Center for Scientific Computing, Theory and Data

Exact solution of the classical and quantum Heisenberg mean field spin glasses

- Markus Müller

- **Inhomogeneous Random systems IRS 2025,**
  IHP Paris   Jan 28/29, 2025

Coauthors

Nikita Kavokine
(CCQ Flatiron NY, MPI Mainz)

Antoine Georges
(Paris, CCQ Flatiron NY, Uni Genève)

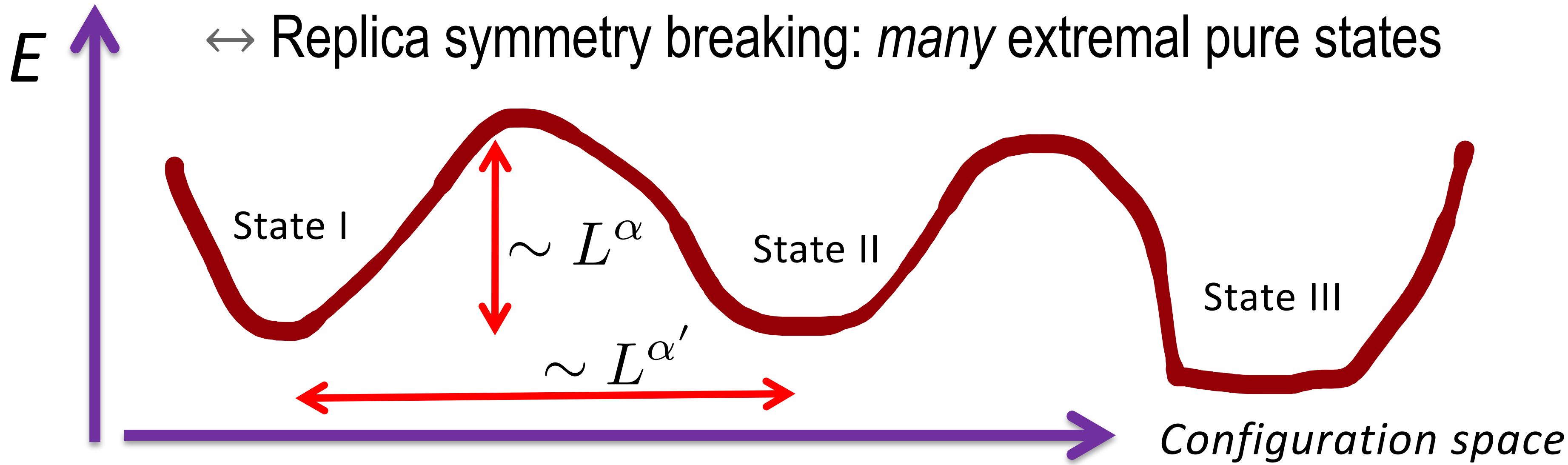
Olivier Parcollet
(CCQ Flatiron NY)

Quantum non–ergodicity: glasses and localization

Glass phases: Frustration and disorder

Fragmentation of phase space: many low energy minima, separated by very high energy barriers

$\leftrightarrow$ Replica symmetry breaking: *many* extremal pure states

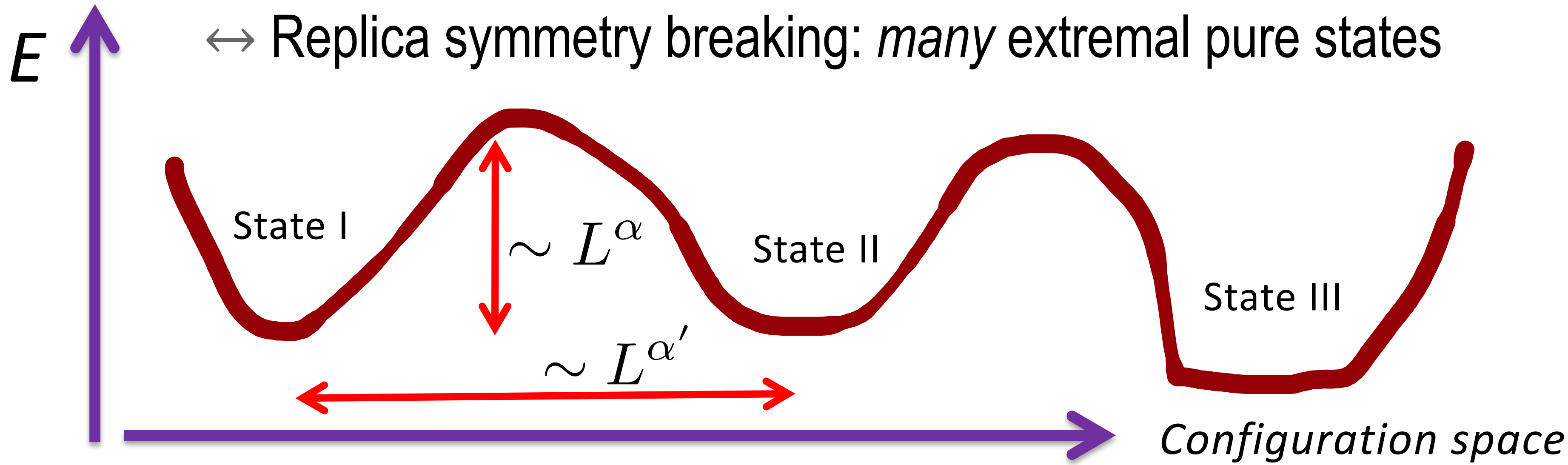


Quantum non–ergodicity: glasses and localization

Glass phases: Frustration and disorder

Fragmentation of phase space: many low energy minima,
separated by very high energy barriers

$\leftrightarrow$ Replica symmetry breaking: *many* extremal pure states

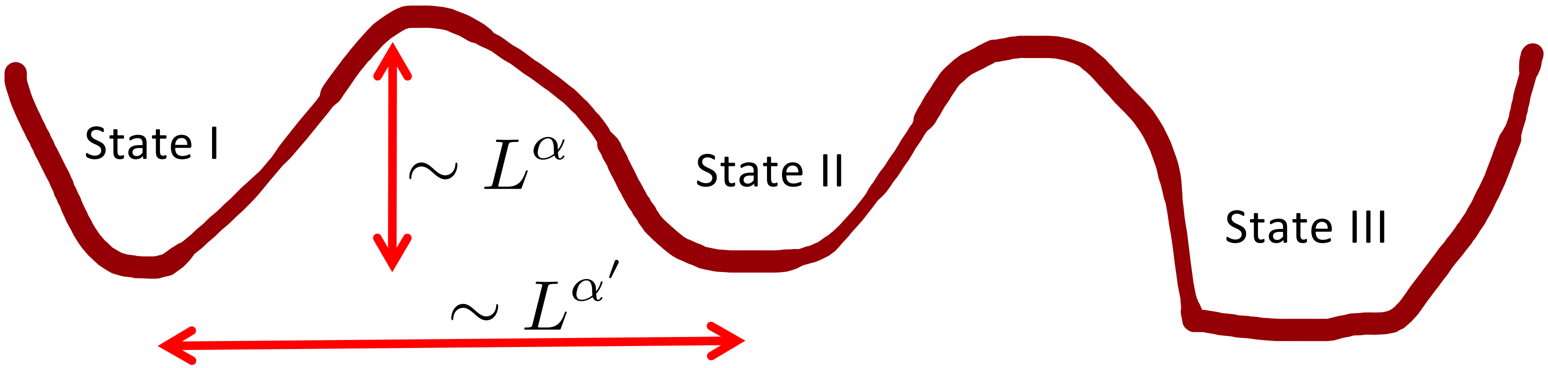


+ Quantum fluctuations / dynamics?

Quantum non–ergodicity: glasses and localization

Glass phases: Frustration and disorder

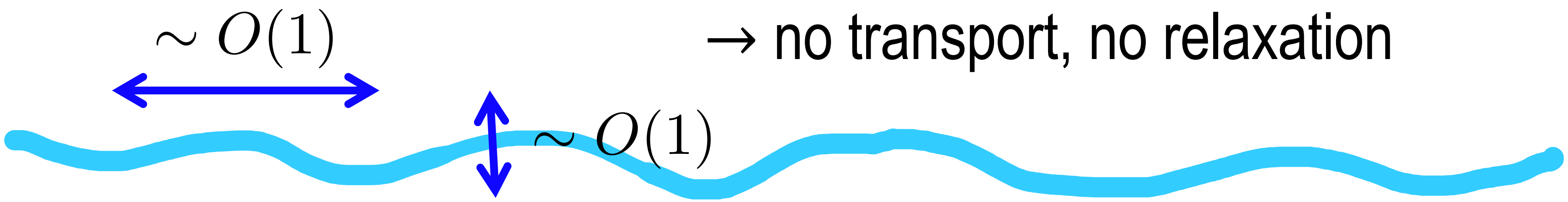
Fragmentation of phase space: many low energy minima,
separated by very high energy barriers
$\leftrightarrow$ Replica symmetry breaking: *many* extremal pure states



Anderson & many-body localization: Disorder + weak tunneling/interaction:

$\rightarrow$ *local* moves almost never resonant:

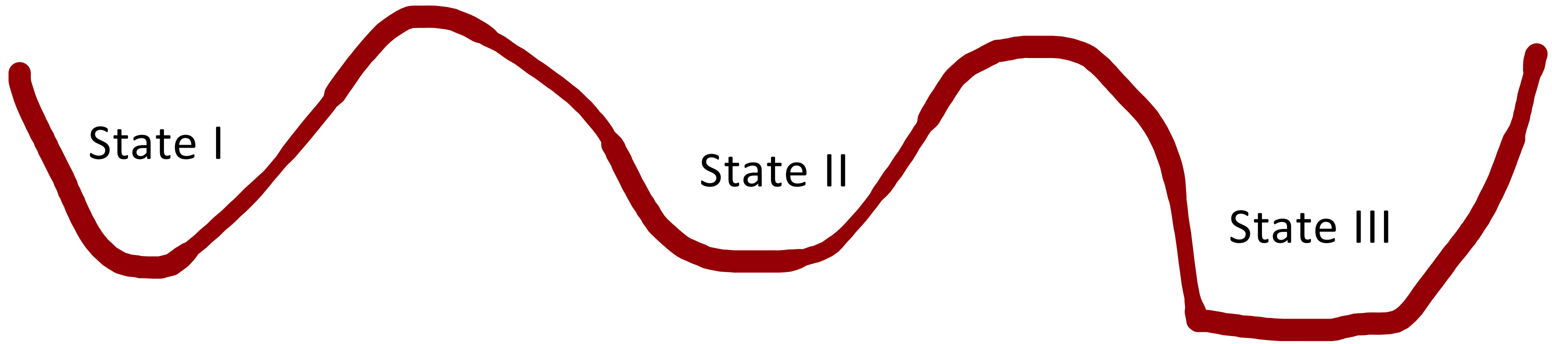
$\rightarrow$ no transport, no relaxation



Interesting question:

How does a glass phase impact the quantum excitations?

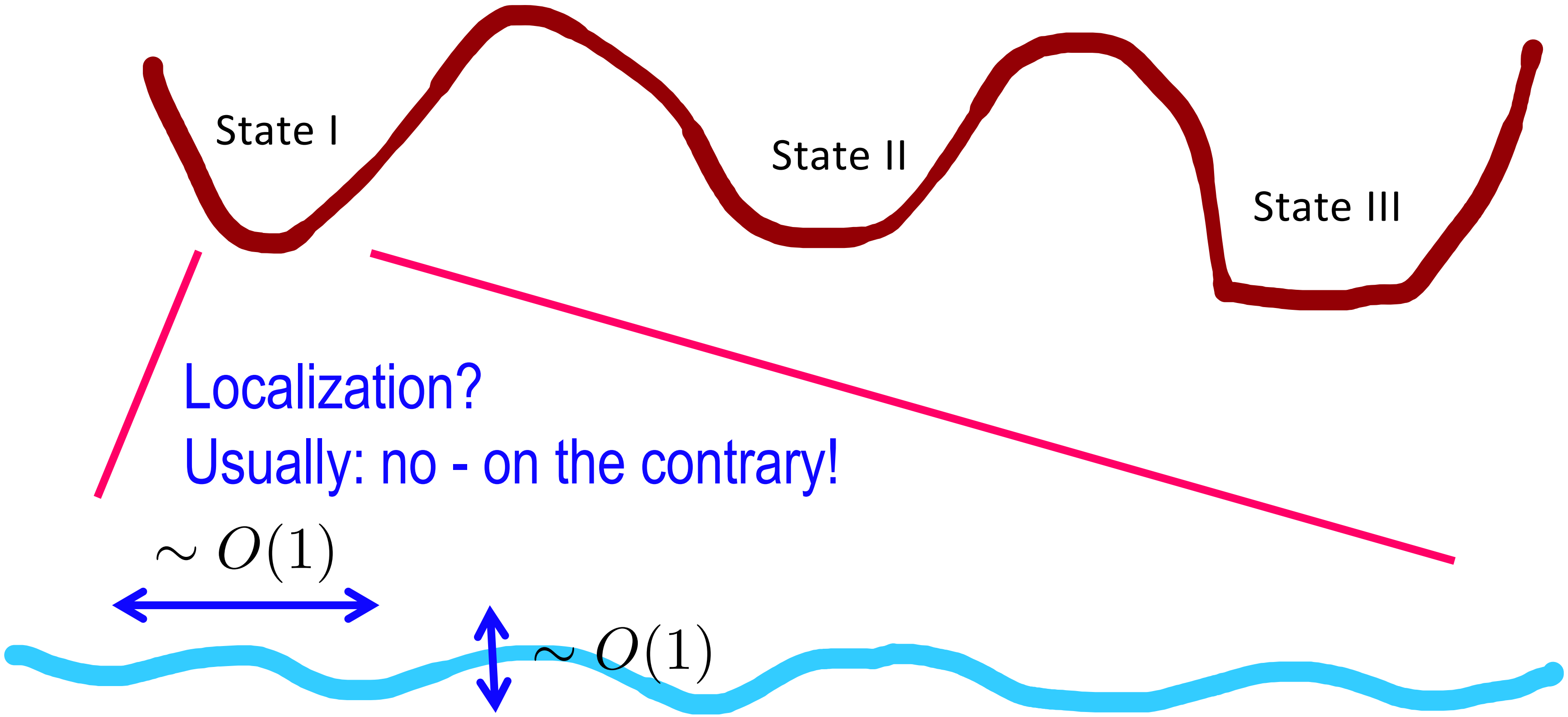
Collective «spin waves» of a quantum glass?
Is there localization?



Interesting question:

How does a glass phase impact the quantum excitations?

Collective «spin waves» of a quantum glass?
Is there localization?



The Heisenberg spin glass

Motivation

- *Ising* glasses well understood (at mean field level):

  Classical Sherrington-Kirkpatrick (SK) model;
  Quantum SK with transverse field

$$\mathcal{H} = \sum_{i<j} J_{ij} S_i^z S_j^z - \Gamma \sum_i S_i^x$$

PAUL SCHERRER INSTITUT

The Heisenberg spin glass

Motivation

- *Ising* glasses well understood (at mean field level):

  Classical Sherrington-Kirkpatrick (SK) model;
  Quantum SK with transverse field

$$\mathcal{H} = \sum_{i<j} J_{ij} S_i^z S_j^z - \Gamma \sum_i S_i^x$$

- Much less known about *vector spins*, both classical and quantum

$$\mathcal{H} = \sum_{i<j} J_{ij} (S_i^z S_j^z + S_i^x S_j^x + S_i^y S_j^y)$$

(E.g.: local moments in randomly doped Mott insulators)

Sachdev-Ye approach: Can the glass be avoided?

Can the quantum spins form a spin fluid that does not break time-reversal symmetry?

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

A. Georges, O. Parcollet, and S. Sachdev, PRL 85, 840 (2000); PRB 63, 134406 (2001)

$$\mathcal{H} = \sum_{i<j} J_{ij}(S_i^z S_j^z + S_i^x S_j^x + S_i^y S_j^y)$$

PAUL SCHERRER INSTITUT PSI

Sachdev-Ye approach: Can the glass be avoided?

Can the quantum spins form a spin fluid that does not break time-reversal symmetry?

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

A. Georges, O. Parcollet, and S. Sachdev, PRL 85, 840 (2000); PRB 63, 134406 (2001)

↔ Random spin *chains* (D. Fisher '85), solvable in 1d:

Spin liquid: random singlet phase, strong disorder fixed point;

What happens in the opposite limit of high connectivity?

$$\mathcal{H} = \sum_{i<j} J_{ij} (S_i^z S_j^z + S_i^x S_j^x + S_i^y S_j^y)$$

Sachdev-Ye approach: Can the glass be avoided?

Can the quantum spins form a spin fluid that does not break time-reversal symmetry?

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

A. Georges, O. Parcollet, and S. Sachdev, PRL 85, 840 (2000); PRB 63, 134406 (2001)

↔ Random spin *chains* (D. Fisher '85), solvable in 1d:

Spin liquid: random singlet phase, strong disorder fixed point;

What happens in the opposite limit of high connectivity?

What is the quantum dynamics in- *and* outside the glass phase?

$$\mathcal{H} = \sum_{i<j} J_{ij}(S_i^z S_j^z + S_i^x S_j^x + S_i^y S_j^y)$$

Sachdev-Ye approach: Can the glass be avoided?

Can the quantum spins form a spin fluid that does not break time-reversal symmetry?

