University of Cologne

# Theory of robust many-body quantum scars

## In long-range interacting systems

arXiv:2309.12504, in press on PRX

Started in January 2020:

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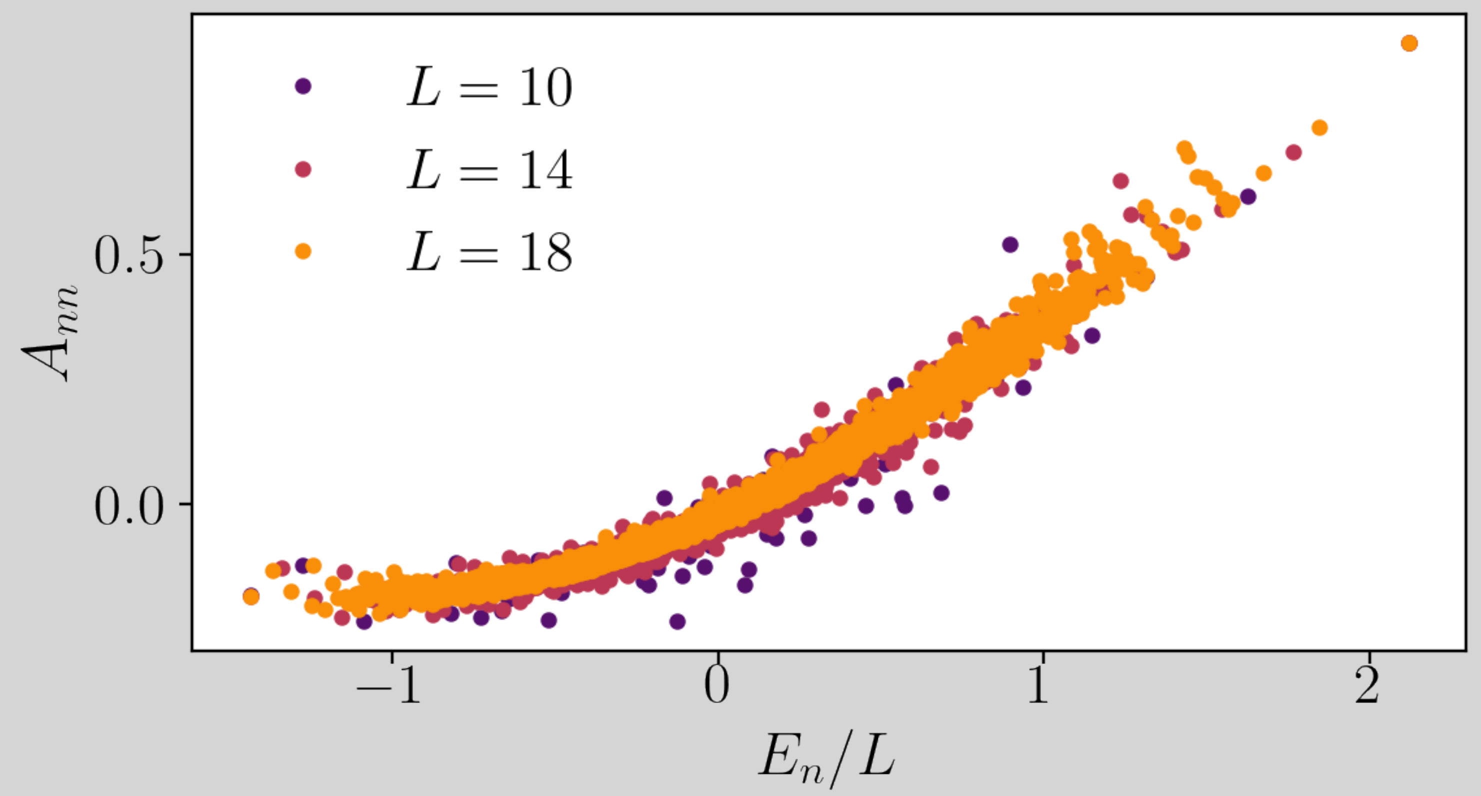
# Non-equilibrium states of matter

Observables $\langle E_n | A | E_n \rangle$ smooth functions of $E_n/L$




Fast thermalization

Quantum Many-Body chaos (Eigenstate Thermalization Hypothesis)

Weak ergodicity breaking

Prethermalization

Fragmentation

Many-Body quantum Scars

**This talk**

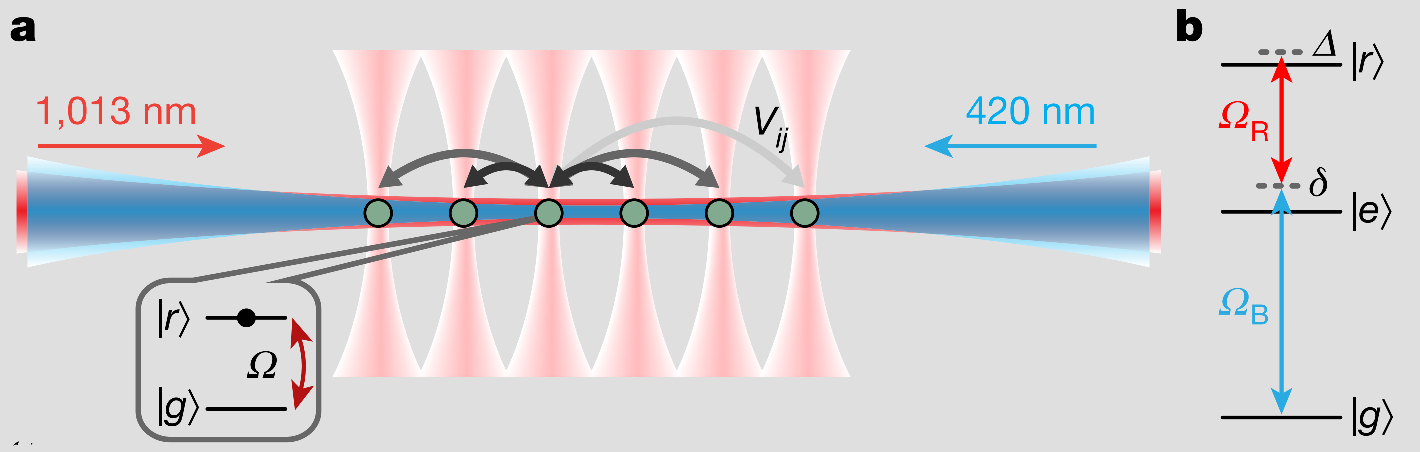
....

