Errico: a colleague and friend

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The Errico's scientific career starts in 1967 with his degree in Physics at University of Roma. Associated professor in the years 71-80 at University of L'Aquila and then in Roma.
Full professor at the University of L'Aquila, Roma 1 and Roma 2 from 1980.
Retired in 2012, then he becomes member of GSSI in L'Aq. From 2014 he is also professor emeritus at Roma 2, Tor Vergata.
Since 2009 he is member of Accademia dei Lincei.
His scientific interest is devoted to the Statistical Mechanics via a rigorous mathematical approach, in the spirit of Ruelle, Lebowitz, Gallavotti and the Russian school.

Arguments: Time Evolution of Particle Systems, Euclidean Quantum Field Theory, Ergodic Theory, Non-equilibrium Statistical Mechanics and Hydrodynamic Limits.
For such problems Errico develops sophisticated mathematical tools as Operator Algebras, Functional Analysis, Probability and Stochastic Processes (more recently also the De Giorgi Geometric Measure Theory).

Remarkable results:
With C. Marchioro and A. Pellegrinotti in 1975, Errico proves the existence of time evolution for a system of infinitely many particles in thermal equilibrium, in any spatial dimensions, extending a previous, well known one-dimensional result due to O. Lanford.
In a rare paper written by himself, then he shows the equivalence between the mechanical notion of the pressure and its thermodynamical characterization (1974).
Then he collaborated with a large roman group of colleagues in 1978 and 1980, proving the ultraviolet stability in models from EQFT.
The remarkable Errico's contributions in Non-Equilibrium Statistical Mechanics starts with A. Galves, C. Kipnis, C. Marchioro, in (1981). They study stationary non-equilibrium measures for spin systems which exhibit a temperature gradient.

This research leads Errico in studying stochastic processes and the connection between the microscopic scales (Statistical Mechanics and Stochastic Processes) and the macroscopic scales (Continuous Mechanics and PDE description).
In a remarkable review
A. De Masi, E. Presutti (1991) Mathematical methods for hydrodynamical limits Lecture Notes in Mathematics 1501, Springer-Verlag
the authors discuss, in a rigorous way, the notion of the hydrodynamic limit, essentially for stochastic systems, providing a rigorous definition. Notice that when the local equilibrium is not simply Maxwellian, as for the Boltzmann equation, this notion is much more delicate and requires new efforts.

Personal considerations. I learnt many fundamental ideas from Errico in a very simple way. A short list.
1) Superstability estimates. I had difficulties in understanding how to proceed. Errico showed me that, assuming preliminarly that the potential is short range, then the proof was simpler and conceptually clear.
2) v-functions. In proving hydrodynamic limits often one has to compare two family of marginals, say f_j and g_j. Suppose for the moment that they factorize. Then the difference $f_j - g_j$ is much larger than the product of the one-particle distributions. Say ε against ε^j. Expanding such a product one obtain a definition which can be used also if g_j do not factorizes. This allows more accurate estimates in problems concerning hydro limits. Errico explained me this and I used this notion (which b.t.w. is a sort of reduced cumulants) in the study of kinetic limits. The v-functions were introduced in the review with A. De Masi already discussed.

3) Doeblin condition. Errico explained me in a very simple way and it was very useful for me.

A final consideration. Errico followed many young researchers, suggesting problems and techniques. Also following their progresses both from a human and technical view point, taking into account their autonomy. I must say that this is not very frequent in the academic world.
Errico was:
very high level researcher and person (from a scientific and moral view point)
but
with a low profile behaviour.