S. Sachdev and J. Ye, PRL 70, 3339 (1993)

A. Georges, O. Parcollet, and S. Sachdev, PRL 85, 840 (2000); PRB 63, 134406 (2001)

↔ Random spin *chains* (D. Fisher '85), solvable in 1d:

Spin liquid: random singlet phase, strong disorder fixed point;

What happens in the opposite limit of high connectivity?

What is the quantum dynamics in- *and* outside the glass phase?

Gaussian, all-to-all

$$\mathcal{H} = \sum_{i<j} J_{ij} (S_i^z S_j^z + S_i^x S_j^x + S_i^y S_j^y)$$

Goal today: Understand physics of mean field limit & compare with Ising

The Heisenberg spin glass

Mean field model

$$\mathcal{H} = -\sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Classical model

(Large S)

$$|\mathbf{S}_i| = 1/2$$

Quantum model

Non-commuting spin components, S=1/2

$$[S_i^\alpha, S_j^\beta] = i\epsilon^{\alpha\beta\gamma}\delta_{ij}S_i^\gamma$$

PAUL SCHERRER INSTITUT

The Heisenberg spin glass

Mean field model

$$\mathcal{H} = -\sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Classical model

(Large S)

$$|\mathbf{S}_i| = 1/2$$

Quantum model

Non-commuting spin components, S=1/2

$$[S_i^{\alpha}, S_j^{\beta}] = i\epsilon^{\alpha\beta\gamma}\delta_{ij}S_i^{\gamma}$$

Continuous glass transition at T_g

$$T_{\mathrm{g}} = JS^2/3$$

Bray, Moore, JPC 14, 2629 (1981).

Gabay, Garel, De Dominicis, JPC 15, 7165 (1982)

The Heisenberg spin glass

Mean field model

$$\mathcal{H} = -\sum_{i<j} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j$$

Classical model

(Large S)

$$|\mathbf{S}_i| = 1/2$$

Quantum model

Non-commuting spin components, S=1/2

$$[S_i^\alpha, S_j^\beta] = i\epsilon^{\alpha\beta\gamma}\delta_{ij}S_i^\gamma$$

Continuous glass transition at T_g

$$T_\mathrm{g} = JS^2/3$$

$$T_\mathrm{g} \approx J\langle \mathbf{S}^2\rangle/3\sqrt{3}$$

Bray, Moore, JPC 14, 2629 (1981).

Bray, Moore, JPC 13, L655-60 (1980).

Gabay, Garel, De Dominicis, JPC 15, 7165 (1982)

Sachdev-Ye approach: Can the glass be avoided?

Challenge of quantum spins: Hard to deal with, even in mean field!

S. Sachdev and J. Ye,
PRL 70, 3339 (1993)

A. Georges, O. Parcollet,
and S. Sachdev,
PRL 85, 840 (2000);
PRB 63, 134406 (2001)

Promote SU(2) spins to SU(M) spins
→ solvable in the limit of large M!

Sachdev-Ye approach: Can the glass be avoided?

SU(2) → SU(M) spins

$$H = \frac{1}{\sqrt{NM}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Solvability in the limit $M \to \infty$!

$$\mathbf{S} = \{S_{\alpha\beta}\} \quad 1 \leq \alpha, \beta \leq M$$

PAUL SCHERRER INSTITUT PSI

Sachdev-Ye approach: Can the glass be avoided?

SU(2) → SU(M) spins

$$H = \frac{1}{\sqrt{NM}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Solvability in the limit $M \to \infty$!

$$\mathbf{S} = \{S_{\alpha\beta}\} \quad 1 \leq \alpha, \beta \leq M$$

Different representations of SU(M) = different models / loc Hilbert space

Abrikosov fermions

$$S_{\alpha\beta} = f_\alpha^\dagger f_\beta$$

$$\Sigma_\alpha f_\alpha^\dagger f_\alpha = q_0 M \, (0 \leqslant q_0 \leqslant 1)$$

S. Sachdev and J. Ye (1993)

PAUL SCHERRER INSTITUT PSI

Sachdev-Ye approach: Can the glass be avoided?

SU(2) → SU(M) spins

$$H = \frac{1}{\sqrt{NM}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Solvability in the limit $M \to \infty$!

$$\mathbf{S} = \{S_{\alpha\beta}\} \quad 1 \leq \alpha, \beta \leq M$$

Different representations of SU(M) = different models / loc Hilbert space

Schwinger bosons		Abrikosov fermions
$S_{\alpha\beta} = b^\dagger_\alpha b_\beta - S\delta_{\alpha\beta}$	↔	$S_{\alpha\beta} = f^\dagger_\alpha f_\beta$
$\Sigma_\alpha b^\dagger_\alpha b_\alpha = SM (0 \leqslant S)$		$\Sigma_\alpha f^\dagger_\alpha f_\alpha = q_0 M (0 \leqslant q_0 \leqslant 1)$
A. Georges, O. Parcollet, and S. Sachdev (2000/01);		*S. Sachdev and J. Ye (1993)*

Sachdev-Ye approach: Can the glass be avoided?

SU(2) → SU(M) spins

$$H = \frac{1}{\sqrt{NM}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Solvability in the limit $M \to \infty$!

$$\mathbf{S} = \{S_{\alpha\beta}\} \quad 1 \le \alpha, \beta \le M$$

Different representations of SU(M) = different models / loc Hilbert space

Schwinger bosons

$$S_{\alpha\beta} = b^\dagger_\alpha b_\beta - S\,\delta_{\alpha\beta}$$

$$\Sigma_\alpha b^\dagger_\alpha b_\alpha = SM (0 \leqslant S)$$

↔

Abrikosov fermions

$$S_{\alpha\beta} = f^\dagger_\alpha f_\beta$$

$$\Sigma_\alpha f^\dagger_\alpha f_\alpha = q_0 M (0 \leqslant q_0 \leqslant 1)$$

A. Georges, O. Parcollet, and S. Sachdev (2000/01);

S. Sachdev and J. Ye (1993)

→ solvable equations for "parton" Green's functions as $M \to \infty$

Sachdev-Ye approach

SU(2) → SU(M) spins

$$H = \frac{1}{\sqrt{NM}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Solvability in the limit $M \to \infty$!

Spin fluid region: (high T or low spin)

Parton Green's function $$G_B^{ab}(\tau) = -\overline{\langle T b^a(\tau) b^{\dagger b}(0) \rangle}$$

Large-M Dyson equation
(almost identical for fermions)

$$(G_B^{-1})^{ab}(i\nu_n) = i\nu_n \delta_{ab} + \lambda^a \delta_{ab} - \Sigma_B^{ab}(i\nu_n),$$

$$\Sigma_B^{ab}(\tau) = J^2 [G_B^{ab}(\tau)]^2 G_B^{ab}(-\tau),$$

$$G_B^{aa}(\tau = 0^-) = -S.$$

PAUL SCHERRER INSTITUT PSI

Sachdev-Ye approach

SU(2) → SU(M) spins

$$H = \frac{1}{\sqrt{NM}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Solvability in the limit $M \to \infty$!

Spin fluid region: (high T or low spin)

Parton Green's function $G_B^{ab}(\tau) = -\overline{\langle T b^a(\tau) b^{\dagger b}(0) \rangle}$

Large-M Dyson equation
(almost identical for fermions)

$$(G_B^{-1})^{ab}(i\nu_n) = i\nu_n \delta_{ab} + \lambda^a \delta_{ab} - \Sigma_B^{ab}(i\nu_n),$$

$$\Sigma_B^{ab}(\tau) = J^2 [G_B^{ab}(\tau)]^2 G_B^{ab}(-\tau),$$

$$G_B^{aa}(\tau = 0^-) = -S.$$

$$\longrightarrow \quad \chi_{\text{loc}}(\tau) = \langle S(\tau) S(0) \rangle = G_B^{aa}(\tau) G_B^{aa}(-\tau)$$

PAUL SCHERRER INSTITUT PSI

Sachdev-Ye approach

SU(2) → SU(M) spins

$$H = \frac{1}{\sqrt{NM}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Solvability in the limit $M \to \infty$!

Spin fluid region: (high T or low spin)

Parton Green's function $G_B^{ab}(\tau) = -\overline{\langle T b^a(\tau) b^{\dagger b}(0) \rangle}$

Large-M Dyson equation
(almost identical for fermions)

$$(G_B^{-1})^{ab}(i\nu_n) = i\nu_n \delta_{ab} + \lambda^a \delta_{ab} - \Sigma_B^{ab}(i\nu_n),$$

$$\Sigma_B^{ab}(\tau) = J^2 [G_B^{ab}(\tau)]^2 G_B^{ab}(-\tau),$$

$$G_B^{aa}(\tau = 0^-) = -S.$$

→ $\chi_{\rm loc}(\tau) = \langle S(\tau) S(0) \rangle = G_B^{aa}(\tau) G_B^{aa}(-\tau)$

$G_B^{aa}(\tau) \sim 1/(J\tau)^{1/2}$ Very slow decay: Non-Fermi liquid of partons

Sachdev-Ye approach

SU(2) → SU(M) spins

$$H = \frac{1}{\sqrt{NM}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Solvability in the limit $M \to \infty$!

Spin fluid region: (high T or low spin)

Parton Green's function $G_B^{ab}(\tau) = -\overline{\langle T b^a(\tau) b^{\dagger b}(0) \rangle}$

Large-M Dyson equation
(almost identical for fermions)

$$(G_B^{-1})^{ab}(i\nu_n) = i\nu_n \delta_{ab} + \lambda^a \delta_{ab} - \Sigma_B^{ab}(i\nu_n),$$

$$\Sigma_B^{ab}(\tau) = J^2 [G_B^{ab}(\tau)]^2 G_B^{ab}(-\tau),$$

$$G_B^{aa}(\tau = 0^-) = -S.$$

$$\longrightarrow \chi_{\text{loc}}(\tau) = \langle S(\tau) S(0) \rangle = G_B^{aa}(\tau) G_B^{aa}(-\tau) \sim 1/(J\tau)^{1 \equiv \theta}$$

$$G_B^{aa}(\tau) \sim 1/(J\tau)^{1/2}$$

Very slow decay: Non-Fermi liquid of partons

Sachdev-Ye approach

SU(2) → SU(M) spins

$$H = \frac{1}{\sqrt{NM}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Solvability in the limit
$M \to \infty$!

→ Model of (constrained) 4-parton interactions

Kitaev (2015): Generalize to random 4-Majorana interactions!

$$H = \frac{6}{N^3} \sum_{i<j<k<l} J_{ijkl} \chi_i \chi_j \chi_k \chi_l.$$

→ SYK model:

- Fast scrambler: saturating bound for chaos exponent ~ $k_B T/\hbar$
- Holographically dual to a low dimensional black hole
- etc ...

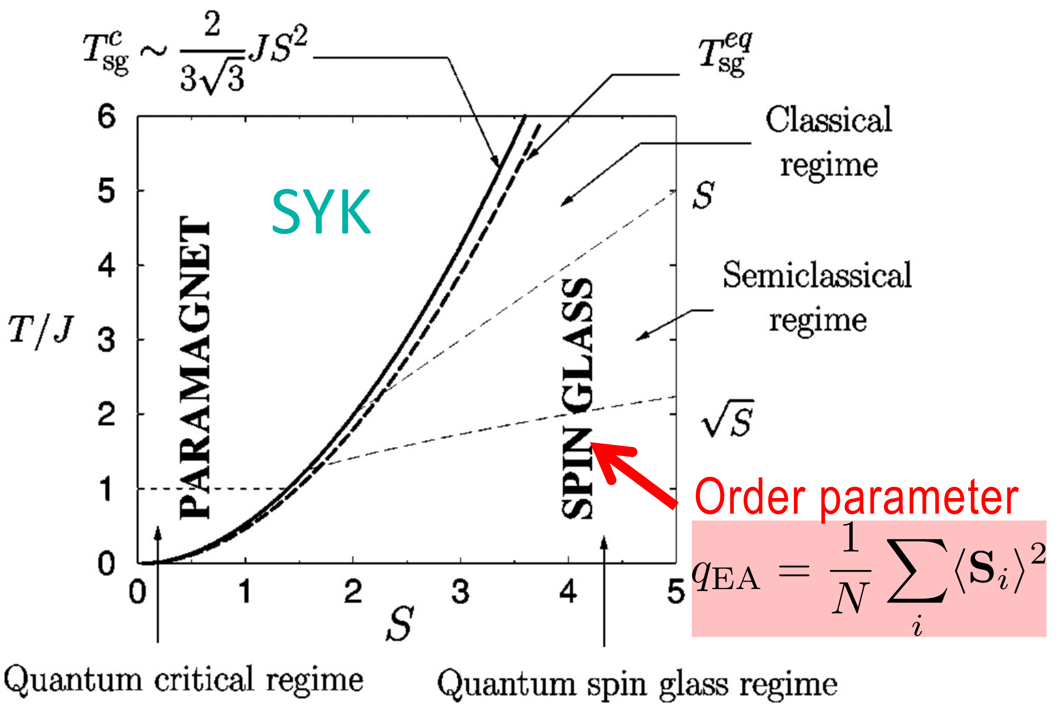
Bosonic SU(M)

Phase diagram (large M): Is there a glass transition?

in mean field ↔ replica symmetry breaking

Bosonic SU(M)

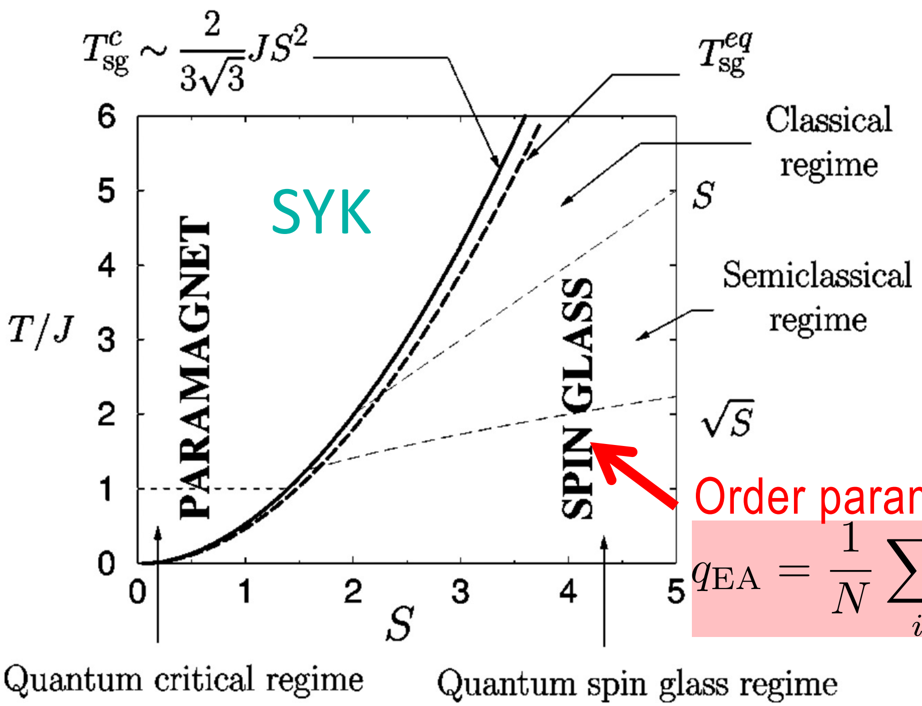
Phase diagram (large M) for bosonic representation



Glass transition: bosons condense

Bosonic SU(M)

Phase diagram (large M) for bosonic representation



Order parameter

$$q_{\rm EA} = \frac{1}{N}\sum_i \langle \mathbf{S}_i \rangle^2$$

Glass transition: one-step RSB (*dynamic* transition due to phase space clustering)

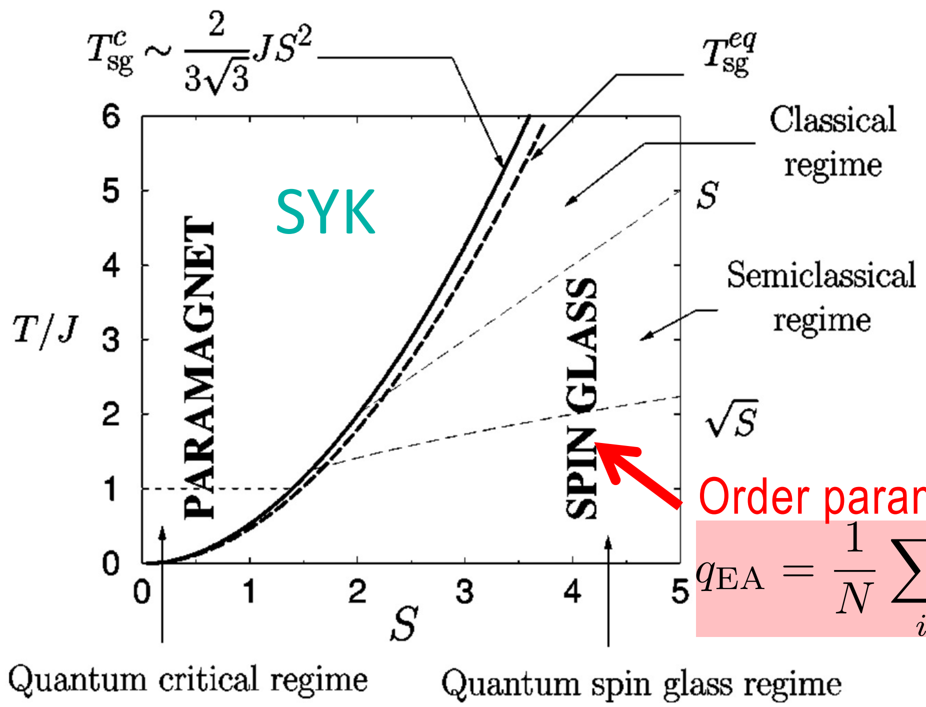
Equilibrium states: gapped; inaccessible