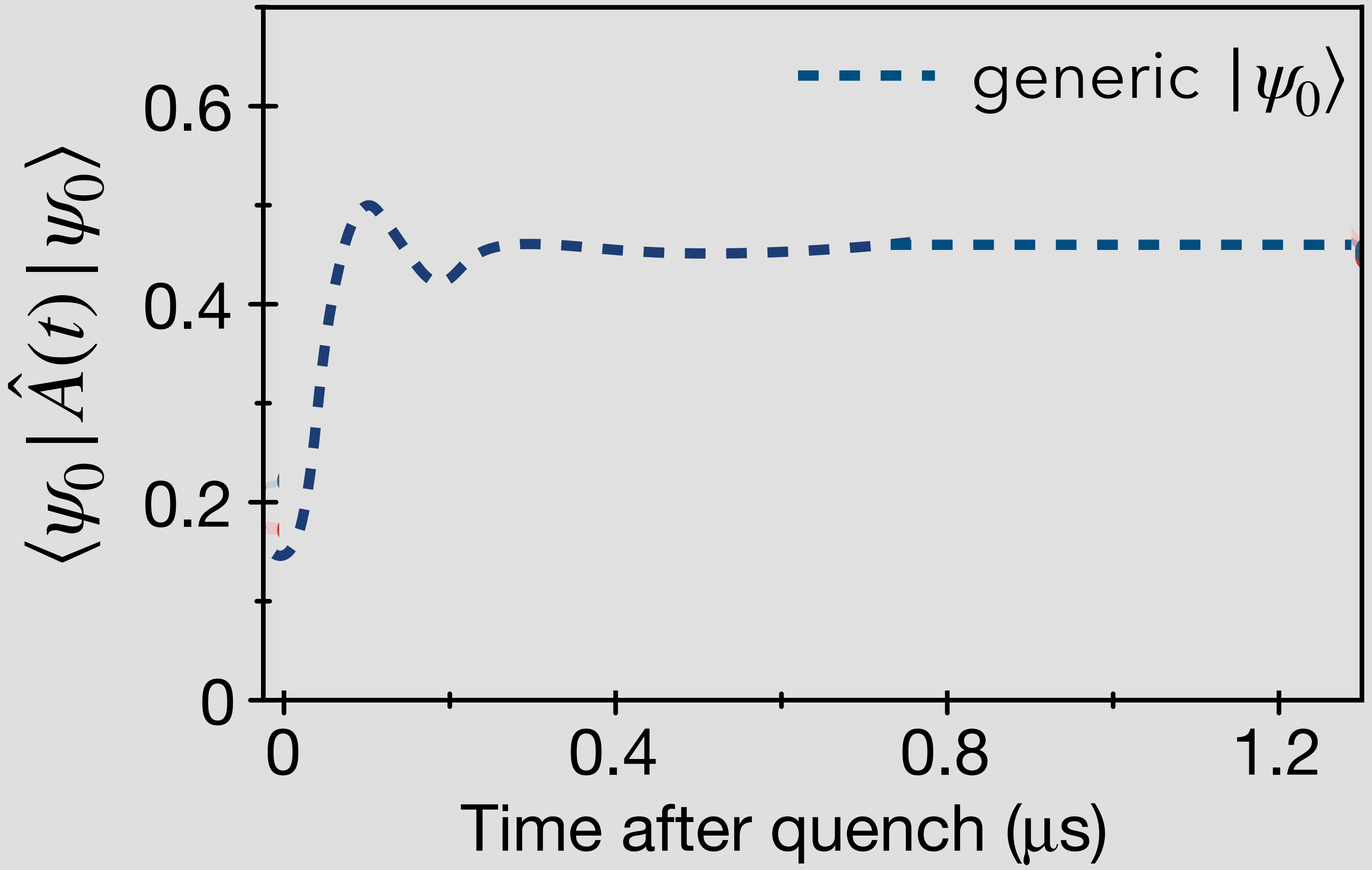
Non-ergodic dynamics

Many-body localization, Quantum glasses

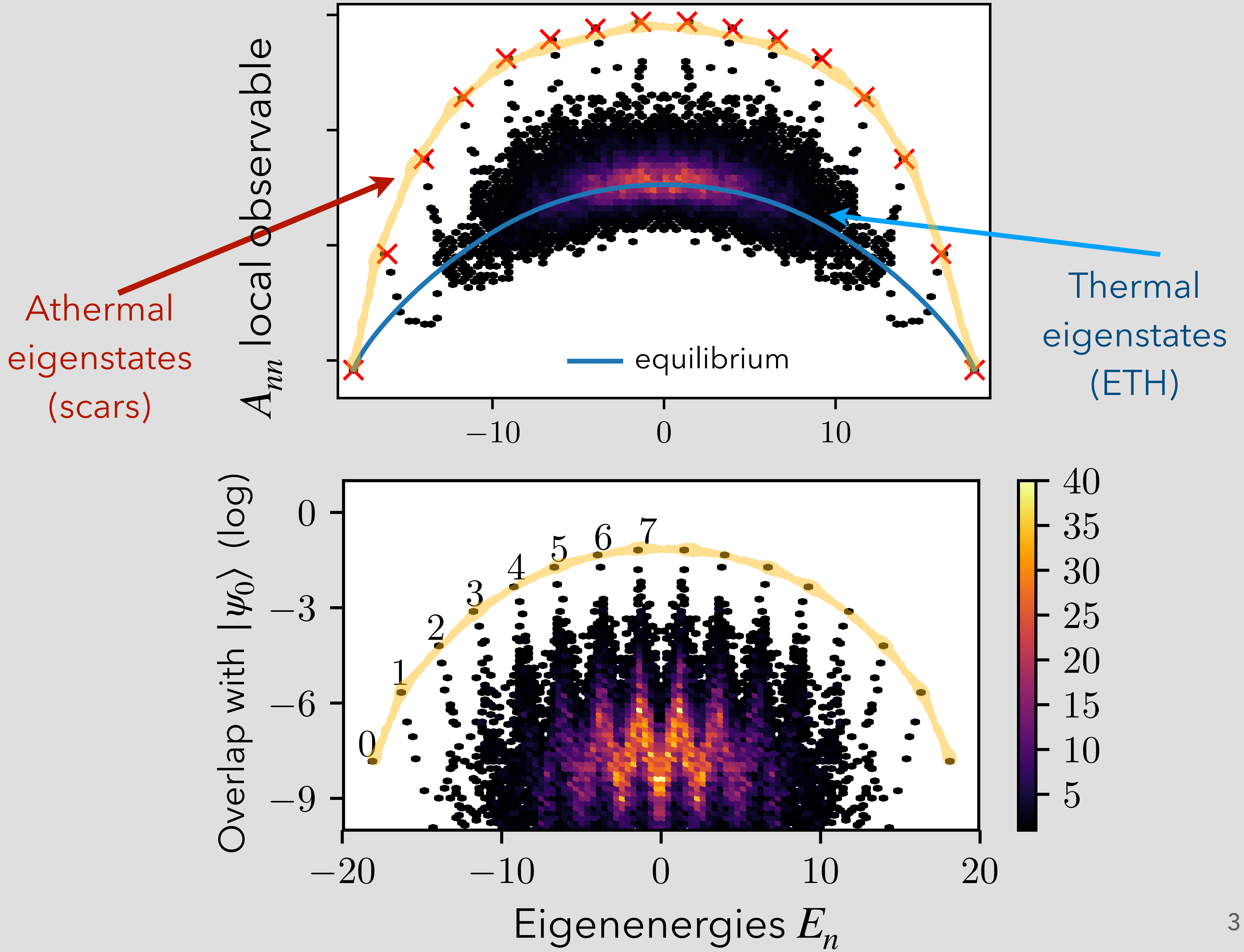
# Many-body quantum scars

Bernien, Schwartz, Keesling, … Lukin – Nature, 2017



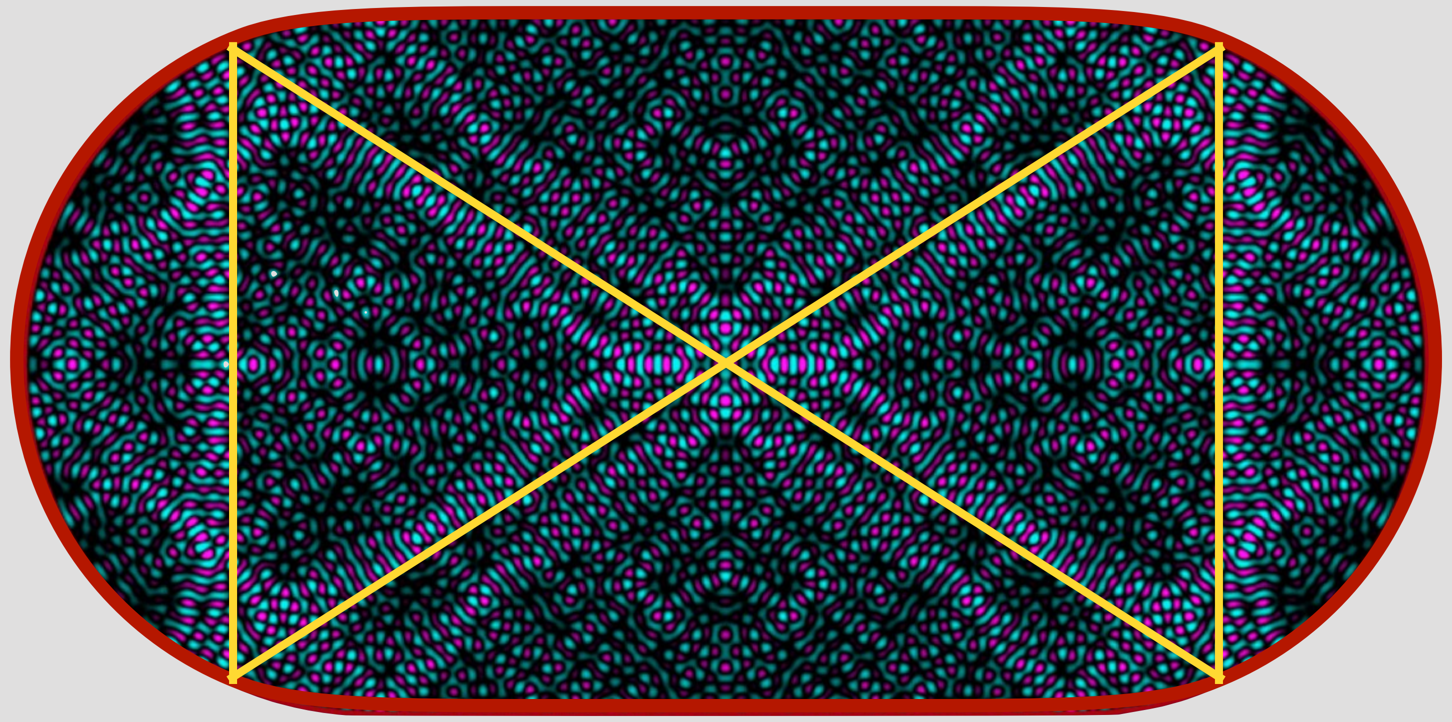




Turner et al. Nat. Phys. 2018



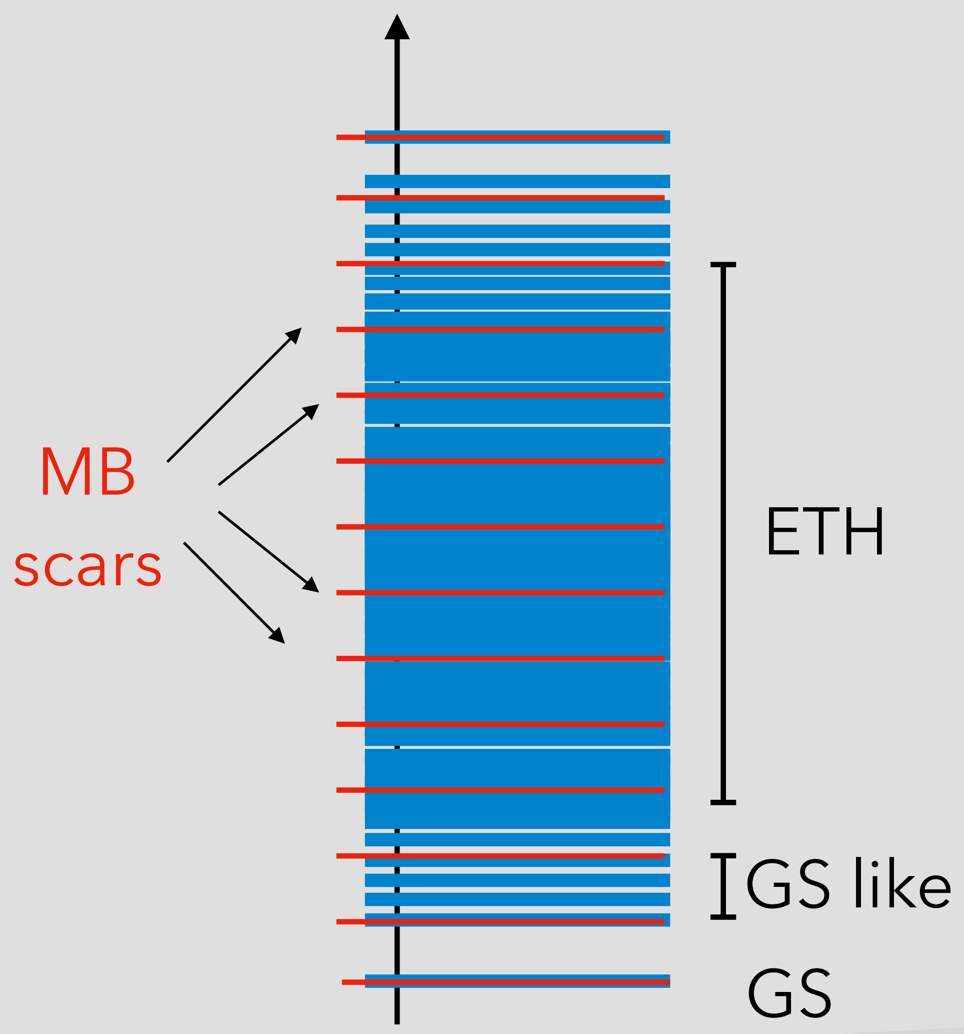

Inspired by

"quantum scars" in billiards $\hbar \ll 1$ Heller 1974



