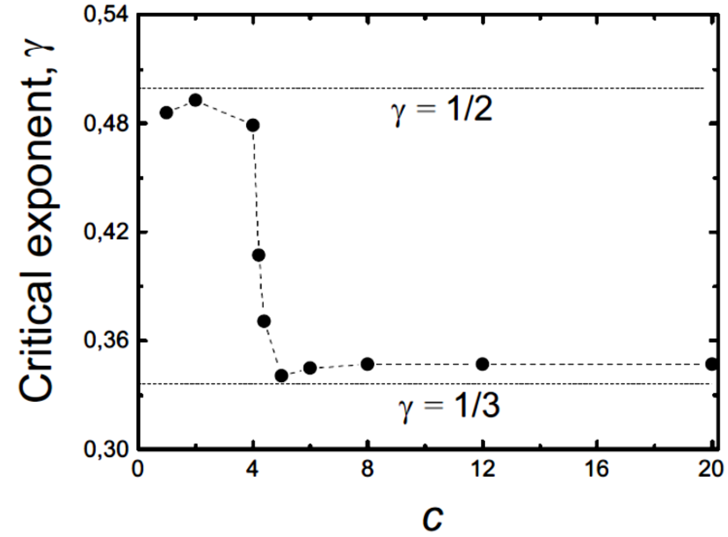
Brownian bridges on supertrees



Phase transition in scaling of fluctuations of Brownian Bridges on large finite trees when $N = c\,K$ (the control parameter is c) – BBP transition?





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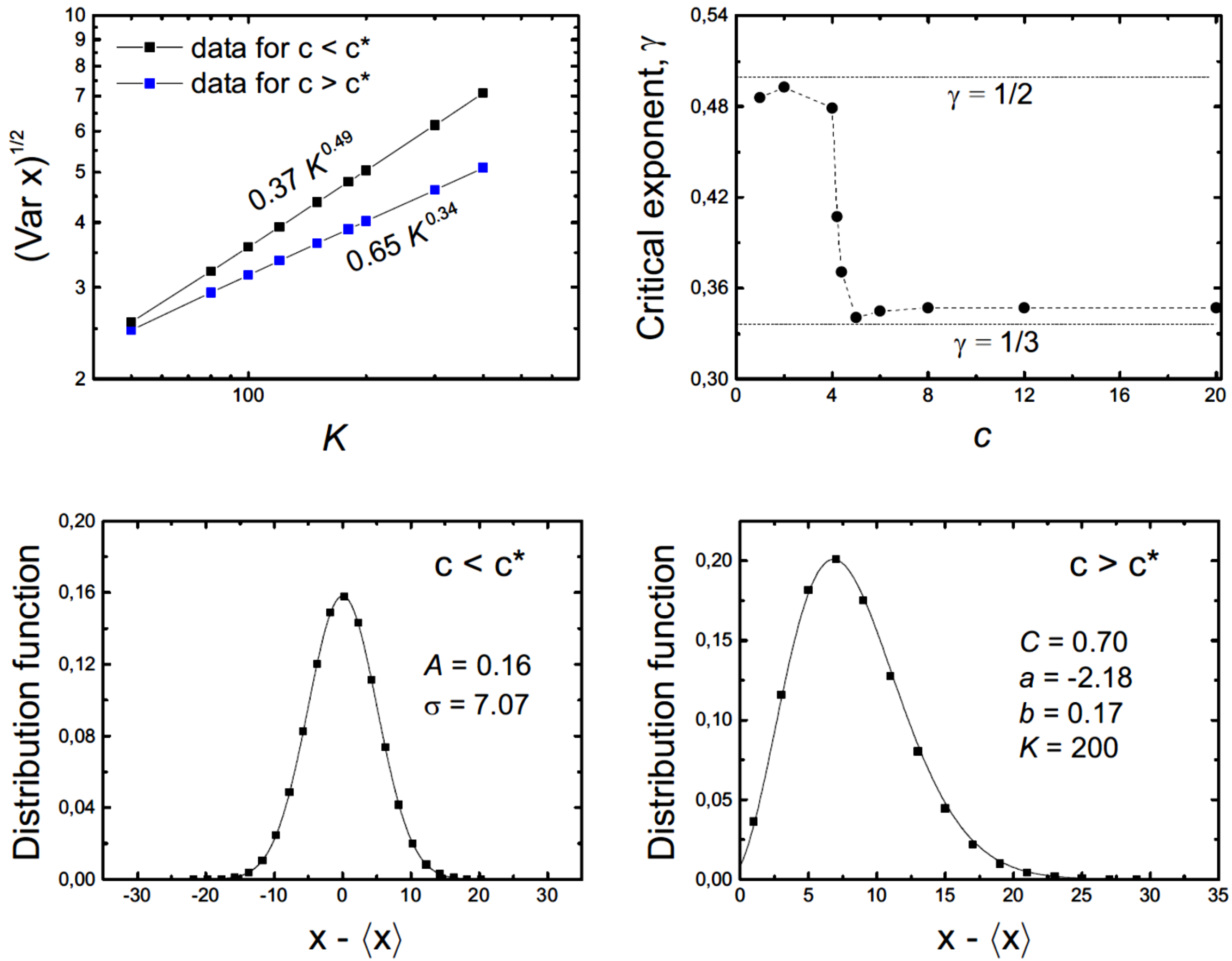

Thanks to collaborators:

- A. Grosberg (NYU, New-York)
- A. Gorsky (IITP, Moscow)
- A. Valov (Inst. of Chem. Phys., Moscow)
- K. Polovnikov (Skoltech, Moscow)
- S. Shlosman (Skoltech and Marseille)
- A. Vladimirov (IITP, Moscow)
- G. Oshanin (Paris-Sorbonne Univ.)
- V. Avetisov (Inst. Chem. Phys., Moscow)

…and to colleagues with whom these topics were discussed:

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- N. Smith (ENS, Paris)
- S. Majumdar (Paris-Saclay Univ.)
- G. Schehr (Paris-Saclay Univ.)
- V. Dotsenko (Paris-Sorbonne Univ.)
- Y. Fyodorov (Kings College, London)