Marginal states: gapless

Glass transition: bosons condense

Bosonic SU(M)

Phase diagram (large M) for bosonic representation



Order parameter

$$q_{\rm EA} = \frac{1}{N}\sum_i \langle \mathbf{S}_i \rangle^2$$

Glass transition: one-step RSB (*dynamic* transition due to phase space clustering)

Equilibrium states: gapped; inaccessible

Marginal states: gapless

Glass transition: bosons condense

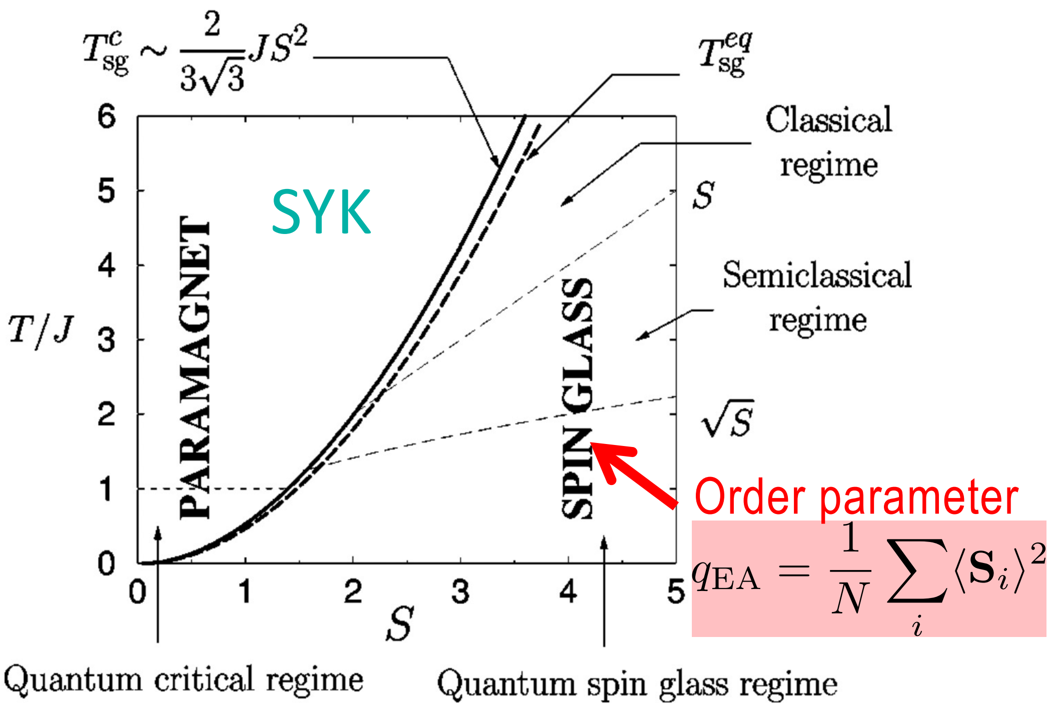
Crossover in dynamics (in gapless marginal states)

$$G_B(\tau)G_B(-\tau) \sim q_{\rm EA} \rightarrow \tau^* = (\omega^*)^{-1} = (q_{\rm EA} J)^{-1}$$

$$G^{aa}_B(\tau) \sim 1/(J\tau)^{1/2}$$

Bosonic SU(M)

Phase diagram (large M) for bosonic representation



Glass transition: one-step RSB (*dynamic* transition due to phase space clustering)

Equilibrium states: gapped; inaccessible

Marginal states: gapless

Glass transition: bosons condense

Crossover in dynamics (in gapless marginal states)

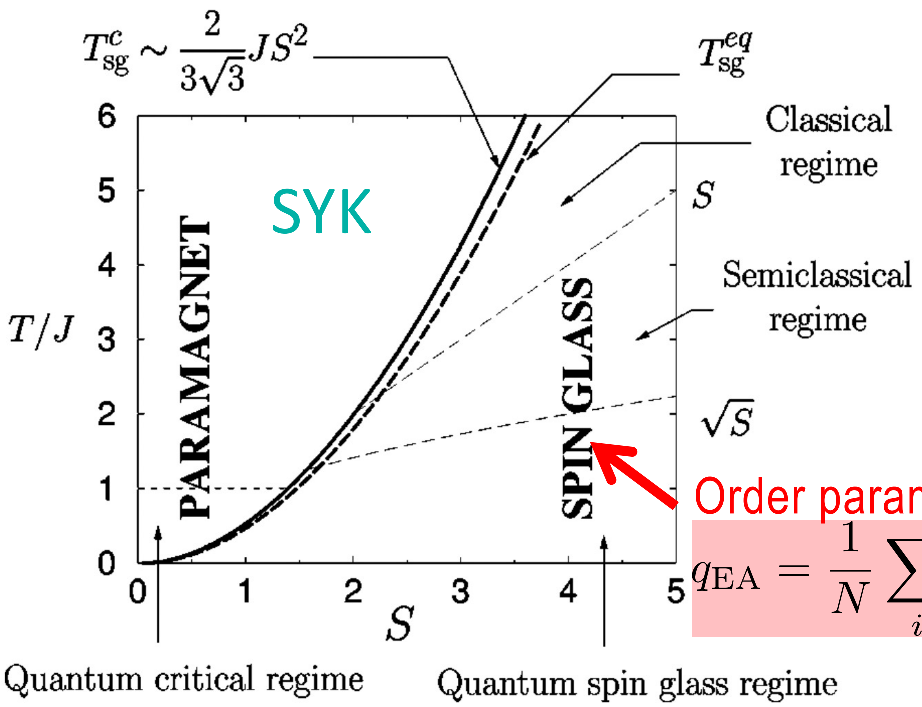
$$G_B(\tau) G_B(-\tau) \sim q_{\mathrm{EA}} \rightarrow \tau^* = (\omega^*)^{-1} = (q_{\mathrm{EA}} J)^{-1}$$

$$G_B^{aa}(\tau) \sim 1/(J\tau)^{1/2} \quad \xrightarrow{\tau > \tau^*} Q(\tau) - q_{\mathrm{EA}} \sim \tau^*/(J\tau^2) \equiv \theta$$

Faster decay typical of MF glasses

Bosonic SU(M)

Phase diagram (large M) for bosonic representation



Order parameter

$$q_{\rm EA} = \frac{1}{N} \sum_i \langle \mathbf{S}_i \rangle^2$$

Glass transition: one-step RSB (*dynamic* transition due to phase space clustering)

Equilibrium states: gapped; inaccessible

Marginal states: gapless

↔ No glass at all for fermionic representation: too strong quantum fluctuations for $M \to \infty$!

PAUL SCHERRER INSTITUT PSI

Fermionic SU(M)

Phase diagram (large M) for fermionic representation

Christos, Haehl, Sachdev, PRB 105, 085120 (2022)

1/M expansion for fermions:

Glass phase appears at

$$T_g \sim \exp[-c\sqrt{M}]$$

but with rather different properties!

↔ No glass at all for fermionic representation: too strong quantum fluctuations for $M \to \infty$!

Fermionic SU(M)

Phase diagram (large M) for fermionic representation

Christos, Haehl, Sachdev, PRB 105, 085120 (2022)

1/M expansion for fermions:

Glass phase appears at

$$T_g \sim \exp[-c\sqrt{M}]$$

but with rather different properties!

Glass transition:
Full RSB instability (continuous freezing, no extensive set of local minima)

Even equilibrium states are marginal and thus gapless

$\rightarrow$ Quantum glass is a critical phase

$\leftrightarrow$ No glass at all for fermionic representation: too strong quantum fluctuations for $M \rightarrow \infty$!

Open questions

- Does any of the large M descriptions capture SU(2)?
  And if so, how?

- Is SYK dynamics found for M = 2? (especially $\theta = 1$)

- Dynamics in the glassy phase?

  - Nature of its collective modes / spin waves?

  - Difference from Ising case
    (where interactions commute)?

PAUL SCHERRER INSTITUT

Recent progress

N. Kavokine, MM, A. Georges, O. Parcollet, PRL ***133****, 016501 (2024)*

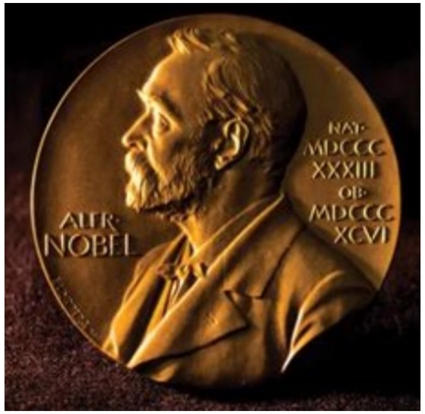
New answers due to:

- Advanced numerical tools for quantum impurity problems for SU(2) spins
 (cont. time quantum Monte Carlo without sign problem)

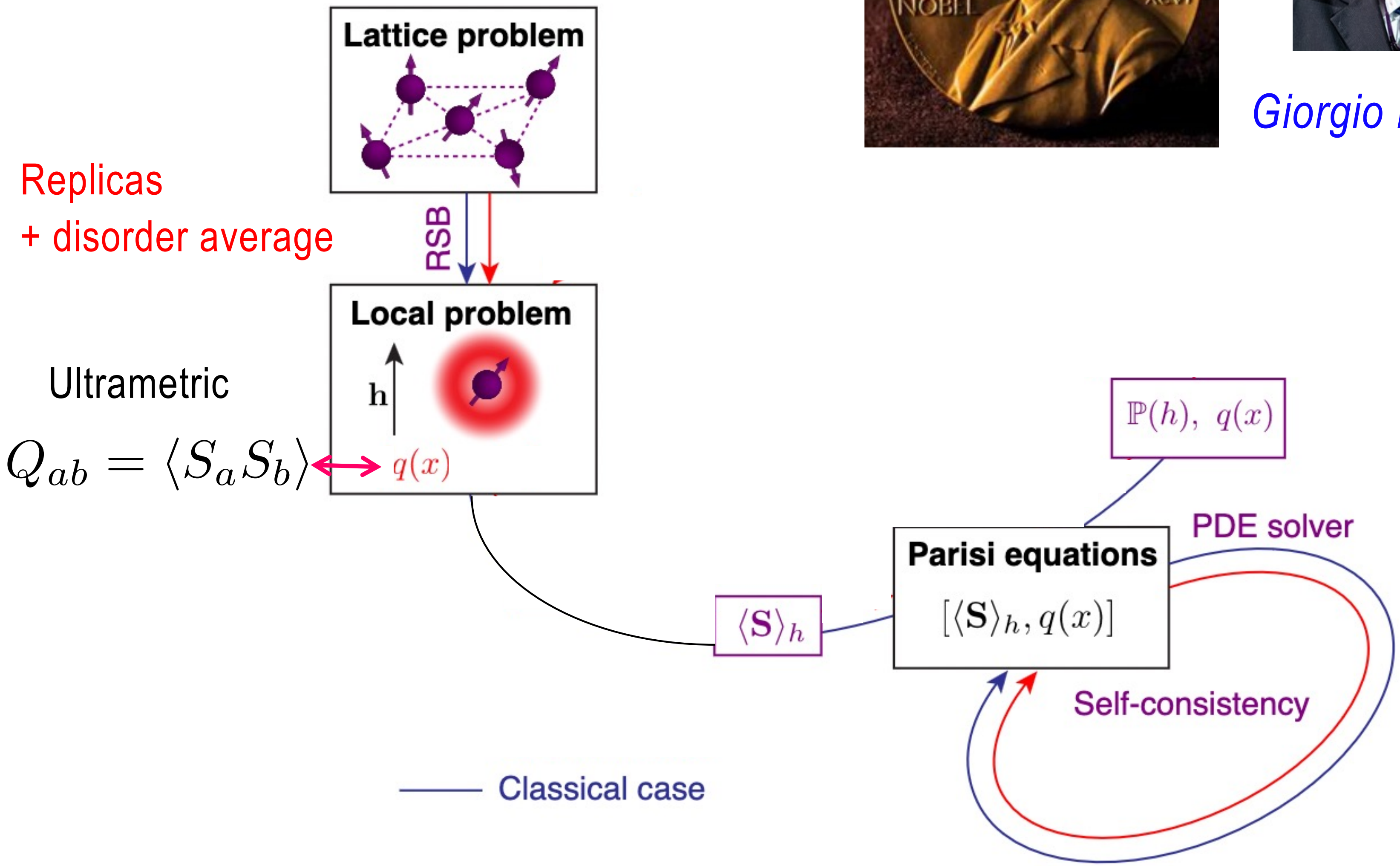
- High precision solver for replica symmetry breaking

Self-consistent solution of the MF equations

Classical glass



Giorgio Parisi, Nobel 2021



Self-consistent solution of the MF equations

Magnetization response to a frozen field

Classical glass

$$\frac{\partial \mathbf{s}}{\partial x} = -\frac{J^2}{2}\frac{\mathrm{d}q}{\mathrm{d}x}\left(\boldsymbol{\nabla}^2\mathbf{s} + 2\beta x(\mathbf{s}\cdot\boldsymbol{\nabla})\mathbf{s}\right)$$

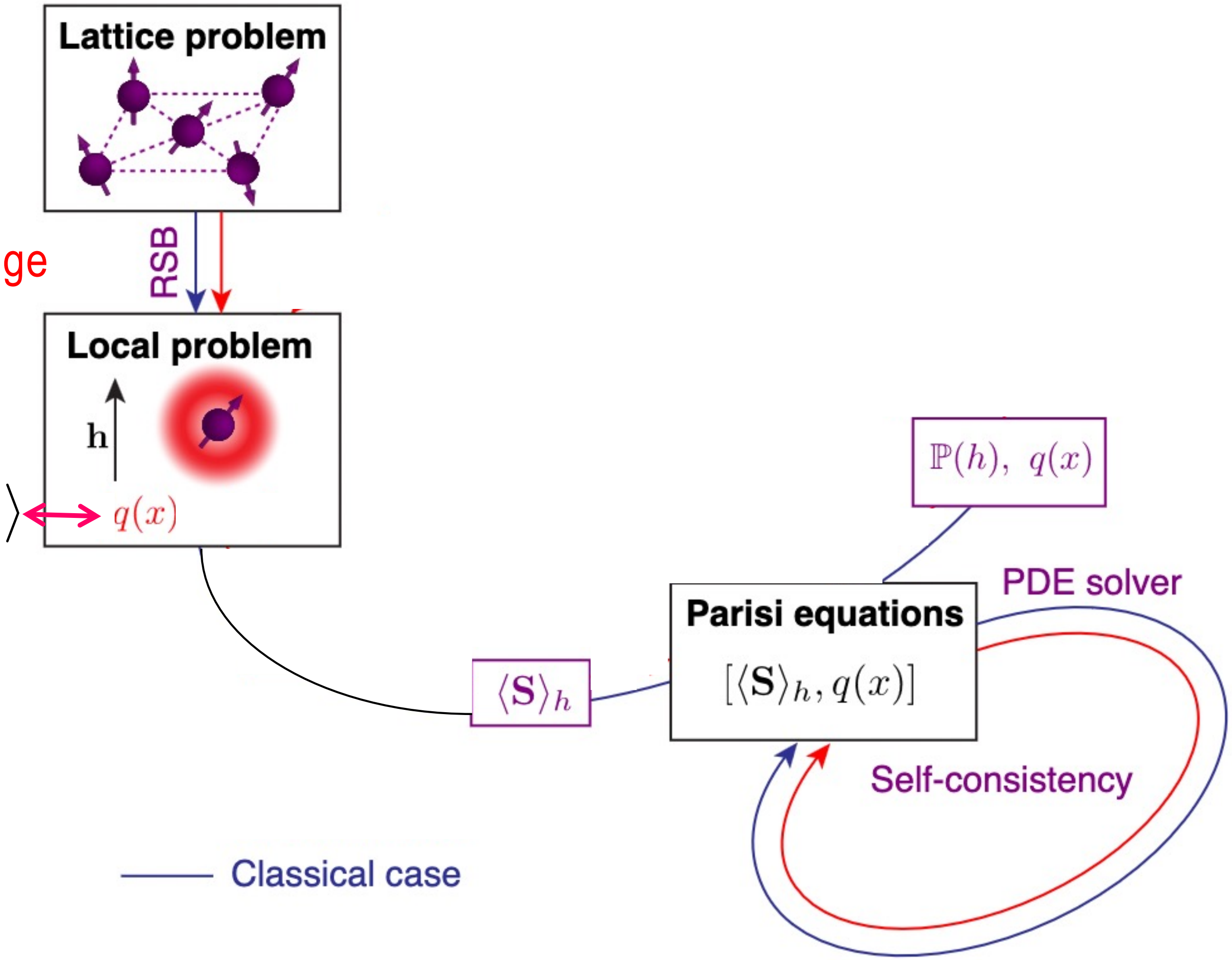
$$\mathbf{s}(1,\mathbf{h}) = \langle \mathbf{S}\rangle_{\mathcal{S}_{\mathrm{loc}}(\mathbf{h}) = \beta\mathbf{h}\cdot\mathbf{S}}$$