# Theory of many-body quantum scars

Turner et al. Nat. Phys. 2018 + Papic, Moudgalya, Chandran, Iadecola, Müller, …

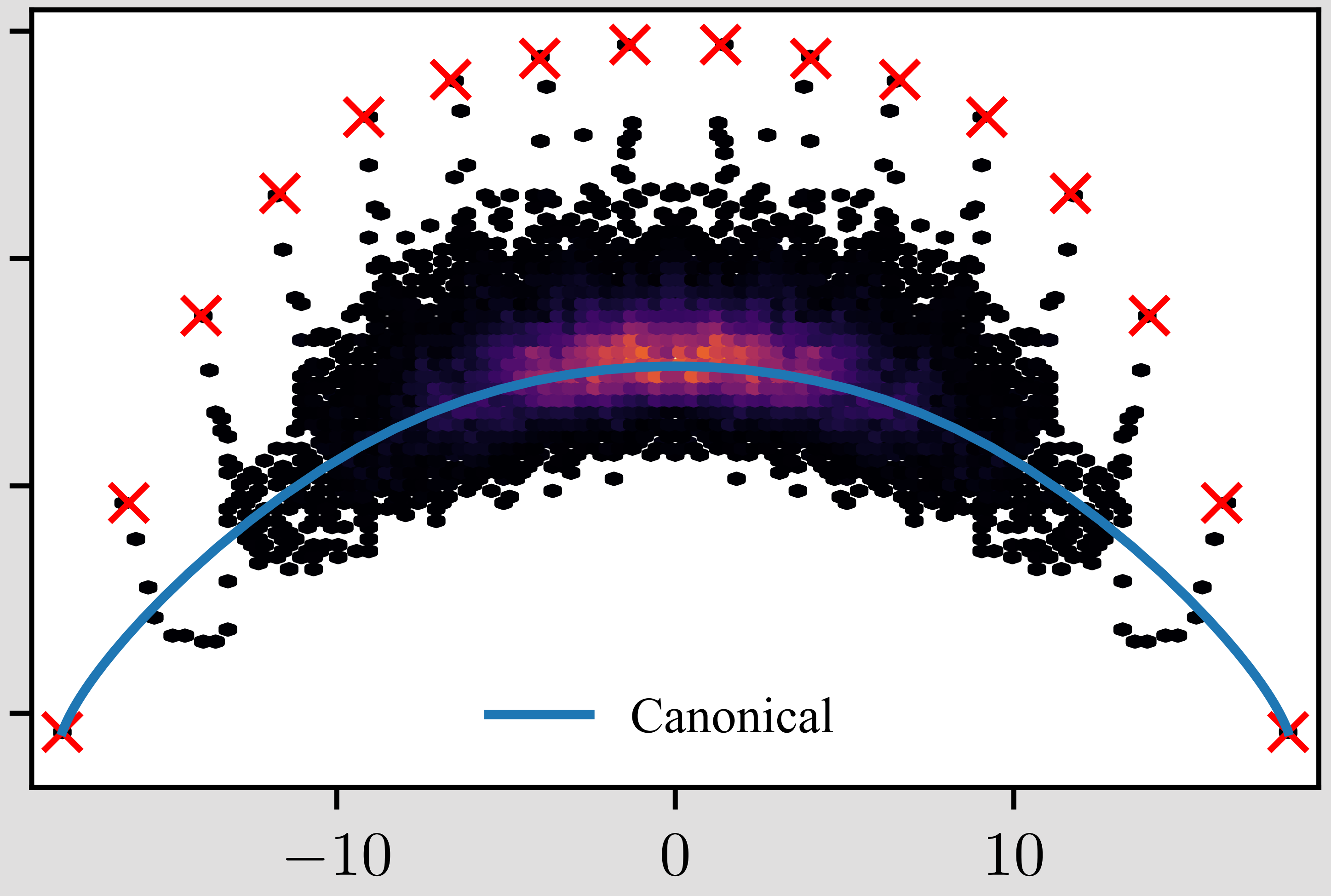


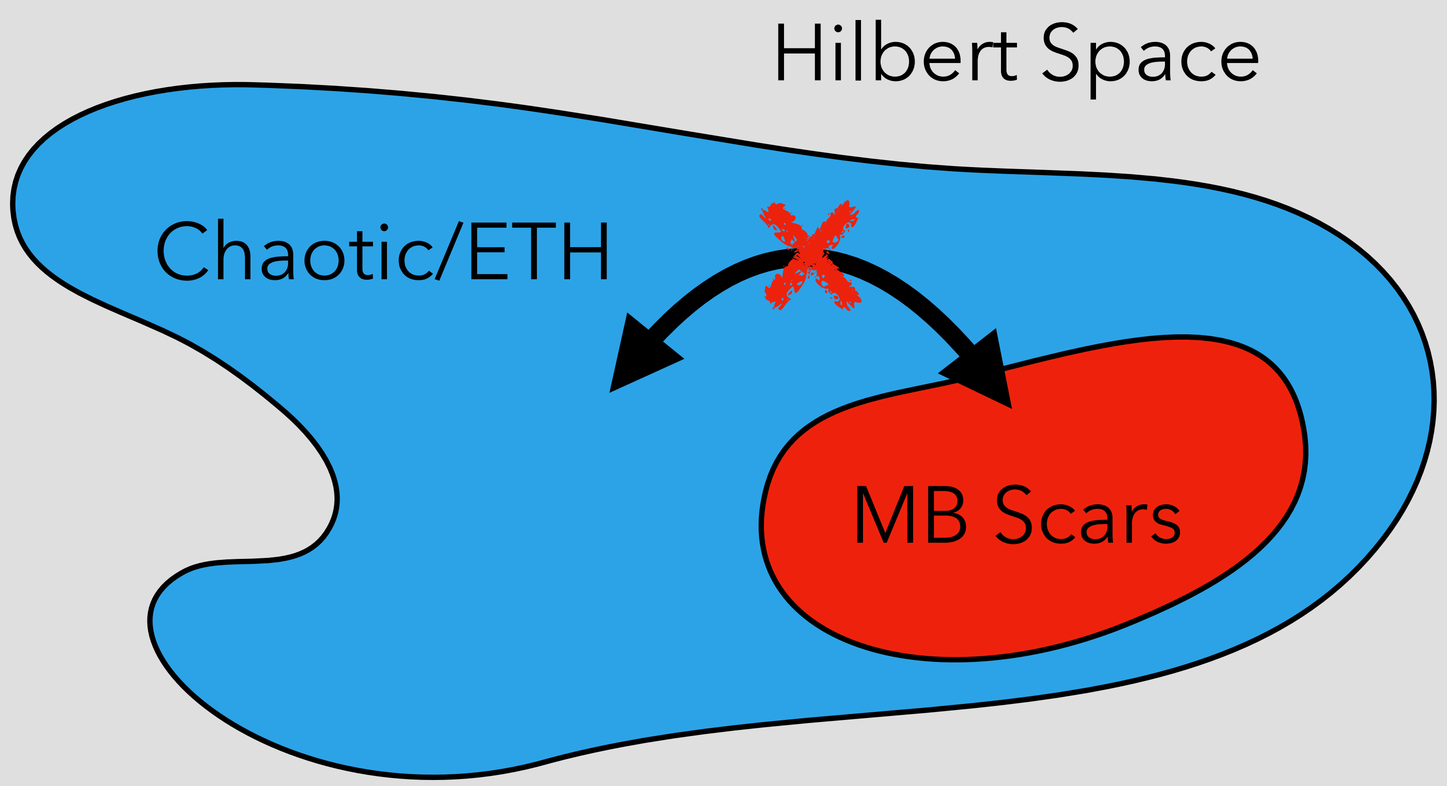

- construction of local $\hat{H}$ with exact scars

  Shiraishi & Mori – PRL (2017)
  Motrunich, Moudgalya, Iadecola, Surace, …

- **unstable** to generic perturbations

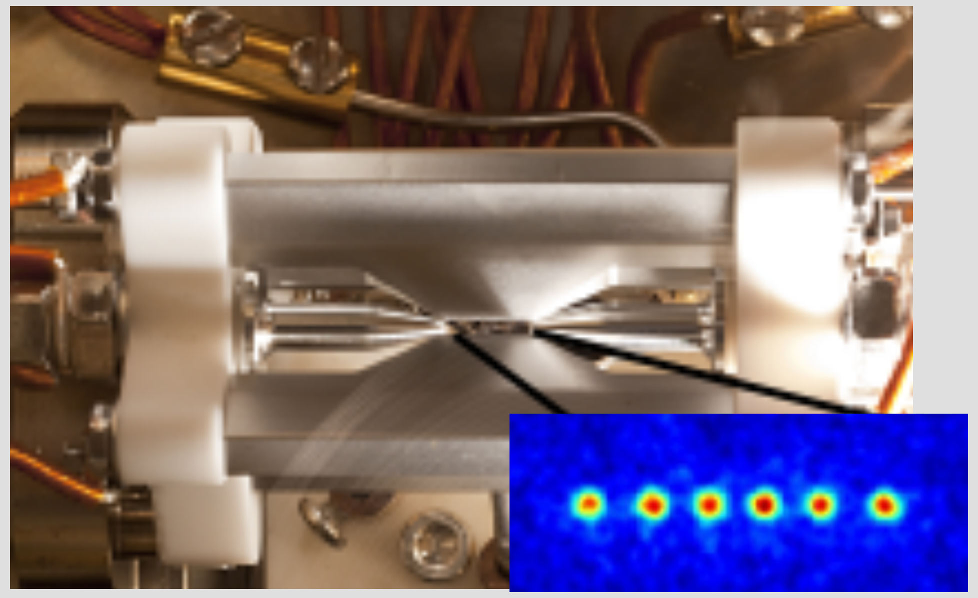
  Lin et al. – PRR (2020), Surace et al. – PRB (2021)




Is there a class of systems with *robust* scars?