$0 \le x \le 1$

Replicas
+ disorder average

Ultrametric

$$Q_{ab} = \langle S_a S_b \rangle$$



Self-consistent solution of the MF equations

Classical glass

Magnetization response to a frozen field

$$\frac{\partial \mathbf{s}}{\partial x} = -\frac{J^2}{2}\frac{\mathrm{d}q}{\mathrm{d}x}\left(\boldsymbol{\nabla}^2\mathbf{s} + 2\beta x(\mathbf{s}\cdot\boldsymbol{\nabla})\mathbf{s}\right)$$

$$\mathbf{s}(1,\mathbf{h}) = \langle \mathbf{S} \rangle_{\mathcal{S}_{\mathrm{loc}}(\mathbf{h}) = \beta \mathbf{h}\cdot\mathbf{S}}$$

$0 \leq x \leq 1$

Distribution of frozen fields

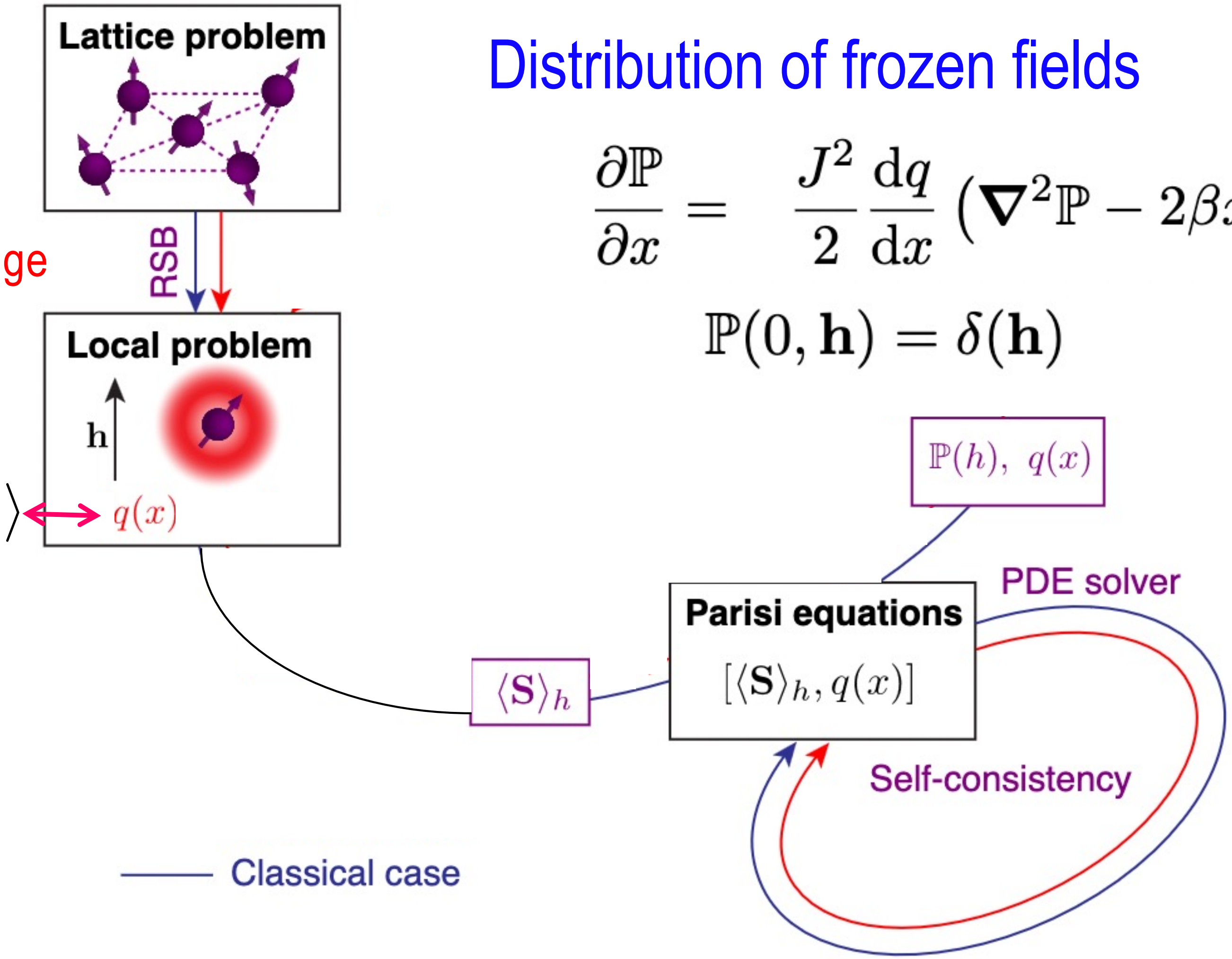
$$\frac{\partial \mathbb{P}}{\partial x} = \frac{J^2}{2}\frac{\mathrm{d}q}{\mathrm{d}x}\left(\boldsymbol{\nabla}^2\mathbb{P} - 2\beta x\boldsymbol{\nabla}(\mathbf{s}\cdot\mathbb{P})\right)$$

$$\mathbb{P}(0,\mathbf{h}) = \delta(\mathbf{h})$$

Replicas
+ disorder average

Ultrametric

$$Q_{ab} = \langle S_a S_b \rangle$$



Self-consistent solution of the MF equations

Quantum glass

Extra complication: self-consistent *dynamic* susceptibility $\chi(\tau)$ within every metastable state

$$\mathcal{S}_{\mathrm{loc}}(\mathbf{h},\chi) = \frac{J^2}{2}\iint_0^\beta \mathrm{d}\tau\mathrm{d}\tau'\chi(\tau-\tau')\mathbf{S}(\tau)\cdot\mathbf{S}(\tau') - \mathbf{h}\int_0^\beta \mathrm{d}\tau\,\mathbf{S}(\tau),$$

$$\mathbf{s}(1,\mathbf{h}) = \langle\mathbf{S}\rangle_{\mathcal{S}_{\mathrm{loc}}(\mathbf{h},\chi)}$$

$$\chi(\tau) = \frac{1}{\ell}\int \mathrm{d}\mathbf{h}\,\mathbb{P}(\mathbf{h})\,\langle\mathbf{S}(0)\mathbf{S}(\tau)\rangle_{\mathcal{S}_{\mathrm{loc}}(\mathbf{h},\chi)} - q(1)$$

Observables computed with continuous time Quantum Monte Carlo (CT-QMC)

Self-consistent solution of the MF equations

Quantum glass

Extra complication: self-consistent *dynamic* susceptibility $\chi(\tau)$ within every metastable state

$$\mathcal{S}_{\rm loc}(\mathbf{h},\chi) = \frac{J^2}{2}\iint_0^\beta \mathrm{d}\tau\mathrm{d}\tau'\chi(\tau-\tau')\mathbf{S}(\tau)\cdot\mathbf{S}(\tau') - \mathbf{h}\int_0^\beta \mathrm{d}\tau\,\mathbf{S}(\tau),$$

$$\mathbf{s}(1,\mathbf{h}) = \langle\mathbf{S}\rangle_{\mathcal{S}_{\rm loc}(\mathbf{h},\chi)}$$

$$\chi(\tau) = \frac{1}{\ell}\int \mathrm{d}\mathbf{h}\,\mathbb{P}(\mathbf{h})\,\langle\mathbf{S}(0)\mathbf{S}(\tau)\rangle_{\mathcal{S}_{\rm loc}(\mathbf{h},\chi)} - q(1)$$

Power law interactions in time!

*cf. Grempel and Rozenberg, PRL **80**, 389 (1998) for paramagnetic regime*

Observables computed with continuous time Quantum Monte Carlo (CT-QMC)

Self-consistent solution of the MF equations

Quantum glass

Extra complication: self-consistent *dynamic* susceptibility $\chi(\tau)$ within every metastable state

$$\mathcal{S}_{\rm loc}(\mathbf{h},\chi) = \frac{J^2}{2}\iint_0^\beta \mathrm{d}\tau\mathrm{d}\tau'\chi(\tau-\tau')\mathbf{S}(\tau)\cdot\mathbf{S}(\tau') - \mathbf{h}\int_0^\beta \mathrm{d}\tau\,\mathbf{S}(\tau),$$

Power law interactions in time!

$$\mathbf{s}(1,\mathbf{h}) = \langle\mathbf{S}\rangle_{\mathcal{S}_{\rm loc}(\mathbf{h},\chi)}$$

$$\chi(\tau) = \frac{1}{\ell}\int \mathrm{d}\mathbf{h}\,\mathbb{P}(\mathbf{h})\,\langle\mathbf{S}(0)\mathbf{S}(\tau)\rangle_{\mathcal{S}_{\rm loc}(\mathbf{h},\chi)} - q(1)$$

Observables computed with continuous time Quantum Monte Carlo (CT-QMC)

*cf. Grempel and Rozenberg, PRL **80**, 389 (1998)* for *paramagnetic regime*

Similar technique developped previously for other quantum glasses:

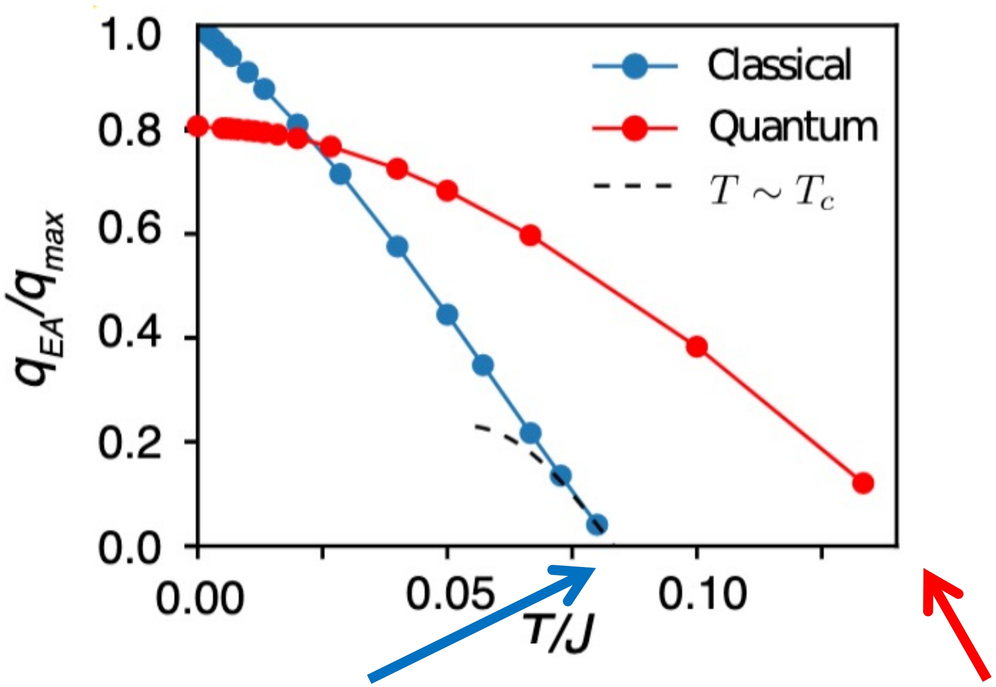
- Bethe lattice quantum Coulomb glass *I. Lovas et al., Phys. Rev. Res. **4**, 023067, 2022*
- Transverse field Ising model, *A. Kiss, et al., Phys. Rev. B **109**, 024431, 2024*

Results

1. Glass transition and Edwards-Anderson order parameter

Continuous transition, with *continuous* replica symmetry breaking

$$q_{\mathrm{EA}} = \frac{1}{N}\sum_i \langle \mathbf{S}_i \rangle^2$$

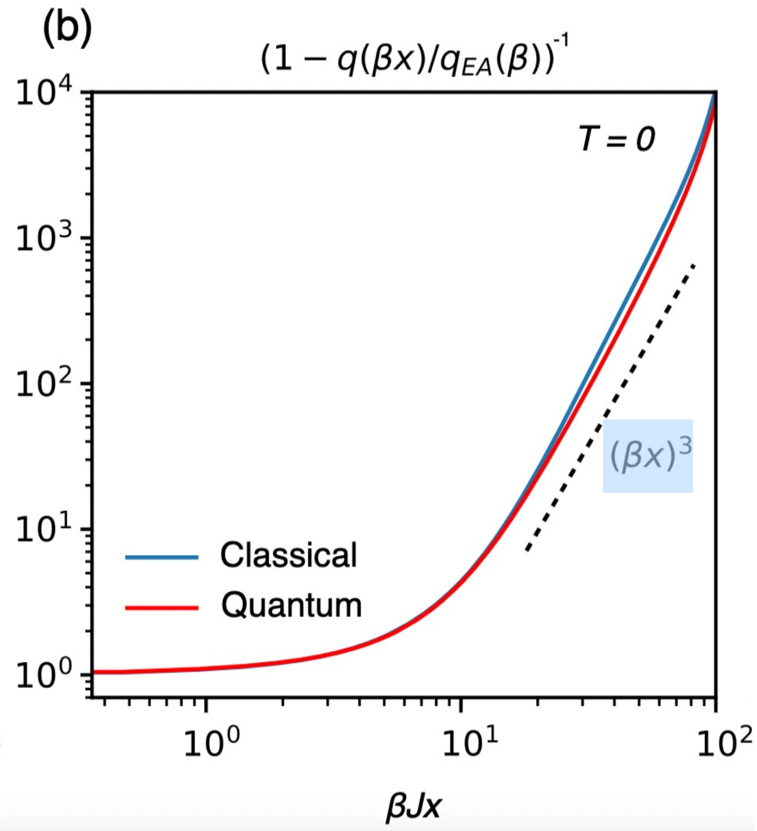


$$T_{\mathrm{g}} = JS^2/3$$

$$T_{\mathrm{g}} \approx J\langle \mathbf{S}^2 \rangle / 3\sqrt{3}$$

Results

2. Structure of replica symmetry breaking:



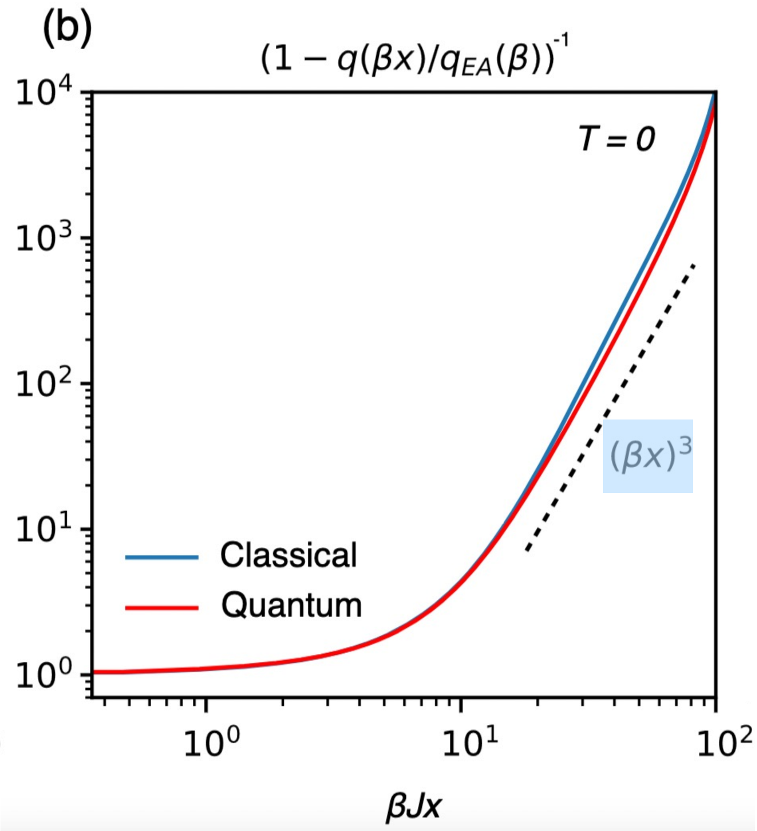
Approach to the plateau $q_{\rm EA} = q(x \to 1)$

$$1 - q(x)/q_{\rm EA} \sim 1/(\beta x)^{\alpha}$$

$\alpha \approx 3 (= n)$ Heisenberg

Results

2. Structure of replica symmetry breaking:



Approach to the plateau $q_{\mathrm{EA}} = q(x \to 1)$

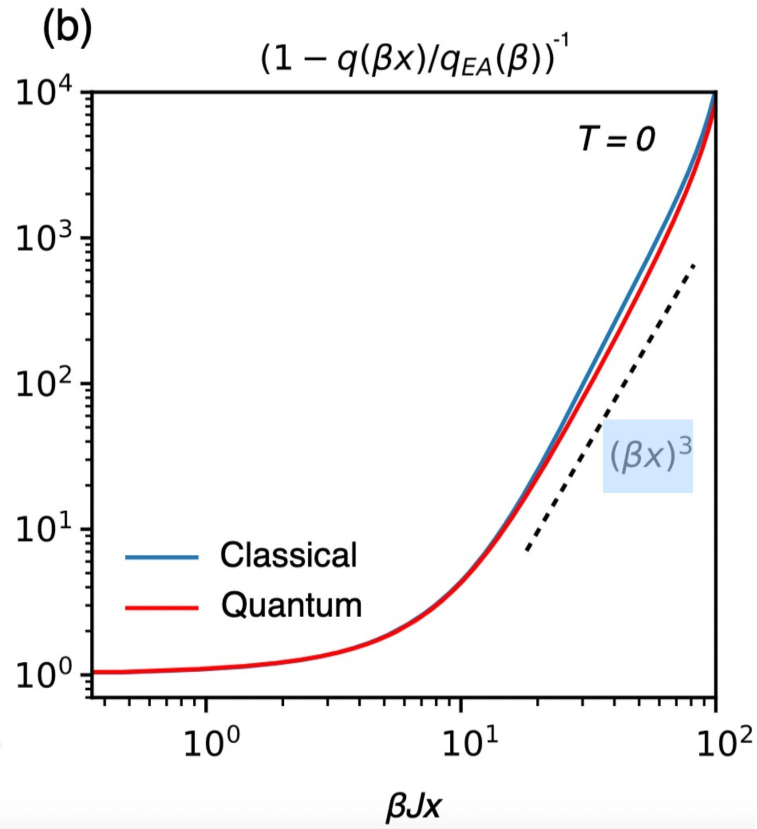
$$1 - \tilde{q}(x)/q_{\mathrm{EA}} \sim 1/(\beta x)^{\alpha}$$

$\alpha \approx 3 (= n)$ Heisenberg

↔

$\alpha = 2$ (for $n = 1$) Ising SK

2. Structure of replica symmetry breaking:



Approach to the plateau $q_{\mathrm{EA}} = q(x \to 1)$

$$1 - q(x)/q_{\mathrm{EA}} \sim 1/(\beta x)^{\alpha}$$

$\alpha \approx 3 (= n)$ Heisenberg

↔

$\alpha = 2$ (for $n = 1$) Ising SK

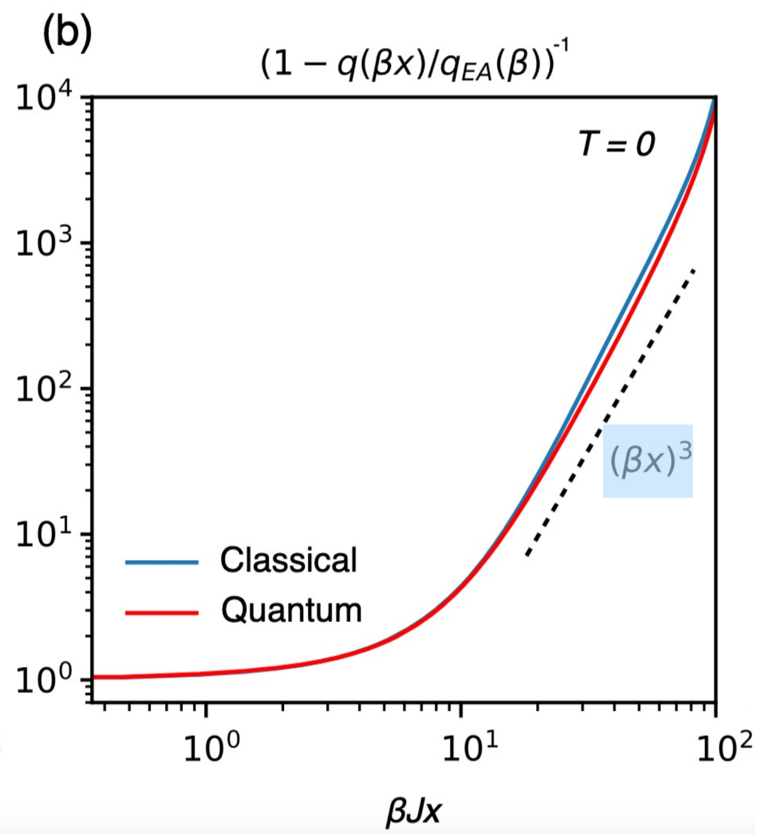
captures distribution of low energy states, and hence controls jumps Δm of magnetization in a field ramp

$$\rho(\Delta m) \sim 1/(\Delta m)^{2/\alpha}$$

Le Doussal, MM, Wiese '10

Results

2. Structure of replica symmetry breaking:



Approach to the plateau $q_{\rm EA} = q(x \to 1)$

$$1 - q(x)/q_{\rm EA} \sim 1/(\beta x)^{\alpha}$$

$\alpha \approx 3 (= n)$ Heisenberg

↔

$\alpha = 2$ (for $n = 1$) Ising SK

captures distribution of low energy states, and hence controls jumps Δm of magnetization in a field ramp

$$\rho(\Delta m) \sim 1/(\Delta m)^{2/\alpha}$$

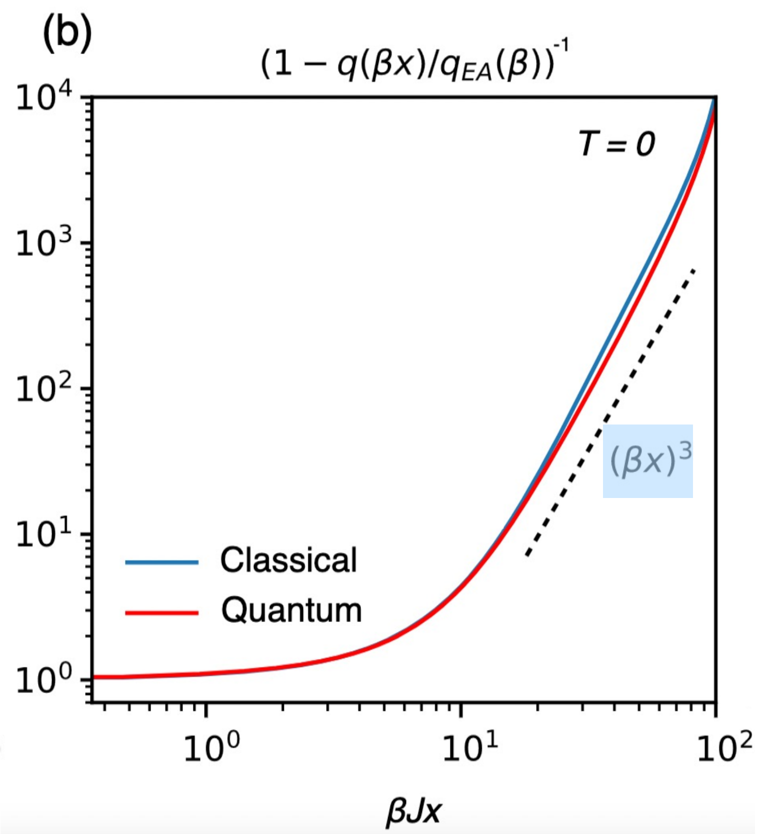
Le Doussal, MM, Wiese '10

Vector spins ($n = \alpha > 2$): jumps **dominated by large avalanches**

Ising spins ($\alpha = 2$): the jumps have a critical power law

Results

2. Structure of replica symmetry breaking:



Approach to the plateau $q_{\rm EA} = q(x \to 1)$

$$1 - q(x)/q_{\rm EA} \sim 1/(\beta x)^{\alpha}$$

$\alpha \approx 3 (= n)$ Heisenberg

↔

$\alpha = 2$ (for $n = 1$) Ising SK

captures distribution of low energy states, and hence controls jumps Δm of magnetization in a field ramp

$$\rho(\Delta m) \sim 1/(\Delta m)^{2/\alpha}$$

Le Doussal, MM, Wiese '10

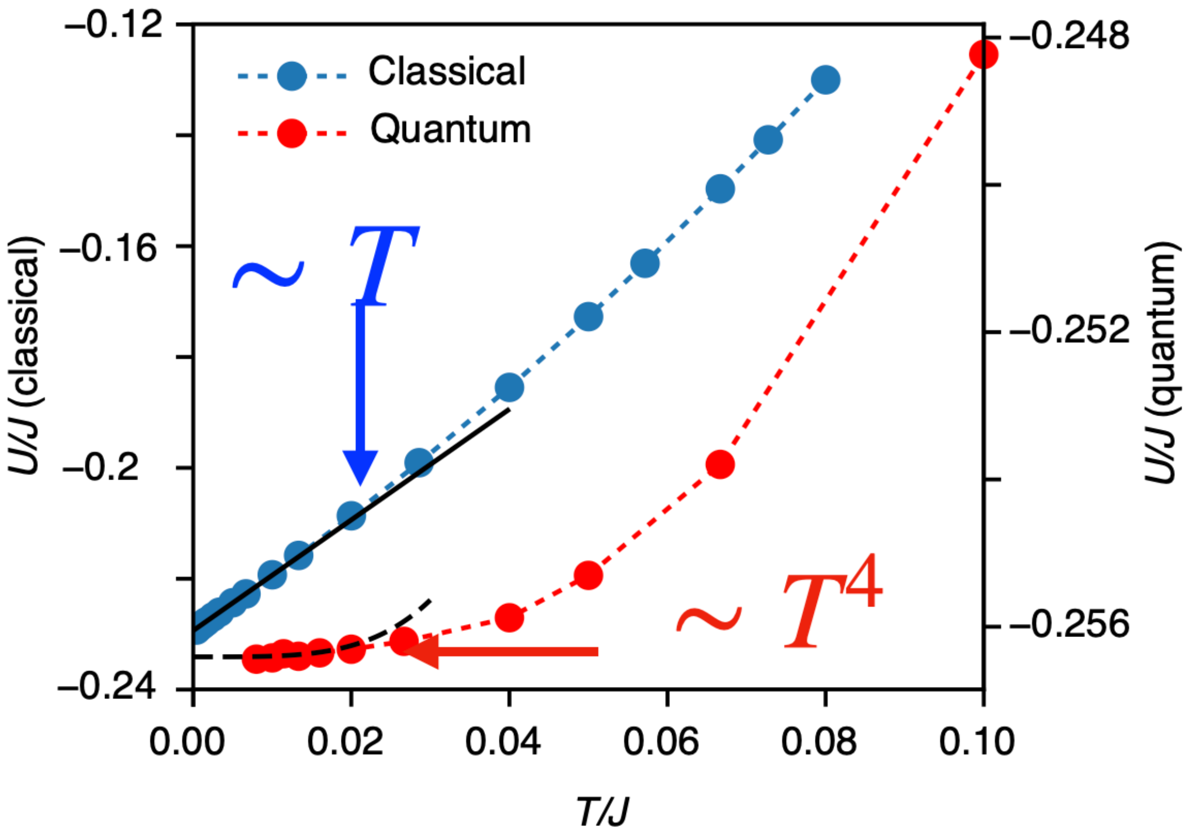
Vector spins ($n = \alpha > 2$): jumps **dominated by large avalanches** *Andreanov, Sharma, MM '14*

Ising spins ($\alpha = 2$): the jumps have a critical power law *Pazmandi, Zarand, Zimanyi '99*

↔ Avalanche statistics in field ramps at T=0 : Ising spins *differ* from vector spins!

Results

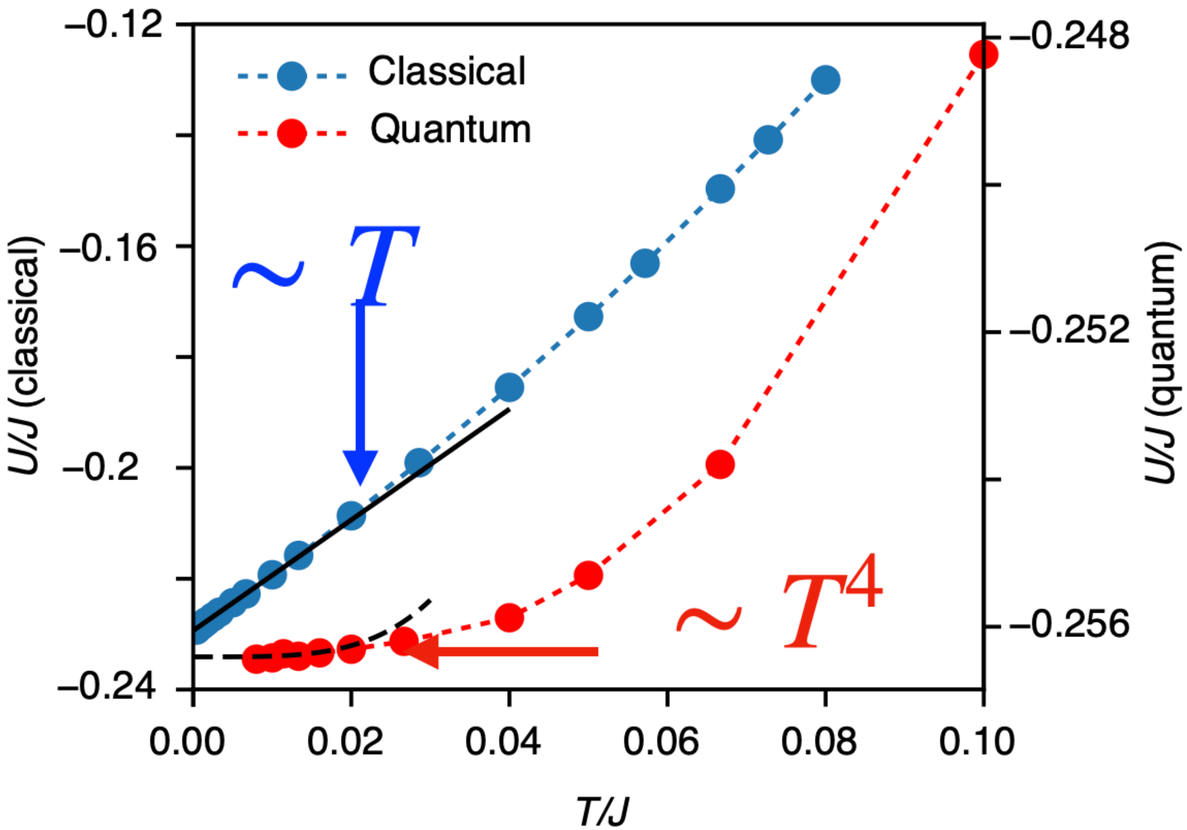
3. Thermodynamics: internal energy U and specific heat C_V



Classical: intrastate specific heat: $C_V \approx 1$ (Du-Long Petit)

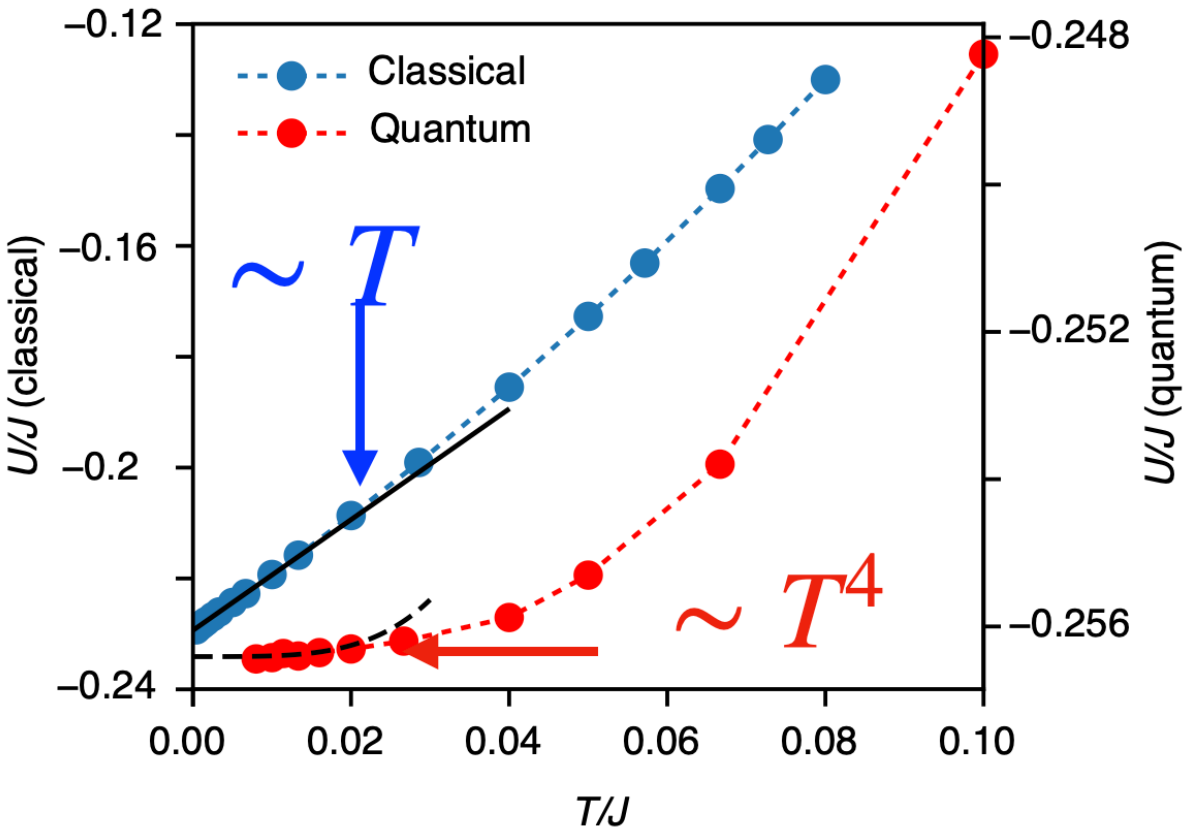
Results

3. Thermodynamics: internal energy U and specific heat C_V



Classical: intrastate specific heat: $C_V \approx 1$ (Du-Long Petit)

3. Thermodynamics: internal energy U and specific heat C_V



Quantum fluctuations harden the spin waves

Compatible with: $C_V \sim T^3$ ⟷ Like in *marginal* states of solvable one-step RSB glasses (*G. Schehr '04,'05*)