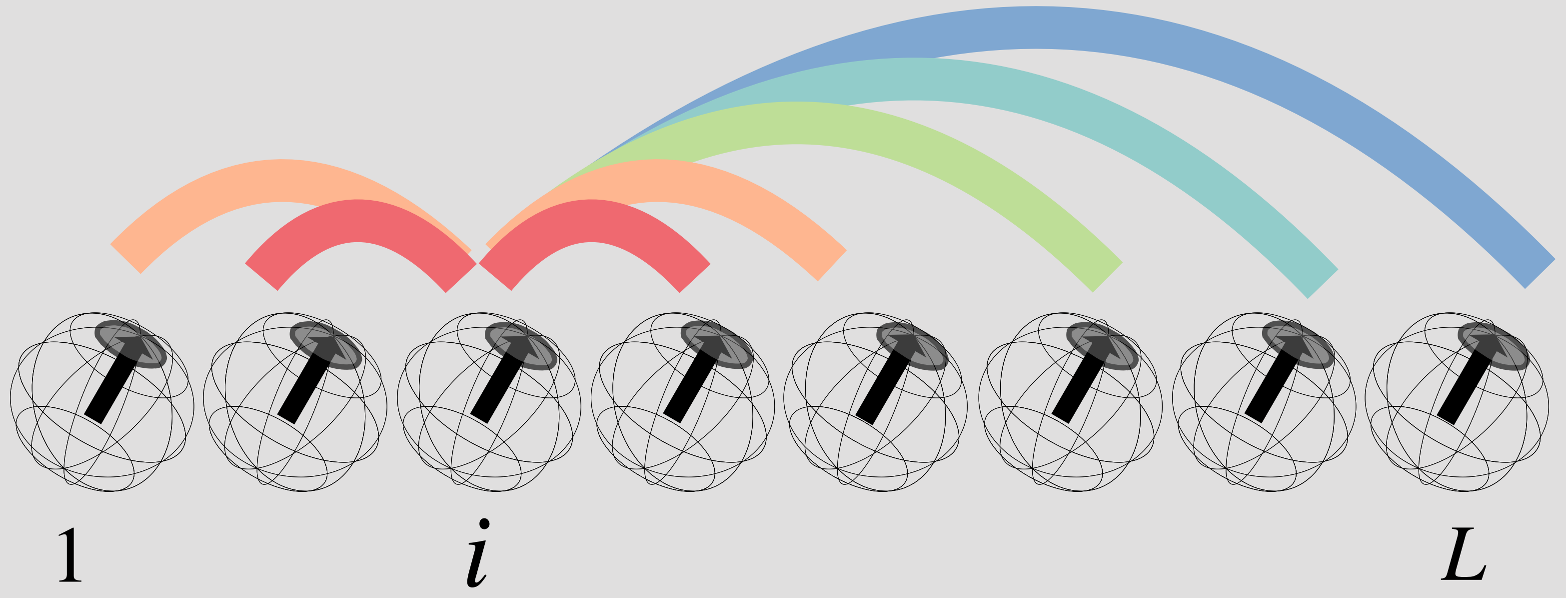
# Long-range interacting systems



$0.5 < \alpha < 1.8$

- Two level systems (spins 1/2 or "qubits")
- Interactions mediated by spatially delocalized degrees of freedom

$$J_{ij} \sim \frac{J}{|i-j|^{\alpha}}$$




**Variable-range quantum Ising chain**

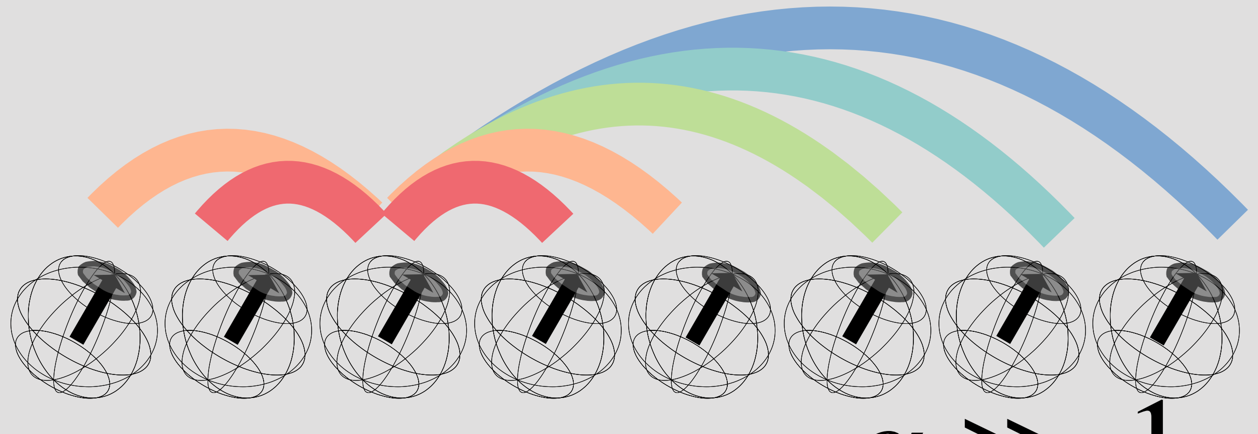
$$H = -\sum_{i<j}^{L} J_{ij} s_i^x s_j^x - h \sum_{i}^{L} s_i^z$$

**Quantum experiments in AMO physics**

**Unusual quantum dynamics:**

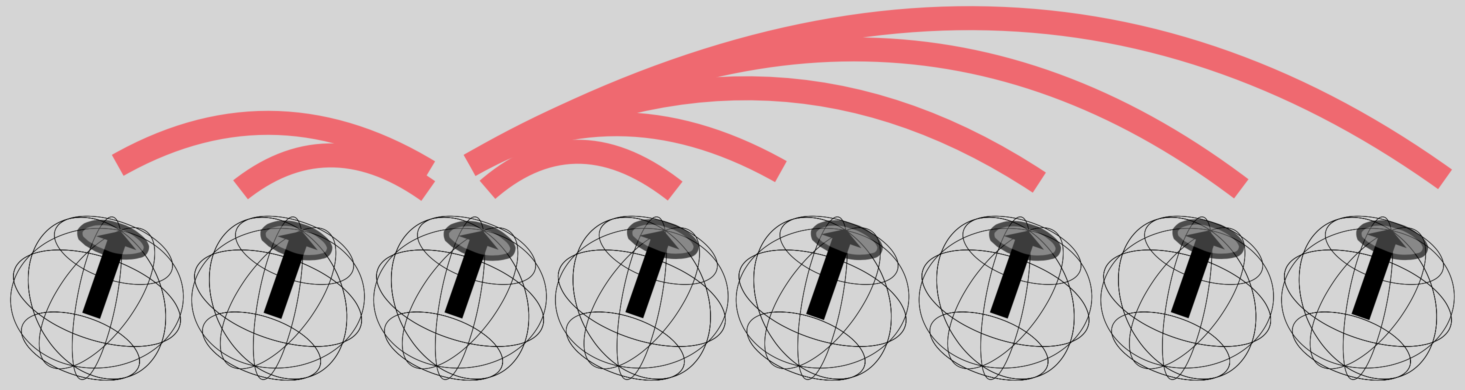
Defenu et al. Rev. Mod. Phys 2023
Defenu, Lerose, **SP** – Phys. Rep. 2024

# Dynamics in long-range systems





**Mean field Dynamics (Solvable)** $J = J_0/N_\alpha$



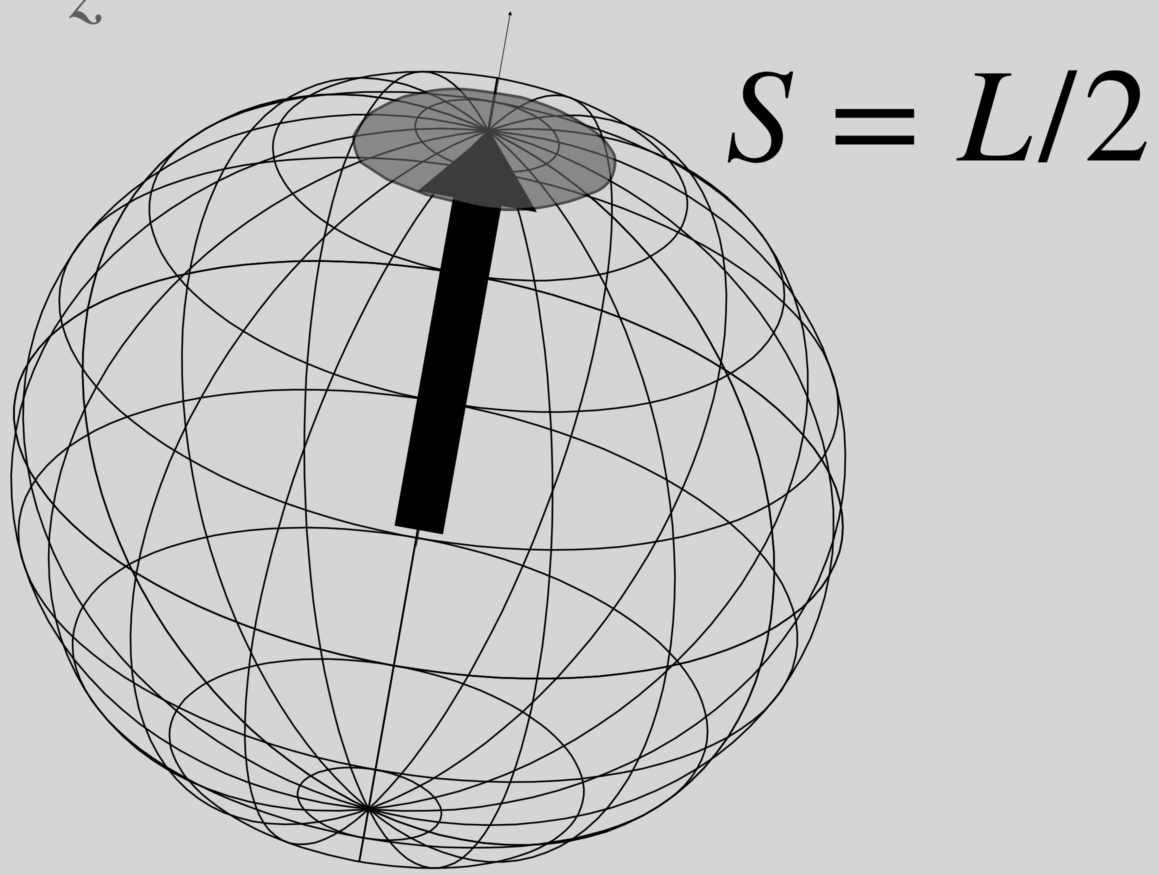
$$\hat{H}_{\alpha=0} = -J\Big(\sum_{ij} s_i^x s_j^x\Big) - h\sum_i s_i^z$$

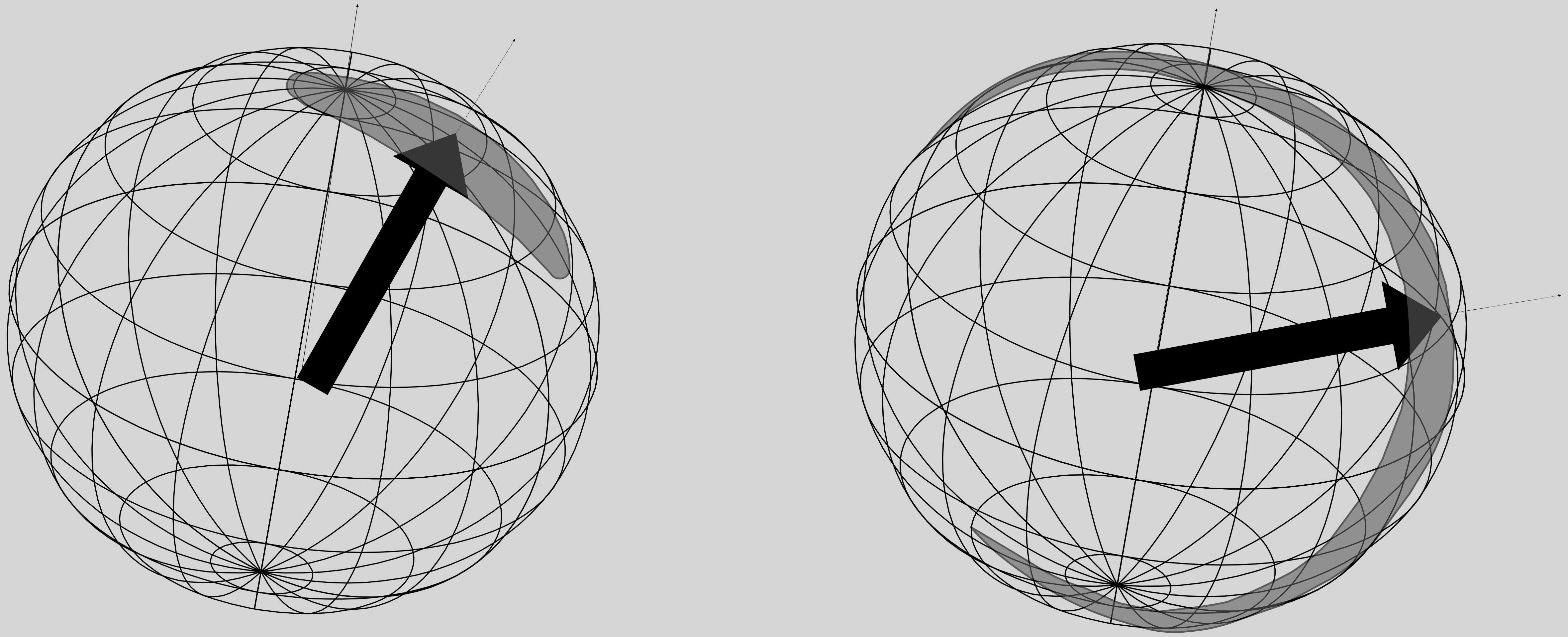
$\sum_{ij} s_i^x s_j^x \to (\hat{S}_x)^2$, $\quad \sum_i s_i^z \to \hat{S}_z$

Semiclassical picture of **collective spin squeezing**

GS and fully polarized states

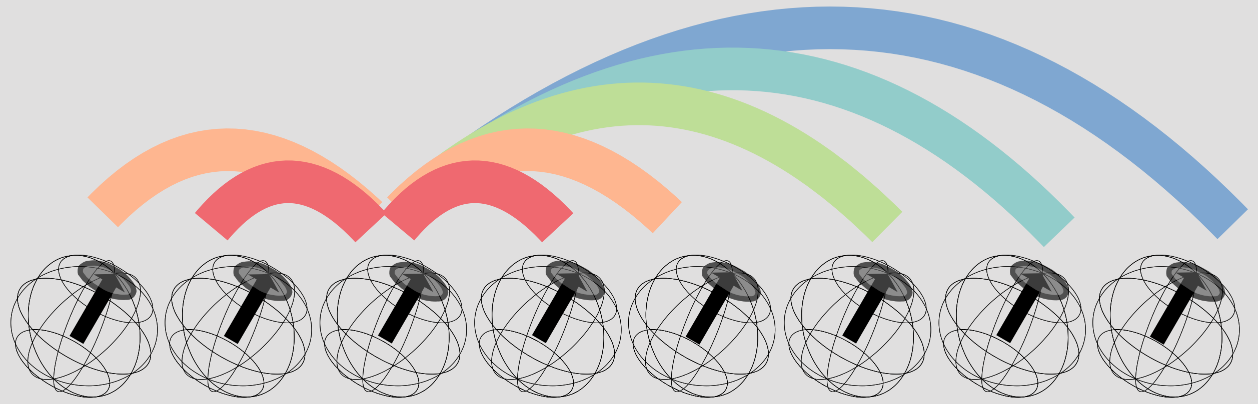
$\hbar_{\text{eff}} \sim \hbar/L$

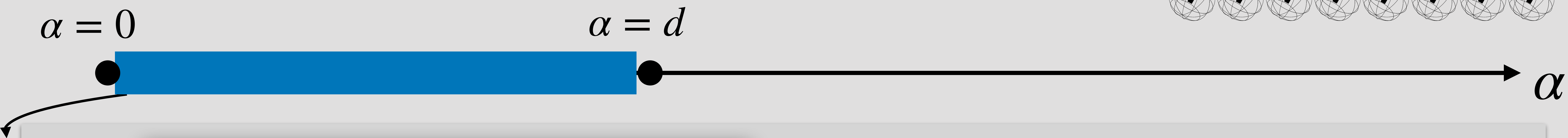


$\Longrightarrow$ **Thermalization impossible** (spin size conserved) $\quad \vec{S}(t) \propto L$

# Dynamics in long-range systems





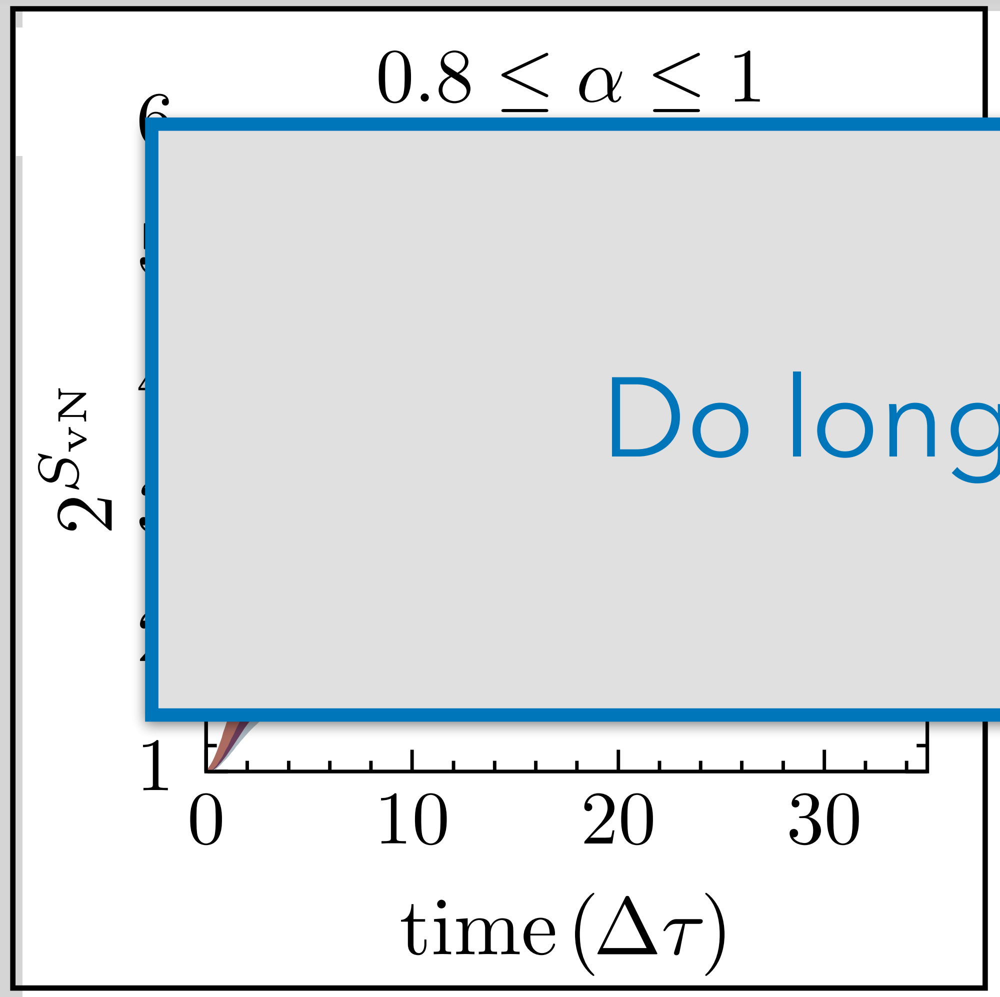

**2013**

PHYSICAL REVIEW X 3, 031015 (2013)