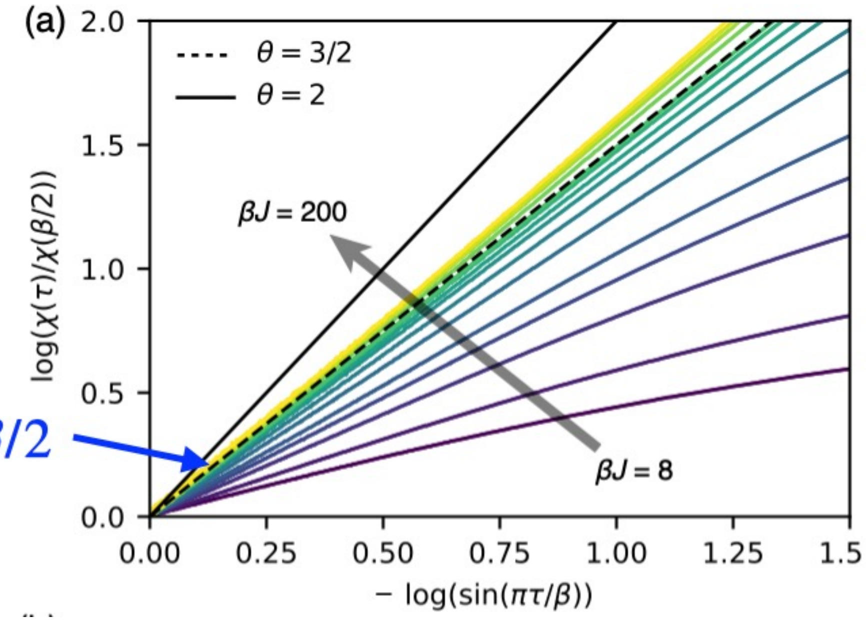
4. Dynamic susceptibility

$$\chi(\tau) = \overline{\langle \mathbf{S}_i(\tau)\mathbf{S}_i(0)\rangle_c}$$

- Power law at low energy / long time τ, cut off at $\frac{1}{\tau} \sim T$
- Fit to conformal form:

$$\chi(\tau) \approx \chi(\beta/2)\left(\frac{1}{\sin(\pi\tau/\beta)}\right)^{\theta(T)}$$

$\tau \sim \beta/2$



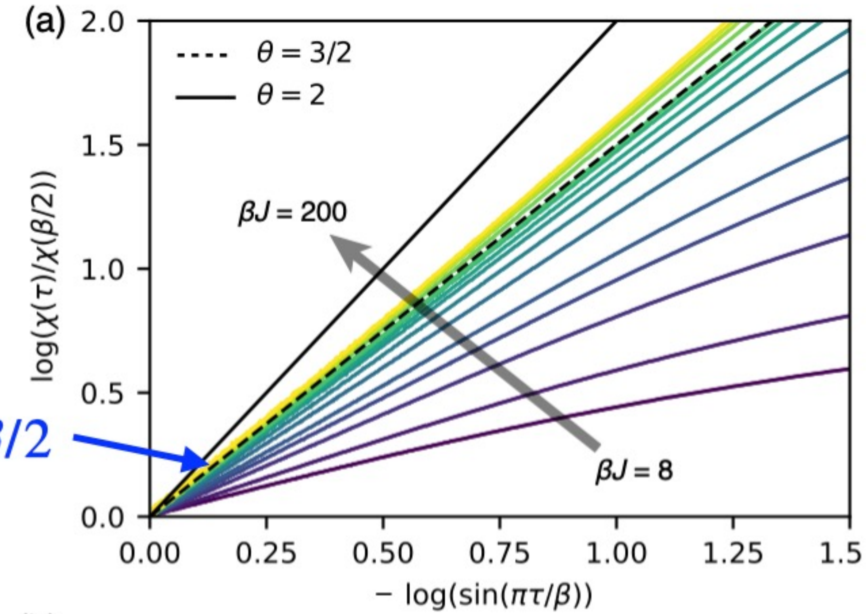
Results

4. Dynamic susceptibility $\chi(\tau) = \overline{\langle \mathbf{S}_i(\tau)\mathbf{S}_i(0)\rangle_c}$

- Power law at low energy / long time τ, cut off at $\frac{1}{\tau} \sim T$
- Fit to conformal form:

$$\chi(\tau) \approx \chi(\beta/2)\left(\frac{1}{\sin(\pi\tau/\beta)}\right)^{\theta(T)}$$

$\tau \sim \beta/2$



- Expect for insulating glasses $\theta_{T=0} = 2$
  Read, Sachdev, Ye ('95) [Landau exp]
  Christos, Haehl, Sachdev ('22) [M≫1]

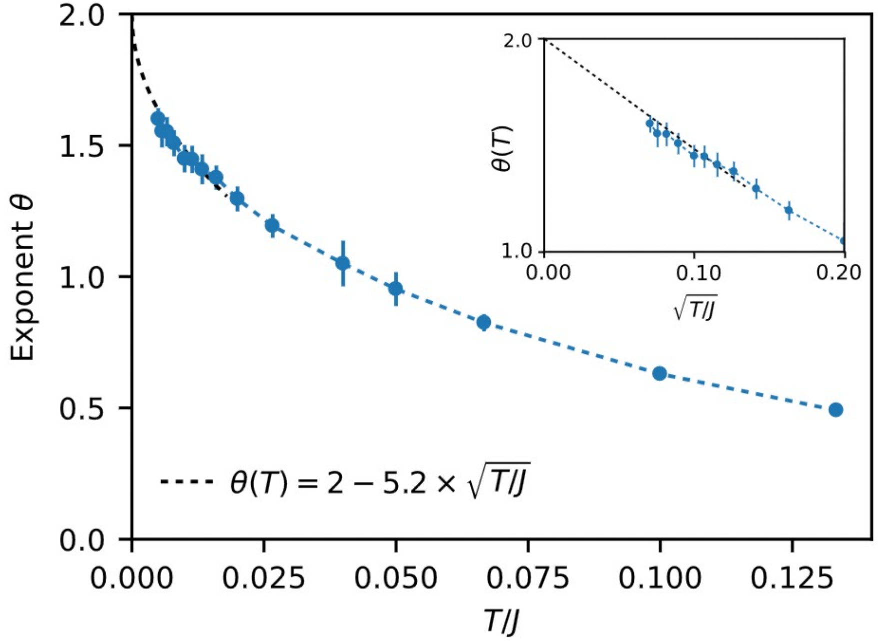
Results

4. Dynamic susceptibility $\chi(\tau) = \overline{\langle \mathbf{S}_i(\tau)\mathbf{S}_i(0)\rangle_c}$

- Power law at low energy / long time τ, cut off at $\frac{1}{\tau} \sim T$
- Fit to conformal form:

$$\chi(\tau) \approx \chi(\beta/2)\left(\frac{1}{\sin(\pi\tau/\beta)}\right)^{\theta(T)}$$

- Expect for insulating glasses $\theta_{T=0} = 2$
  Read, Sachdev, Ye ('95) [Landau exp]
  Christos, Haehl, Sachdev ('22) [M≫1]



Very slow creep toward $\theta_{T=0} = 2$
Similar as in the Quantum *Ising* SK!

Results

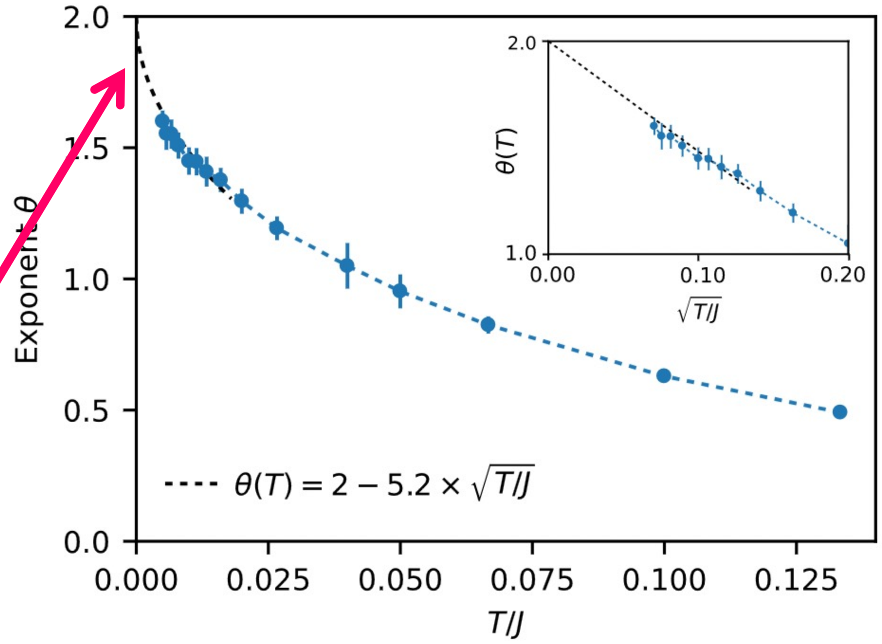
4. Dynamic susceptibility

$$\chi(\tau) = \overline{\langle \mathbf{S}_i(\tau)\mathbf{S}_i(0)\rangle_c}$$

- Power law at low energy / long time τ, cut off at $\frac{1}{\tau} \sim T$
- Fit to conformal form:

$$\chi(\tau) \approx \chi(\beta/2)\left(\frac{1}{\sin(\pi\tau/\beta)}\right)^{\theta(T)}$$

- Expect for insulating glasses $\theta_{T=0} = 2$
  Read, Sachdev, Ye ('95) [Landau exp]
  Christos, Haehl, Sachdev ('22) [M≫1]



Very slow creep toward $\theta_{T=0} = 2$
Similar as in the Quantum *Ising* SK!

Compatible with $\chi''(\omega) \sim \omega^{1=\theta-1}$ ⟷ Like in *marginal* states of solvable 1-step RSB glasses (*G. Schehr '04,'05*)

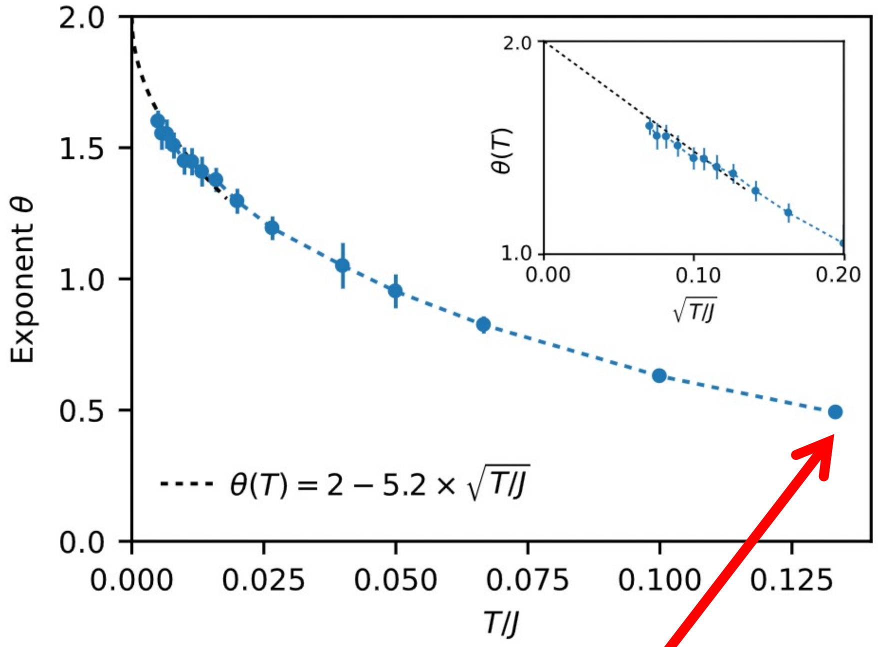
Results

4. Dynamic susceptibility $\chi(\tau) = \overline{\langle \mathbf{S}_i(\tau)\mathbf{S}_i(0)\rangle_c}$

- Power law at low energy / long time τ, cut off at $\frac{1}{\tau} \sim T$
- Fit to conformal form:

$$\chi(\tau) \approx \chi(\beta/2)\left(\frac{1}{\sin(\pi\tau/\beta)}\right)^{\theta(T)}$$

- Expect for insulating glasses $\theta_{T=0} = 2$
  Read, Sachdev, Ye ('95) [Landau exp]
  Christos, Haehl, Sachdev ('22) [M≫1]



Very slow creep toward $\theta_{T=0} = 2$
Similar as in the Quantum *Ising* SK!

Note: no trace of the SY-K dynamics $(\theta = 1)$ above $T_c \sim 0.14J$

Interpreting dynamics and specific heat

PAUL SCHERRER INSTITUT

Interpreting dynamics and specific heat

- What are the collective modes/spin waves of a quantum spin glass?

A wide open question in finite dimensions!

Mean field: some physical insight is possible

Interpreting dynamics and specific heat

- What are the collective modes/spin waves of a quantum spin glass?

- What hides behind the super-universal forms of dynamics

$$\chi(\tau) = \overline{\langle \mathbf{S}_i(\tau)\mathbf{S}_i(0)\rangle_c} \sim \frac{1}{\tau^2}$$

$\chi''(\omega) \sim \omega$ (Ohmic spectral function)

and the specific heat scaling

$$C_V \sim T^3$$

found in so many (marginal) states of mean field glasses?

Interpreting dynamics and specific heat

- What are the collective modes/spin waves of a quantum spin glass?

- What hides behind the super-universal forms of dynamics

$$\chi(\tau) = \overline{\langle \mathbf{S}_i(\tau)\mathbf{S}_i(0)\rangle_c} \sim \frac{1}{\tau^2}$$

$$\chi''(\omega) \sim \omega \qquad \text{(Ohmic spectral function)}$$

and the specific heat scaling

$$C_V \sim T^3$$

found in so many (marginal) states of mean field glasses?

For concreteness and non-trivial predictions:
Consider transverse field Ising SK model
as representative of insulating (non-metallic) spin glasses

PAUL SCHERRER INSTITUT

Quantum long range spin glasses: Quantum SK model

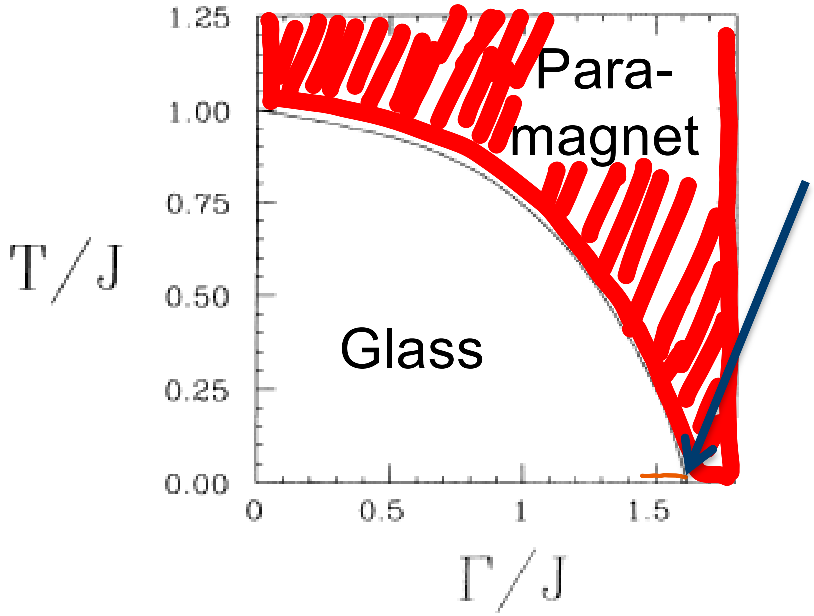
How does marginal stability affect quantum dynamics?
Collective modes?

$$H = -\Gamma \sum_i s_i^x - \sum_{i,j} J_{i,j} s_i^z s_j^z$$

Quantum long range spin glasses: Quantum SK model

How does marginal stability affect quantum dynamics?
Collective modes?

$$H = -\Gamma \sum_i s_i^x - \sum_{i,j} J_{i,j} s_i^z s_j^z$$



Spectral gap closes *(Miller, Huse PRL 1993)*

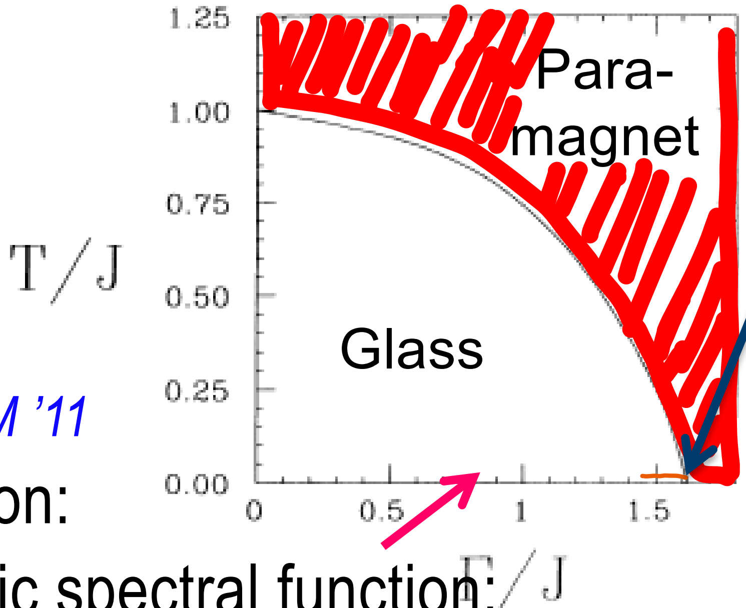
& remains closed in the glass phase!