**Entanglement Growth in Quench Dynamics with Variable Range Interactions**

J. Schachenmayer,[1] B. P. Lanyon,[2] C. F. Roos,[2] and A. J. Daley[1]

$S_{\mathrm{vN}} = -\operatorname{Tr}(\rho_{N/2} \ln \rho_{N/2})$

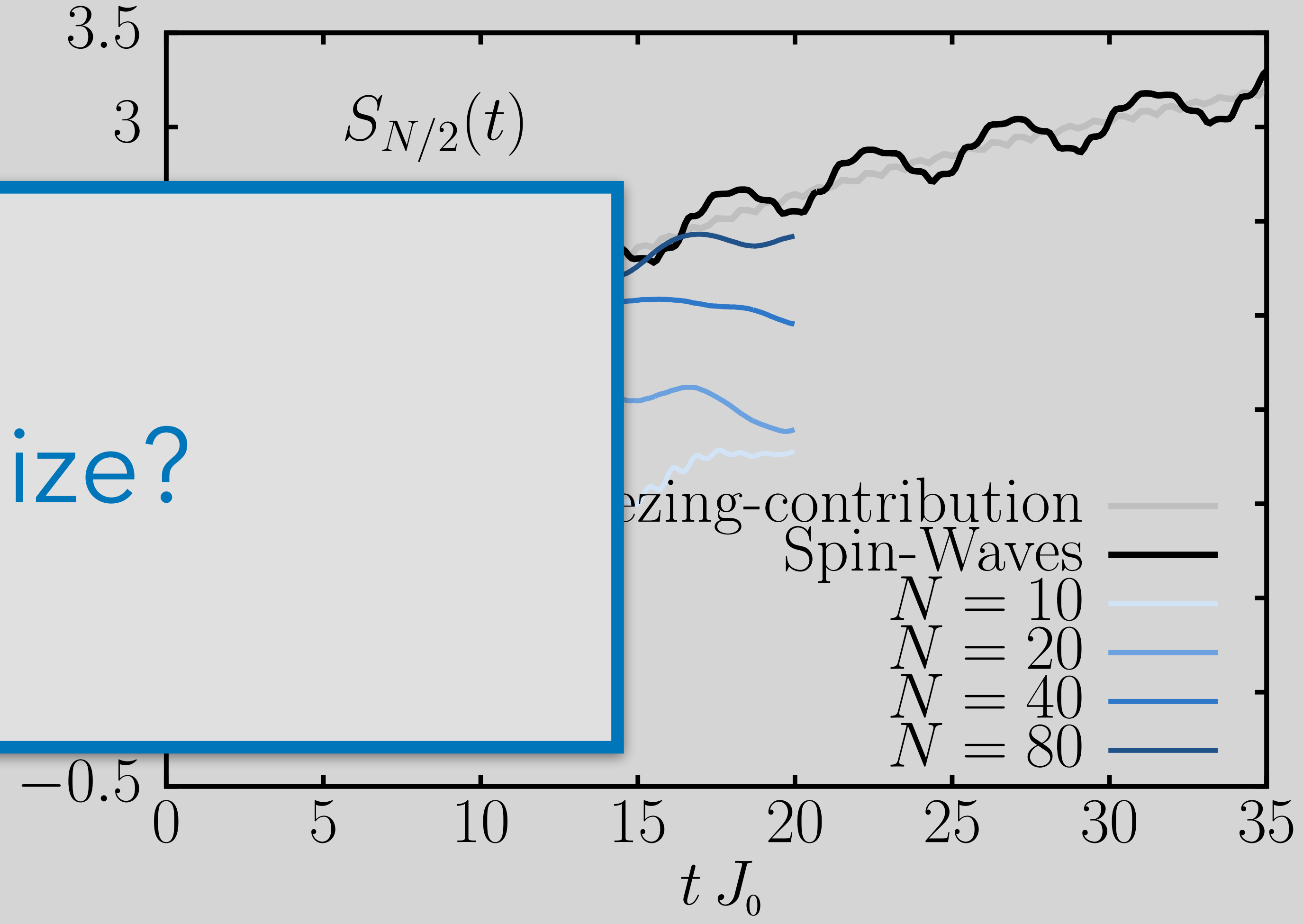



Do long-range systems ultimately thermalize?

different behavior. Counterintuitively, quenches above the critical point for these long-range interactions lead only to a logarithmic increase of bipartite entanglement in time, so that in this regime, long-range interactions produce a slower growth of entanglement than short-range interactions.