Read, Sachdev, Ye, PRL (1993)

Quantum long range spin glasses: Quantum SK model

How does marginal stability affect quantum dynamics?
Collective modes?

$$H = -\Gamma \sum_i s_i^x - \sum_{i,j} J_{i,j} s_i^z s_j^z$$



A. Andreanov, MM '11

Spectral gap closes *(Miller, Huse PRL 1993)*

& **remains closed** in the glass phase!

Read, Sachdev, Ye, PRL (1993)

Replica solution:

Gapless Ohmic spectral function:

$$\chi''(\omega \to 0) \equiv \frac{1}{\pi N} \sum_i \mathrm{Im} \langle s_i^z(\omega) s_i^z(-\omega) \rangle |_{\omega \to \omega - i\delta} = \frac{B\omega}{\pi}$$

Quantum SK model

A. Andreanov, MM '11
L. Cugliandolo, MM '23 (Review on Quantum glasses)

Physical interpretation: [applies to ALL insulating meanfield glasses]

cf P. Urbani's talk !

Quantum SK model

A. Andreanov, MM '11
L. Cugliandolo, MM '23 (Review on Quantum glasses)

Physical interpretation: [applies to ALL insulating meanfield glasses]

cf P. Urbani's talk !

Marginally stable energy landscape $G(\{m_i\})$

Minima: gapless semicircular spectrum of Hessian $\mathcal{H}_{ij} = \frac{\delta^2 G}{\delta m_i \delta m_j}$

$$\rho(\lambda) \sim \frac{\sqrt{\lambda \Gamma}}{J^2}$$

Quantum SK model

A. Andreanov, MM '11
L. Cugliandolo, MM '23 (Review on Quantum glasses)

Physical interpretation: [applies to ALL insulating meanfield glasses]

cf P. Urbani's talk !

Marginally stable energy landscape $G(\{m_i\})$

Minima: gapless semicircular spectrum of Hessian $\mathcal{H}_{ij} = \frac{\delta^2 G}{\delta m_i \delta m_j}$

$$\rho(\lambda) \sim \frac{\sqrt{\lambda\Gamma}}{J^2}$$

→ Collective harmonic oscillators of mass $M \sim \Gamma^{-1}$ $\quad \omega = \sqrt{\lambda/M}$

$$\to \rho(\omega) \sim \frac{\omega^2}{\Gamma J^2}$$

PAUL SCHERRER INSTITUT PSI

Quantum SK model

A. Andreanov, MM '11
L. Cugliandolo, MM '23 (Review on Quantum glasses)

Physical interpretation: [applies to ALL insulating meanfield glasses]

cf P. Urbani's talk !

Marginally stable energy landscape $G(\{m_i\})$
Minima: gapless semicircular spectrum of Hessian $\mathcal{H}_{ij} = \frac{\delta^2 G}{\delta m_i \delta m_j}$

$$\rho(\lambda) \sim \frac{\sqrt{\lambda \Gamma}}{J^2}$$

→ Collective harmonic oscillators of mass $M \sim \Gamma^{-1}$ $\omega = \sqrt{\lambda / M}$

$$\rightarrow \rho(\omega) \sim \frac{\omega^2}{\Gamma J^2}$$

$$U = \int_0^\infty d\omega \rho(\omega) \frac{\hbar\omega}{e^{\hbar\omega/T} - 1} \sim T^4 \longrightarrow \boxed{C_V \sim T^3}$$

Quantum SK model

A. Andreanov, MM '11
L. Cugliandolo, MM '23 (Review on Quantum glasses)

Physical interpretation: [applies to ALL insulating meanfield glasses]

cf P. Urbani's talk !

Marginally stable energy landscape $G(\{m_i\})$
Minima: gapless semicircular spectrum of Hessian $\mathcal{H}_{ij} = \frac{\delta^2 G}{\delta m_i \delta m_j}$

$$\rho(\lambda) \sim \frac{\sqrt{\lambda \Gamma}}{J^2}$$

→ Collective harmonic oscillators of mass $M \sim \Gamma^{-1}$ $\omega = \sqrt{\lambda/M}$

$$\to \rho(\omega) \sim \frac{\omega^2}{\Gamma J^2}$$

Gapless spectral function

$$\chi''(\omega) \sim x_\omega^2 \rho(\omega)$$

PAUL SCHERRER INSTITUT PSI

Quantum SK model

A. Andreanov, MM '11
L. Cugliandolo, MM '23 (Review on Quantum glasses)

Physical interpretation: [applies to ALL insulating meanfield glasses]

cf P. Urbani's talk !

Marginally stable energy landscape $G(\{m_i\})$

Minima: gapless semicircular spectrum of Hessian $\mathcal{H}_{ij} = \frac{\delta^2 G}{\delta m_i \delta m_j}$

$$\rho(\lambda) \sim \frac{\sqrt{\lambda\Gamma}}{J^2}$$

→ Collective harmonic oscillators of mass $M \sim \Gamma^{-1}$ $\omega = \sqrt{\lambda/M}$

$$\to \rho(\omega) \sim \frac{\omega^2}{\Gamma J^2}$$

$$\langle x_\omega^2 \rangle = (M\omega)^{-1}$$

Gapless spectral function

$$\chi''(\omega) \sim x_\omega^2 \rho(\omega)$$

PAUL SCHERRER INSTITUT PSI

Quantum SK model

A. Andreanov, MM '11
L. Cugliandolo, MM '23 (Review on Quantum glasses)

Physical interpretation: [applies to ALL insulating meanfield glasses]

cf P. Urbani's talk !

Marginally stable energy landscape $G(\{m_i\})$
Minima: gapless semicircular spectrum of Hessian $\mathcal{H}_{ij} = \frac{\delta^2 G}{\delta m_i \delta m_j}$

$$\rho(\lambda) \sim \frac{\sqrt{\lambda \Gamma}}{J^2}$$

→ Collective harmonic oscillators of mass $M \sim \Gamma^{-1}$ $\omega = \sqrt{\lambda/M}$

$$\rightarrow \rho(\omega) \sim \frac{\omega^2}{\Gamma J^2}$$

$$\langle x_\omega^2 \rangle = (M\omega)^{-1}$$

Gapless spectral function

$$\chi''(\omega) \sim x_\omega^2 \rho(\omega) \sim \frac{\Gamma}{\omega} \frac{\omega^2}{\Gamma J^2} \sim \frac{\omega}{J^2}$$

Non-trivial check: Independent of Q-fluctuation strength Γ!!

PAUL SCHERRER INSTITUT PSI

Quantum SK model

A. Andreanov, MM '11
L. Cugliandolo, MM '23 (Review on Quantum glasses)

Physical interpretation: [applies to ALL insulating meanfield glasses]

cf P. Urbani's talk !

Marginally stable energy landscape: $G(\{m_i\})$

Minima: gapless semicircular spectrum of Hessian $\mathcal{H}_{ij} = \frac{\delta^2 G}{\delta m_i \delta m_j}$

$$\rho(\lambda) \sim \frac{\sqrt{\lambda \Gamma}}{J^2}$$

→ Collective harmonic oscillators of mass $M \sim \Gamma^{-1}$ $\omega = \sqrt{\lambda / M}$

$\langle x_\omega^2 \rangle = (M\omega)^{-1}$

$$\rightarrow \rho(\omega) \sim \frac{\omega^2}{\Gamma J^2}$$

Gapless spectral function from solving RSB:

$$\chi''_{\mathrm{Ising}}(\omega) \approx 0.5 \frac{\omega}{J^2}$$

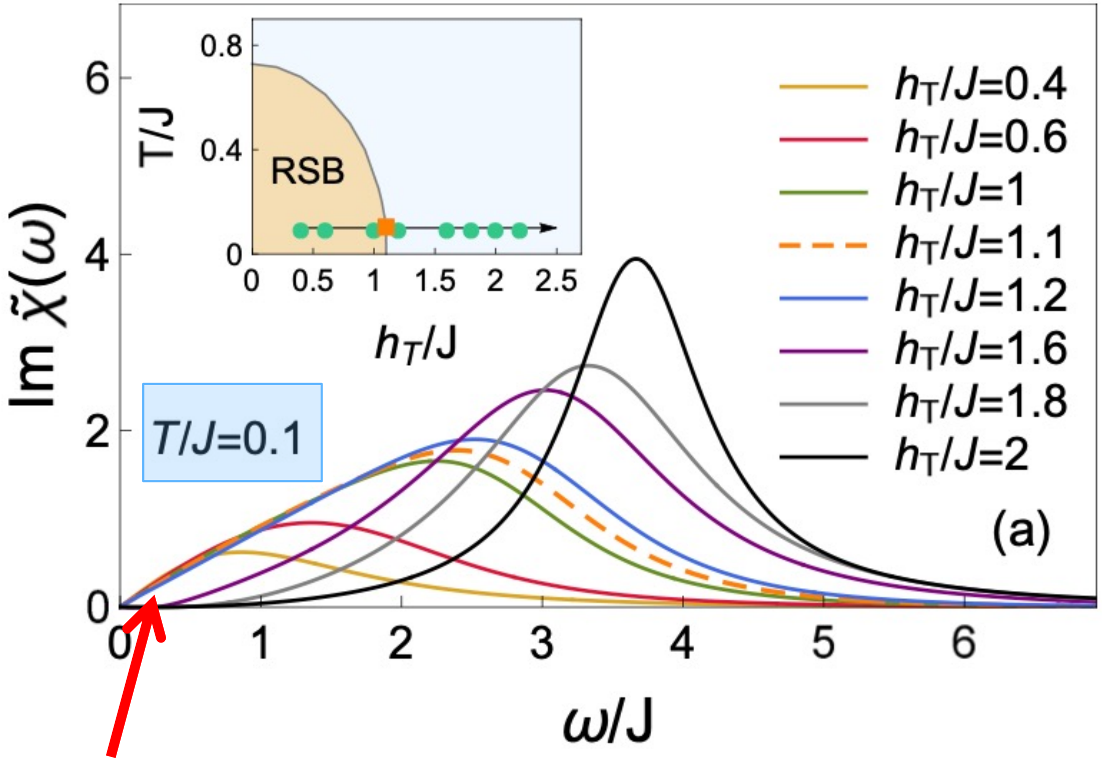
✓

Non-trivial check: Independent of Q-fluctuation strength Γ!!

Quantum Ising spin glass: spectral function, soft modes

A. Kiss, G.Zarand, I. Lovas, PRB 109, 024431 ('24)

Solution combining RSB and DMFT

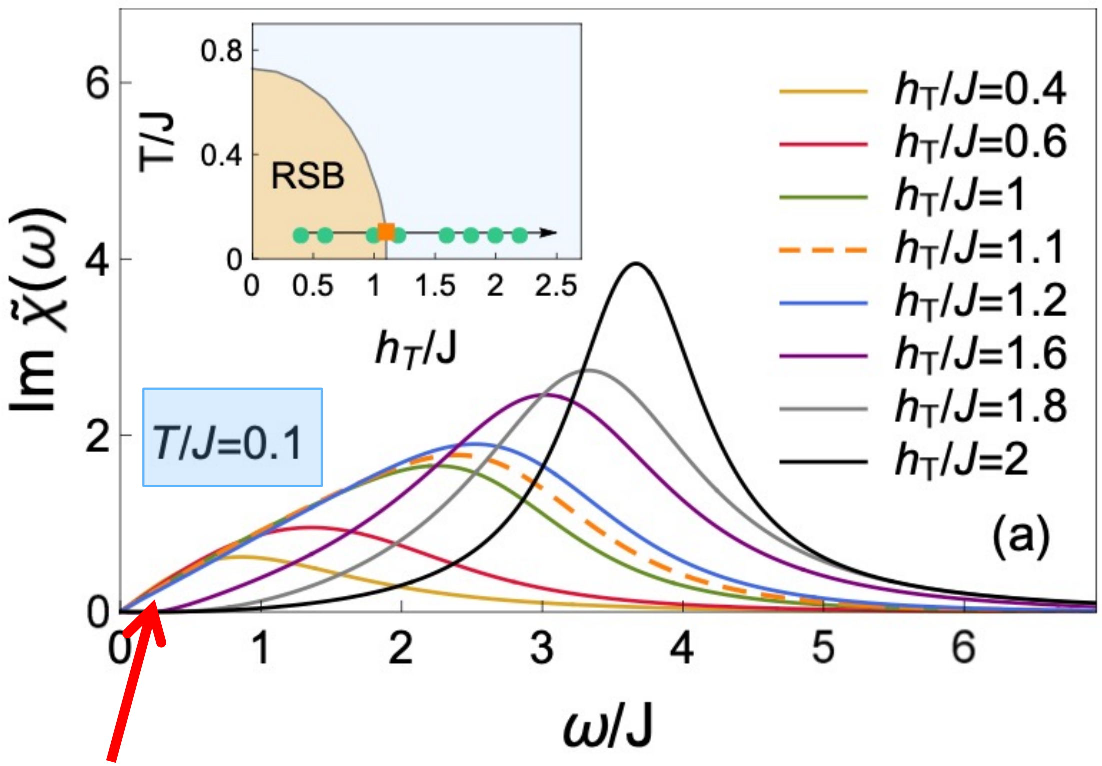


- Spectral function independent of small $h_T \equiv \Gamma$

Quantum Ising spin glass: spectral function, soft modes

A. Kiss, G.Zarand, I. Lovas, PRB 109, 024431 ('24)

Solution combining RSB and DMFT



- Spectral function independent of small $h_T \equiv \Gamma$
- Paramagnet $h_T > h_c$: gap!
- Glass phase : marginal → everywhere gapless

    Substantial spectral transfer to low ω

Quantum spin glasses: Heisenberg vs Ising

$$H_{\mathrm{Hb}} = \frac{1}{\sqrt{N}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \quad \longleftrightarrow \quad H_{\mathrm{Ising}} = -\Gamma \sum_i s_i^x + \frac{1}{\sqrt{N}} \sum_{i<j} J_{ij} s_i^z s_j^z$$

Quantum spin glasses: Heisenberg vs Ising

$$H_{\mathrm{Hb}} = \frac{1}{\sqrt{N}} \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \quad \longleftrightarrow \quad H_{\mathrm{Ising}} = -\Gamma \sum_i s_i^x + \frac{1}{\sqrt{N}} \sum_{i<j} J_{ij} s_i^z s_j^z$$

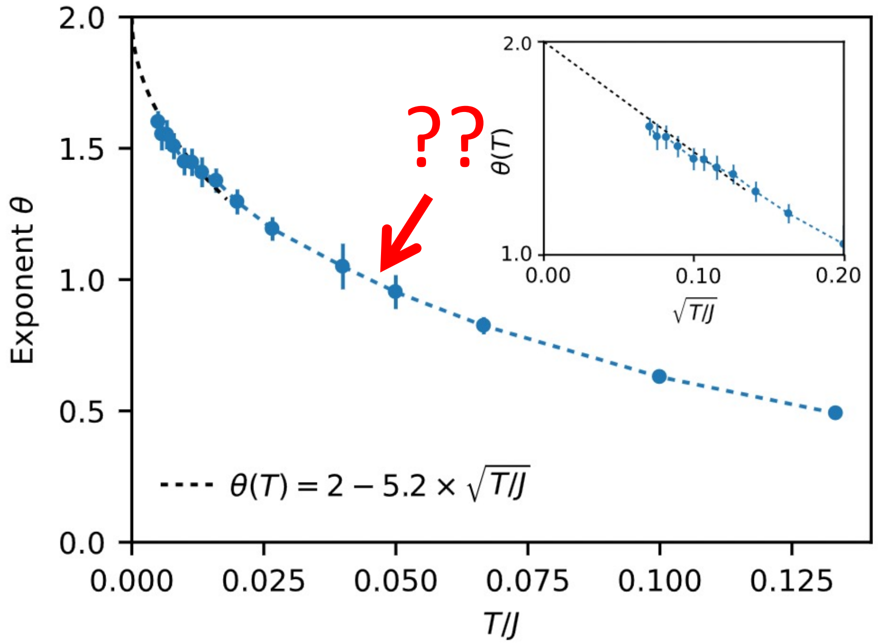
$$\chi''_{\mathrm{Hb}}(\omega) \approx 3.5 \frac{\omega}{J^2} \qquad \chi''_{\mathrm{Ising}}(\omega) \approx 0.5 \frac{\omega}{J^2}$$

The Heisenberg glass is significantly softer than an Ising glass at equal couplings J_{ij}

Open challenge

Slow T-dependence of dynamics?

$$\chi(\tau) \approx \chi(\beta/2)\left(\frac{1}{\sin(\pi\tau/\beta)}\right)^{\theta(T)}$$



- Mode coupling and dissipation??
- Effective friction induced by finite mode occupation??

Very slow creep toward $\theta_{T=0} = 2$

Similar as in the *quantum Ising (SK) mean field glass!*

Melting the quantum spin glass at T = 0 ?