**2020**

Theory of entanglement entropy dynamics for $0 \leq \alpha \leq d$




NO Semiclassical chaos

$\implies$ Slow (log) growth for $0 < t \ll N^{\beta}$

Semiclassical chaos

$\implies$ Fast (linear) growth for $0 < t \ll \log N$

# In this talk:

*Long-range* interacting quantum spin systems

$\updownarrow$

*Robust* quantum many-body scars

1. Integrability of the classical limit $\alpha = 0$ (KAM-like)

2. Sufficiently long-range interactions $0 < \alpha < d$

I. Numerical analysis

II. Analytical theory

III. New theory predictions

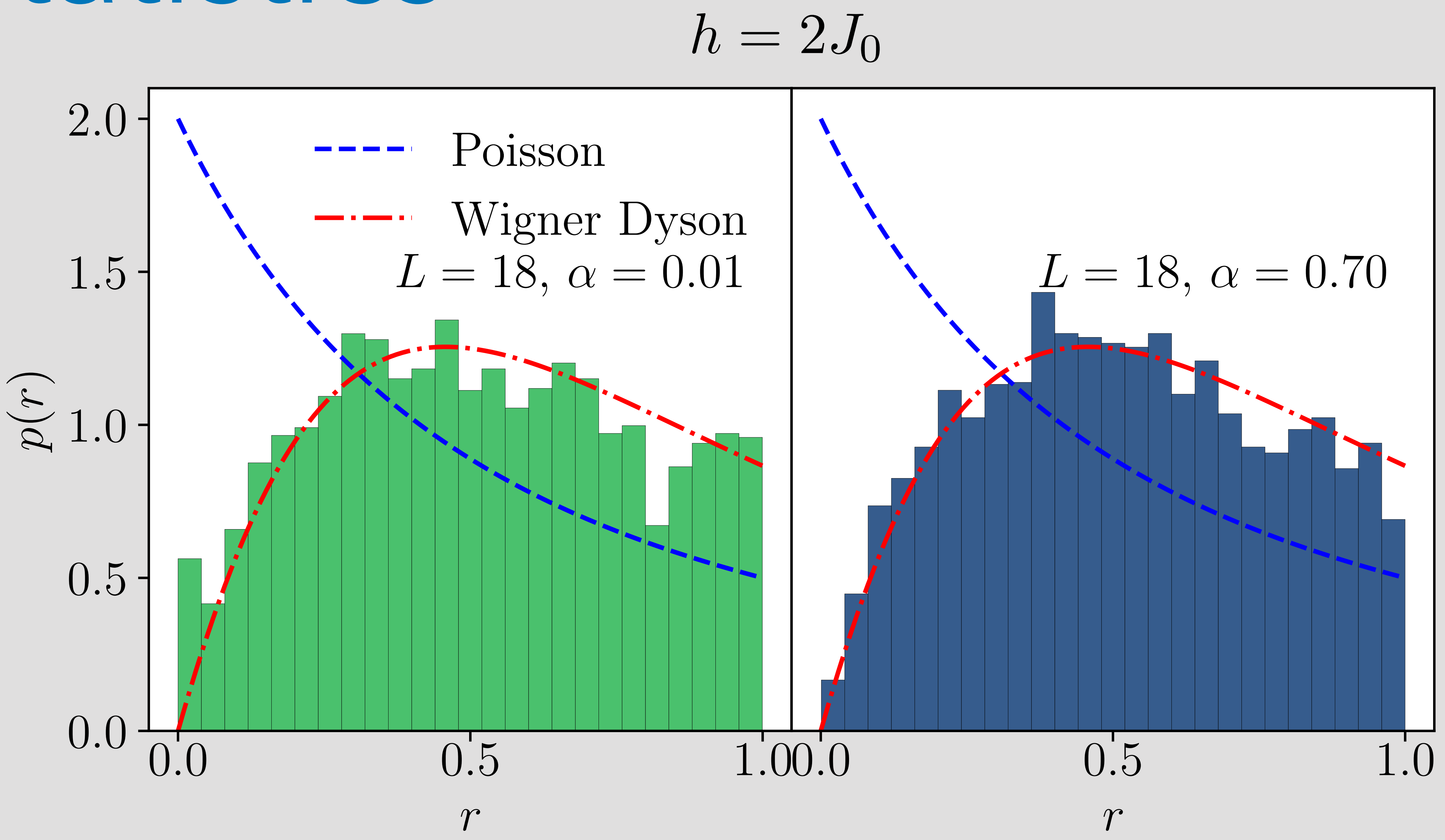
# Numerics: level spacing statistics

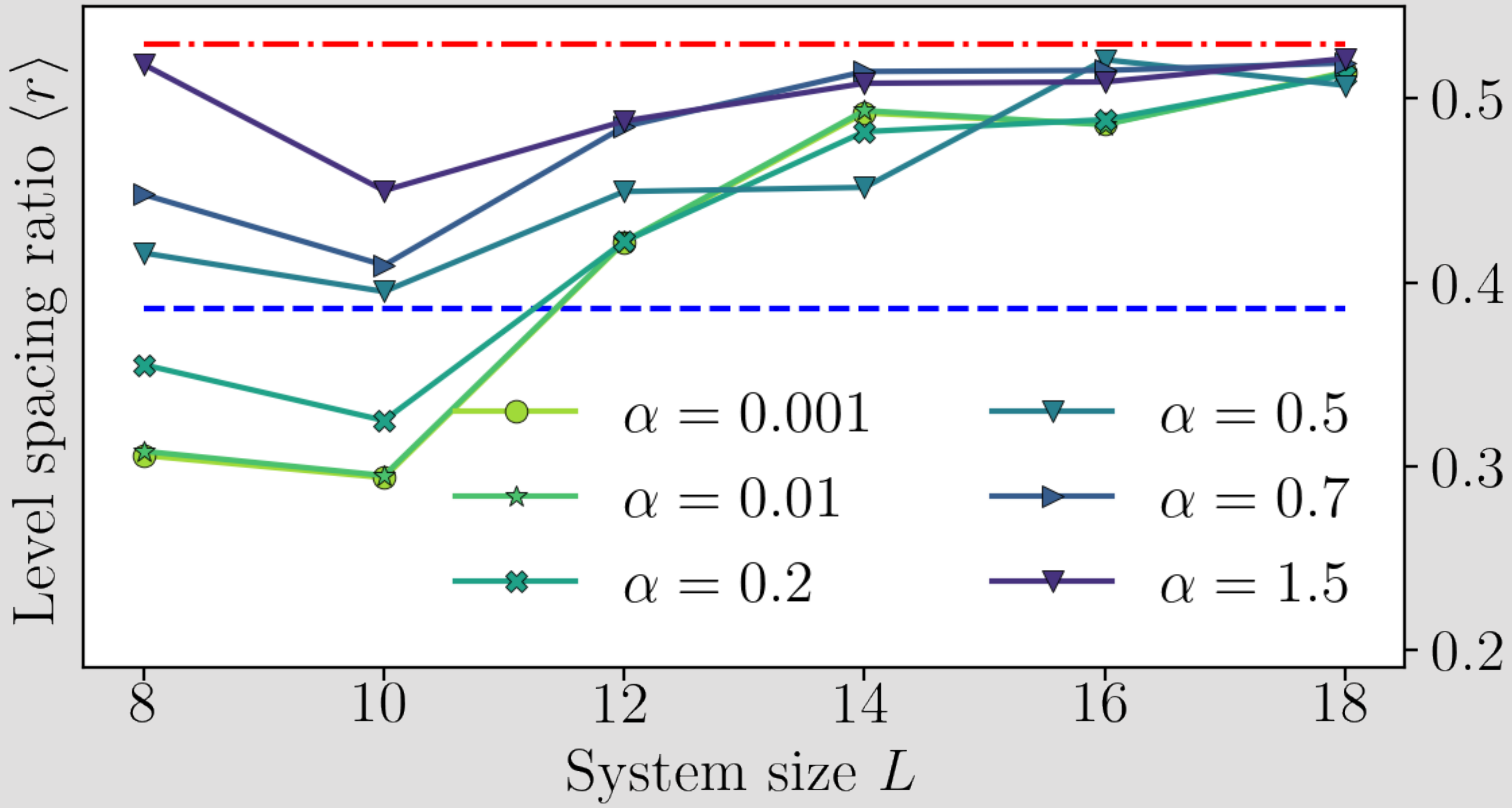
Std metric of quantum chaos/ergodicity

Level spacing ratio

$$r_n = \frac{\min(\Delta E_{n+1}, \Delta E_n)}{\max(\Delta E_{n+1}, \Delta E_n)}$$

- chaos: Wigner Dyson
- Interability: Poisson




Level repulsion for infinitesimal $\alpha > 0 \Longrightarrow$ quantum ergodicity?

Russomanno et al. PRB 2020

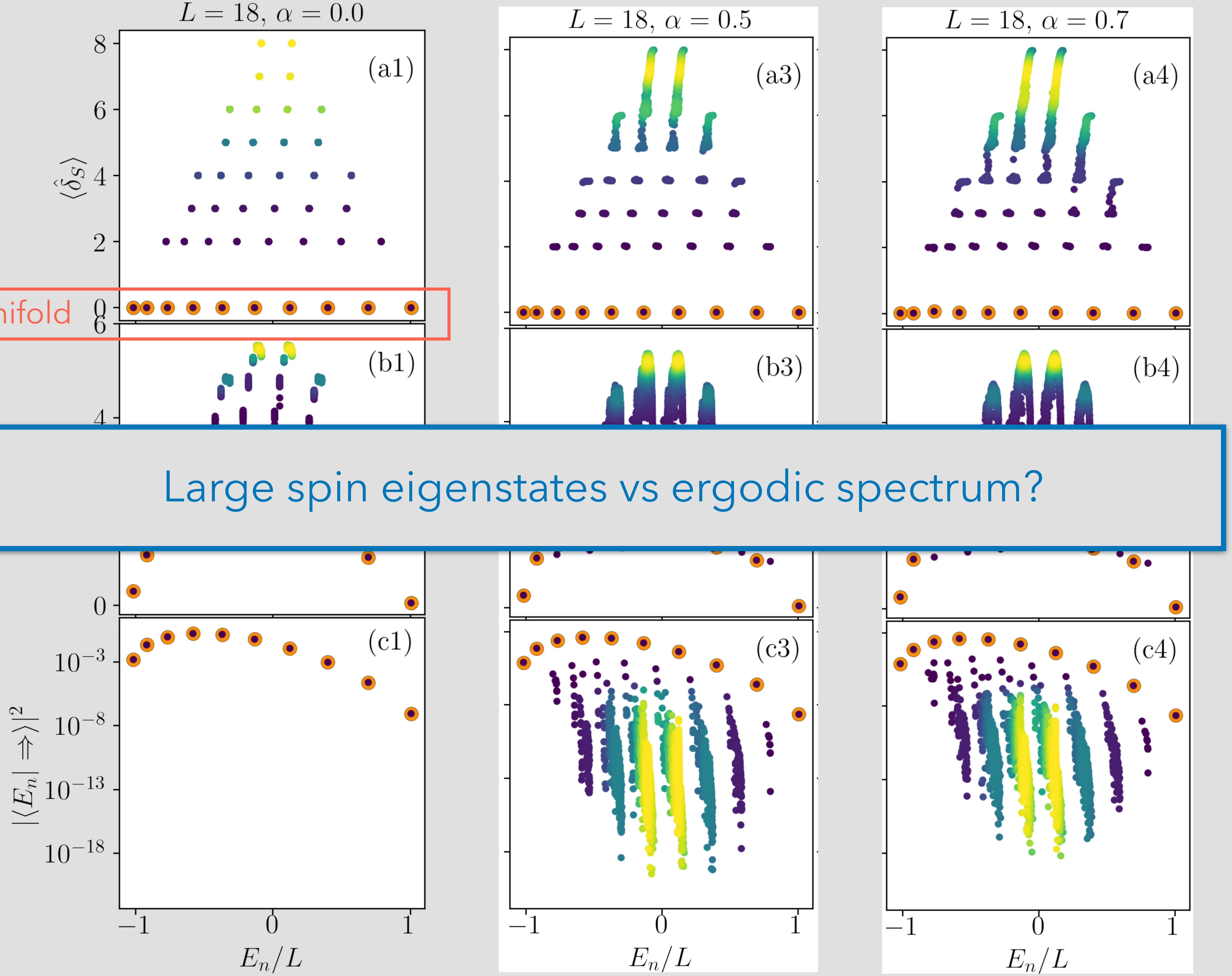
# Numerics: individual eigenstates

Collective spin size depletion
$\delta_S = L/2 - S$

$\delta_S = 0$: Dicke manifold

Half-chain entanglement entropy
$S_{L/2} = -\mathrm{Tr}(\rho_{L/2} \ln \rho_{L/2})$

Overlap with spin-coherent state

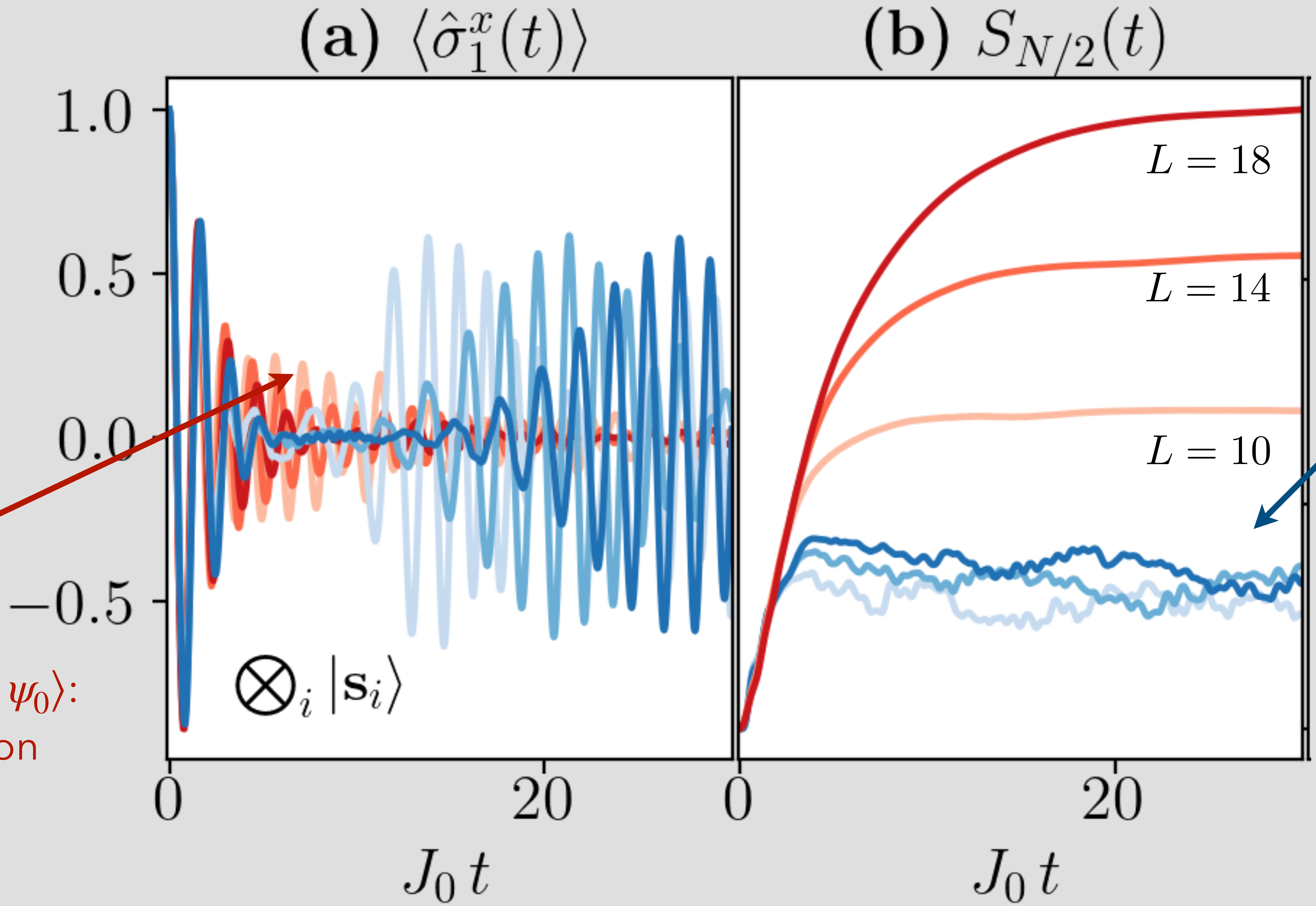



Large spin eigenstates vs ergodic spectrum?