SYK’s marginal Fermi liquid from
comes back -
but via a different mechanism

Melting the quantum spin glass: doping

Doped Mott insulator: $p = \langle n_\uparrow + n_\downarrow \rangle - 1$

$$\mathcal{H} = -\sum_{ij,\sigma=\uparrow,\downarrow} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$n_{i\sigma} = c^\dagger_{i\sigma} c_{i\sigma}$$

$$\mathbf{S}^a_i = c^\dagger_{i\sigma} \sigma^a_{\sigma\sigma'} c_{i\sigma'}$$

Explicit spin-spin interaction added (on top of exchange)

$$U = 4t \text{ and } J = 0.5t$$

Solve again with selfconsistent mean field method + RSB.

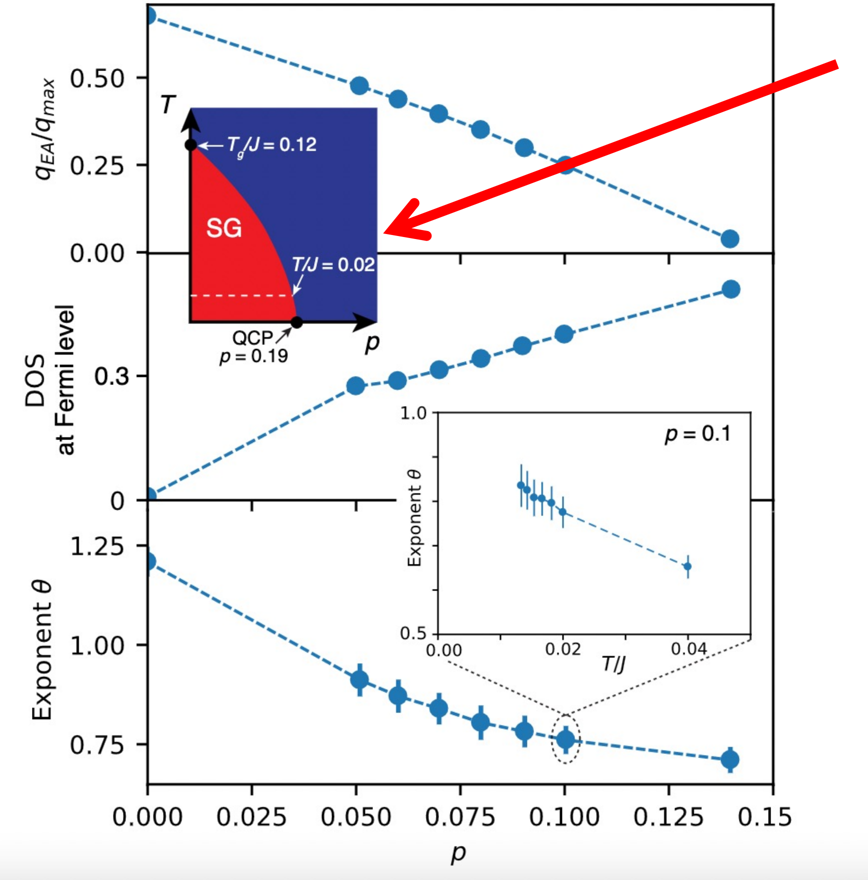
Previous work in the *paramagnet*: non Fermi liquid, Planckian relaxation and $\theta \approx 1$ close to Q-glass transition!

Dumitrescu, Wentzell, Georges, Parcollet PRB '22

Melting the quantum spin glass: doping

Doped Mott insulator: $p = \langle n_\uparrow + n_\downarrow \rangle - 1$

$$\mathcal{H} = -\sum_{ij,\sigma=\uparrow,\downarrow} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

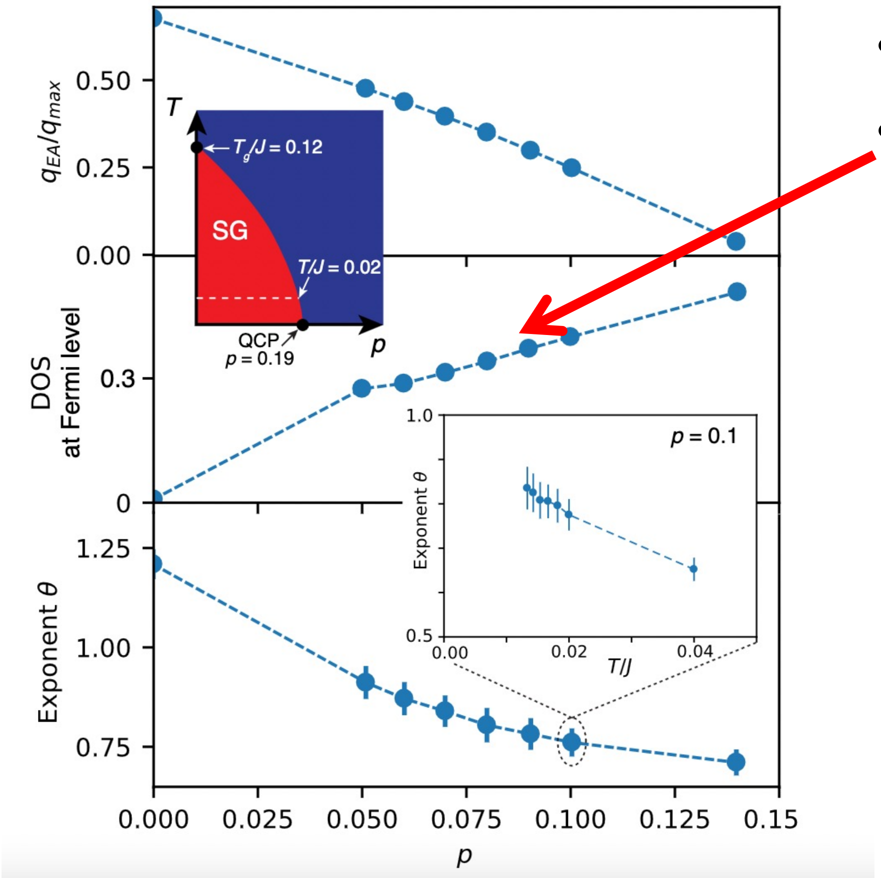


- Doping $p>p_c= 0.19$ melts the glass

Melting the quantum spin glass: doping

Doped Mott insulator: $p = \langle n_\uparrow + n_\downarrow \rangle - 1$

$$\mathcal{H} = - \sum_{ij,\sigma=\uparrow,\downarrow} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

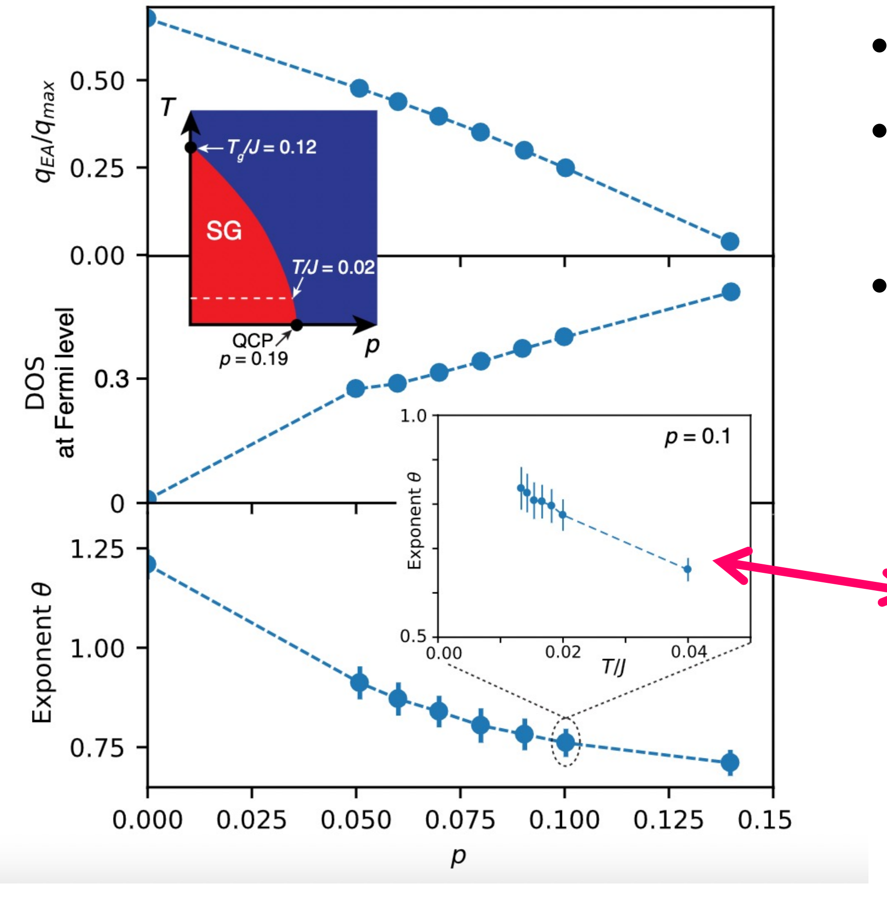


- Doping $p>p_c= 0.19$ melts the glass
- $p>0$ introduces a Fermi surface → metallic spin glass

Melting the quantum spin glass: doping

Doped Mott insulator: $p = \langle n_\uparrow + n_\downarrow \rangle - 1$

$$\mathcal{H} = -\sum_{ij,\sigma=\uparrow,\downarrow} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U\sum_i n_{i\uparrow} n_{i\downarrow} - \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$



- Doping $p>p_c= 0.19$ melts the glass
- $p>0$ introduces a Fermi surface → metallic spin glass
- Landau theory // picture of *Ohmically* overdamped random spin waves *both* would suggest

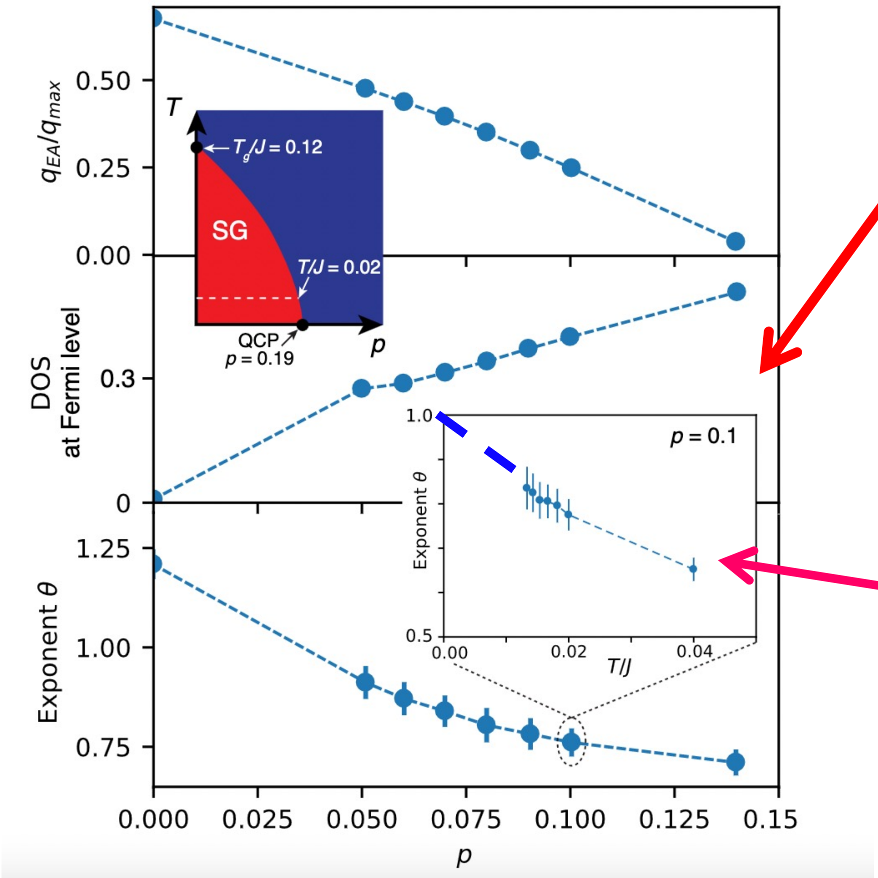
$$\chi''(\omega) \sim \sqrt{\omega} \equiv \omega^{\theta-1}$$

i.e., $\theta = 3/2$.

Melting the quantum spin glass: doping

Doped Mott insulator: $p = \langle n_\uparrow + n_\downarrow \rangle - 1$

$$\mathcal{H} = -\sum_{ij,\sigma=\uparrow,\downarrow} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \sum_{i<j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$



- Doping $p > p_c = 0.19$ melts the glass
- $p > 0$ introduces a Fermi surface → metallic spin glass
- Landau theory // picture of *Ohmically* overdamped random spin waves *both* would suggest

$$\chi''(\omega) \sim \sqrt{\omega} \equiv \omega^{\theta - 1}$$

i.e., $\theta = 3/2$.

- **Instead, we find** $\theta \leq 1$ **!**

Yet slower dynamics! $\theta(T \to 0) \to 1?$

→ Non-Ohmic friction from a non-Fermi liquid ??

PAUL SCHERRER INSTITUT

Self-consistent super-Ohmic bath of oscillators

MM, unpublished

Conjectured scenario for entire metallic glass phase:

Could spin waves constitute a dissipative bath for themselves?

Effective friction of bath of oscillators on given oscillator (coupling prop. to doping *p* [?])

$$\eta_\omega \equiv \frac{\chi''(\omega)}{\omega} \sim \omega^{\theta-2}$$

Self-consistent super-Ohmic bath of oscillators

MM, unpublished

Conjectured scenario for entire metallic glass phase:
Could spin waves constitute a dissipative bath for themselves?

Effective friction of bath of oscillators on given oscillator (coupling prop. to doping *p* [?])

$$\eta_\omega \equiv \frac{\chi''(\omega)}{\omega} \sim \omega^{\theta-2}$$

Typical frequency $$\eta \dot{x} = -\lambda x \rightarrow \lambda_\omega \sim \eta_\omega \omega \sim \omega^{\theta-1}$$

Mode density $$\rho(\lambda) \sim \lambda^{1/2} \rightarrow \rho(\omega) \sim \omega^{3/2(\theta-1)-1}$$

Typical displacement $$\lambda_\omega \langle x^2 \rangle_{\omega,\mathrm{typ}} \sim \hbar\omega \rightarrow \langle x^2 \rangle_{\omega,\mathrm{typ}} \sim \frac{1}{\omega^{\theta-2}}$$

PAUL SCHERRER INSTITUT

Self-consistent super-Ohmic bath of oscillators

MM, unpublished

Conjectured scenario for entire metallic glass phase:
Could spin waves constitute a dissipative bath for themselves?

Effective friction of bath of oscillators on given oscillator (coupling prop. to doping *p* [?])

$$\eta_\omega \equiv \frac{\chi''(\omega)}{\omega} \sim \omega^{\theta-2}$$

Typical frequency $\quad \eta \dot{x} = -\lambda x \rightarrow \lambda_\omega \sim \eta_\omega \omega \sim \omega^{\theta-1}$

Mode density $\quad \rho(\lambda) \sim \lambda^{1/2} \rightarrow \rho(\omega) \sim \omega^{3/2(\theta-1)-1}$

Typical displacement $\quad \lambda_\omega \langle x^2 \rangle_{\omega,\mathrm{typ}} \sim \hbar\omega \rightarrow \langle x^2 \rangle_{\omega,\mathrm{typ}} \sim \dfrac{1}{\omega^{\theta-2}}$

→ Selfconsistency condition on bath spectral function

$$\chi''(\omega) \sim \omega^{\theta-1} \overset{!}{\sim} \rho(\omega) \langle x^2 \rangle_{\omega,\mathrm{typ}} \sim \omega^{(\theta-1)/2}$$

Self-consistent super-Ohmic bath of oscillators

MM, unpublished

Conjectured scenario for entire metallic glass phase:

Could spin waves constitute a dissipative bath for themselves?

Effective friction of bath of oscillators on given oscillator (coupling prop. to doping *p* [?])

$$\eta_\omega \equiv \frac{\chi''(\omega)}{\omega} \sim \omega^{\theta-2}$$

Typical frequency $\quad \eta \dot{x} = -\lambda x \to \lambda_\omega \sim \eta_\omega \omega \sim \omega^{\theta-1}$

Mode density $\quad \rho(\lambda) \sim \lambda^{1/2} \to \rho(\omega) \sim \omega^{3/2(\theta-1)-1}$

Typical displacement $\quad \lambda_\omega \langle x^2 \rangle_{\omega,\mathrm{typ}} \sim \hbar\omega \to \langle x^2 \rangle_{\omega,\mathrm{typ}} \sim \frac{1}{\omega^{\theta-2}}$

→ Selfconsistency condition on bath spectral function

$$\chi''(\omega) \sim \omega^{\theta-1} \overset{!}{\sim} \rho(\omega) \langle x^2 \rangle_{\omega,\mathrm{typ}} \sim \omega^{(\theta-1)/2}$$

$$\to \theta_{\mathrm{met.gl}} = 1\,!$$

= marginal Fermi liquid exponent, robust in entire metallic glass phase!

Open questions

Already at mean field level open questions remain:

- origin of finite T spectral function, creeping of $\theta(T)$
- verification of self-consistently overdamped oscillators

Even more so in real space:

Collective spin waves:

- Spectral density and spatial structure
- Effect on conductivity (R(T) linear in T?)

...

Summary

- Solution of MF Heisenberg glass: high precision RSB + CT-QMC
- Rougher landscape than Ising glasses: vector spins feature large avalanches upon perturbation
- Dynamics and C_V evolve slowly (with $T \rightarrow 0$) to super-universal independent random spin waves
- Doping toward quantum melting: unexpectedly slow dynamics; fermions become non-Fermi liquid