# Numerics: Dynamics at $\alpha = 0.7$

Is it a persistent effect?

Dynamics which strongly depends on the initial state:



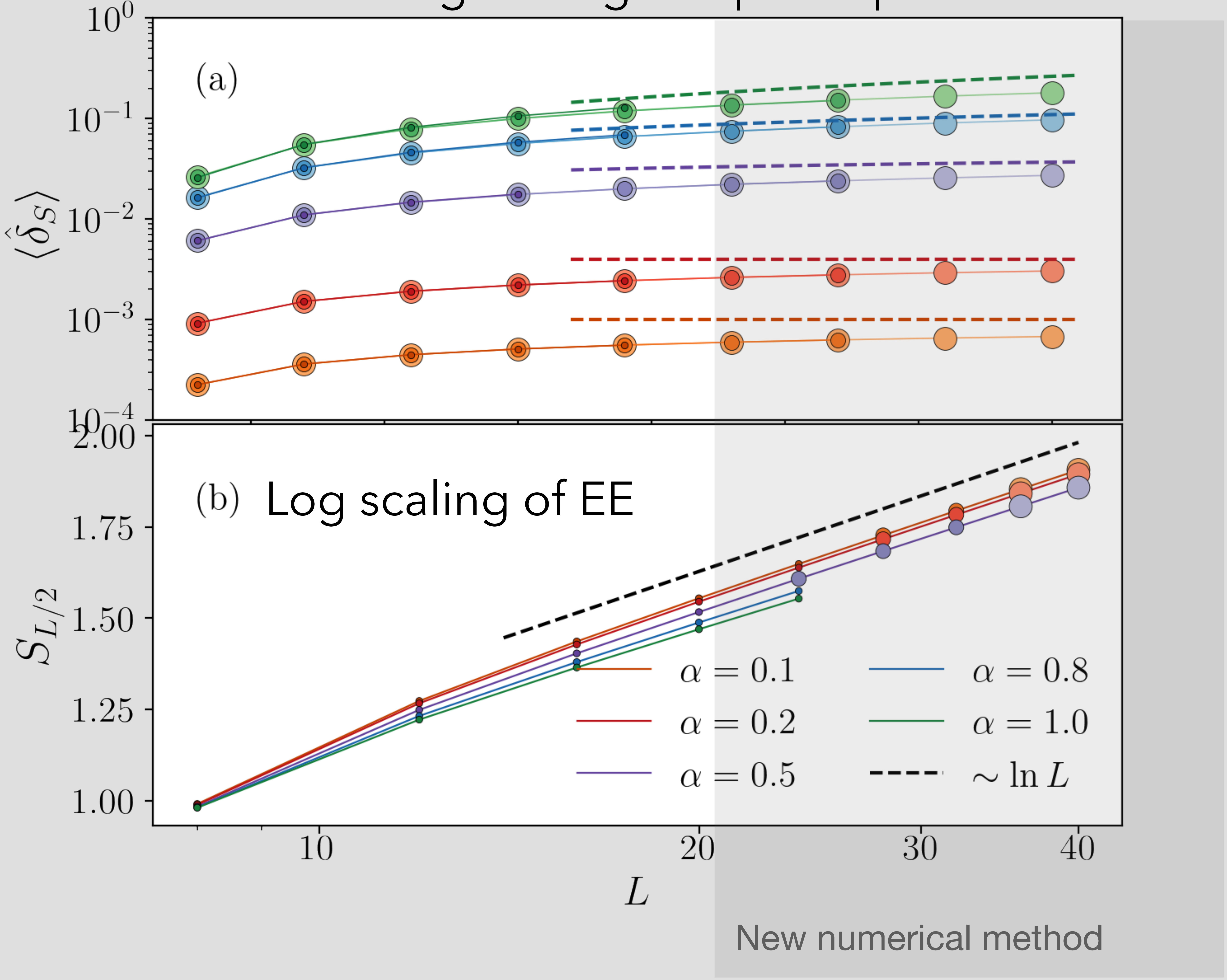

Random product $|\psi_0\rangle$: fast thermalization

Polarized (coherent) $|\psi_0\rangle$: regurrences and log EE

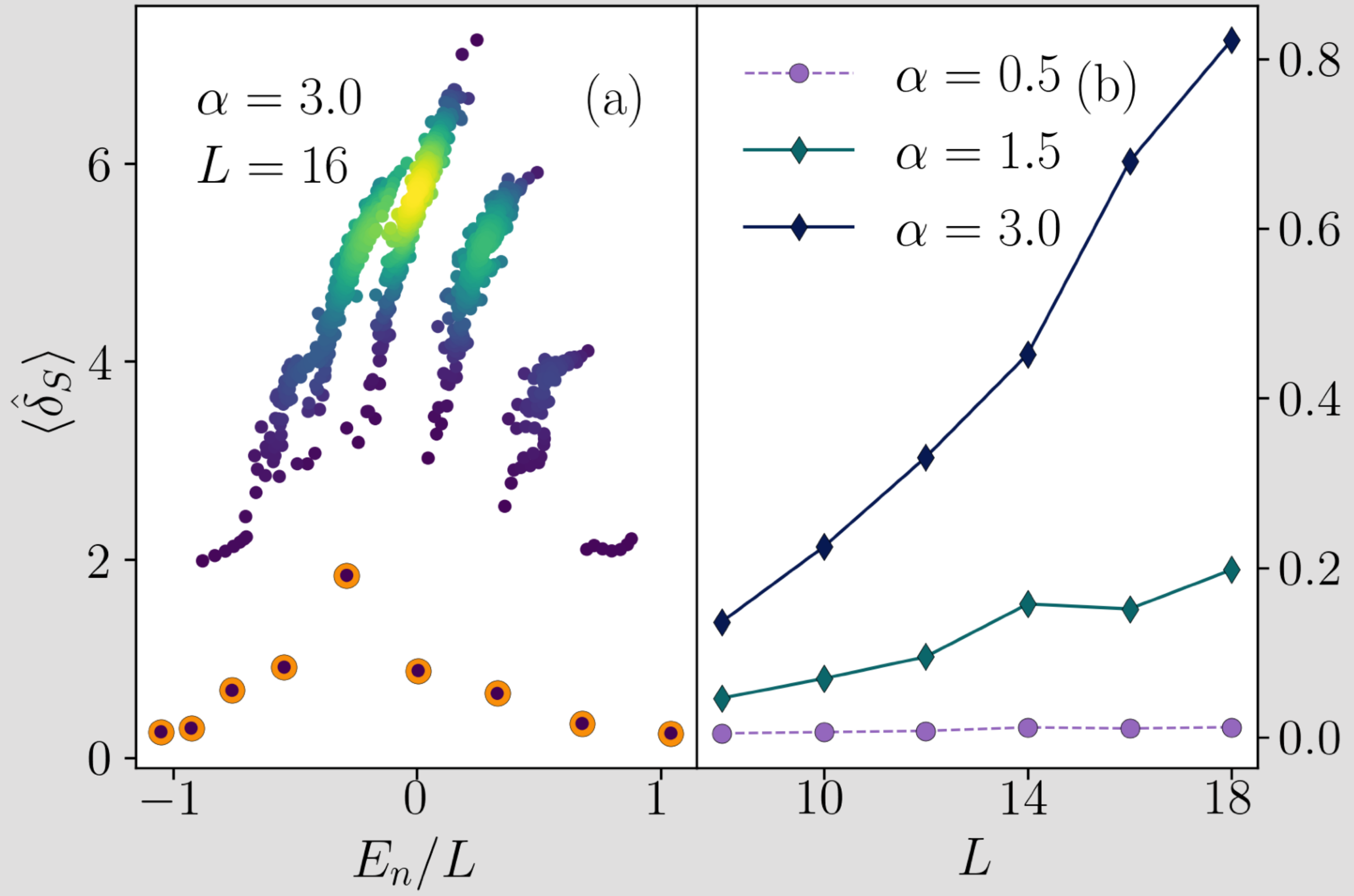
Daley et al. – PRX 2013,
Lerose and SP - PRR 2020
Lerose and SP- PRA 2020

# Numerics: scaling with the system size?

Subleading scaling of spin depletion




Larger $\alpha$ :




Stability of eigenstates for $0 < \alpha \lesssim 1$?

# Theory: expansion of the mean-field spectrum

$$H_\alpha = -J\sum_{i<j}^{L}\frac{\sigma_i^x\sigma_j^x}{|i-j|^\alpha} - h\sum_{i}^{L}\sigma_i^z = H_{\alpha=0} + V_\alpha$$

**$k=0$ part:**

$$H_{\alpha=0} = -J(S^x)^2 - gS^z$$

- Quantum numbers:

$$\delta_S = L/2 - S$$

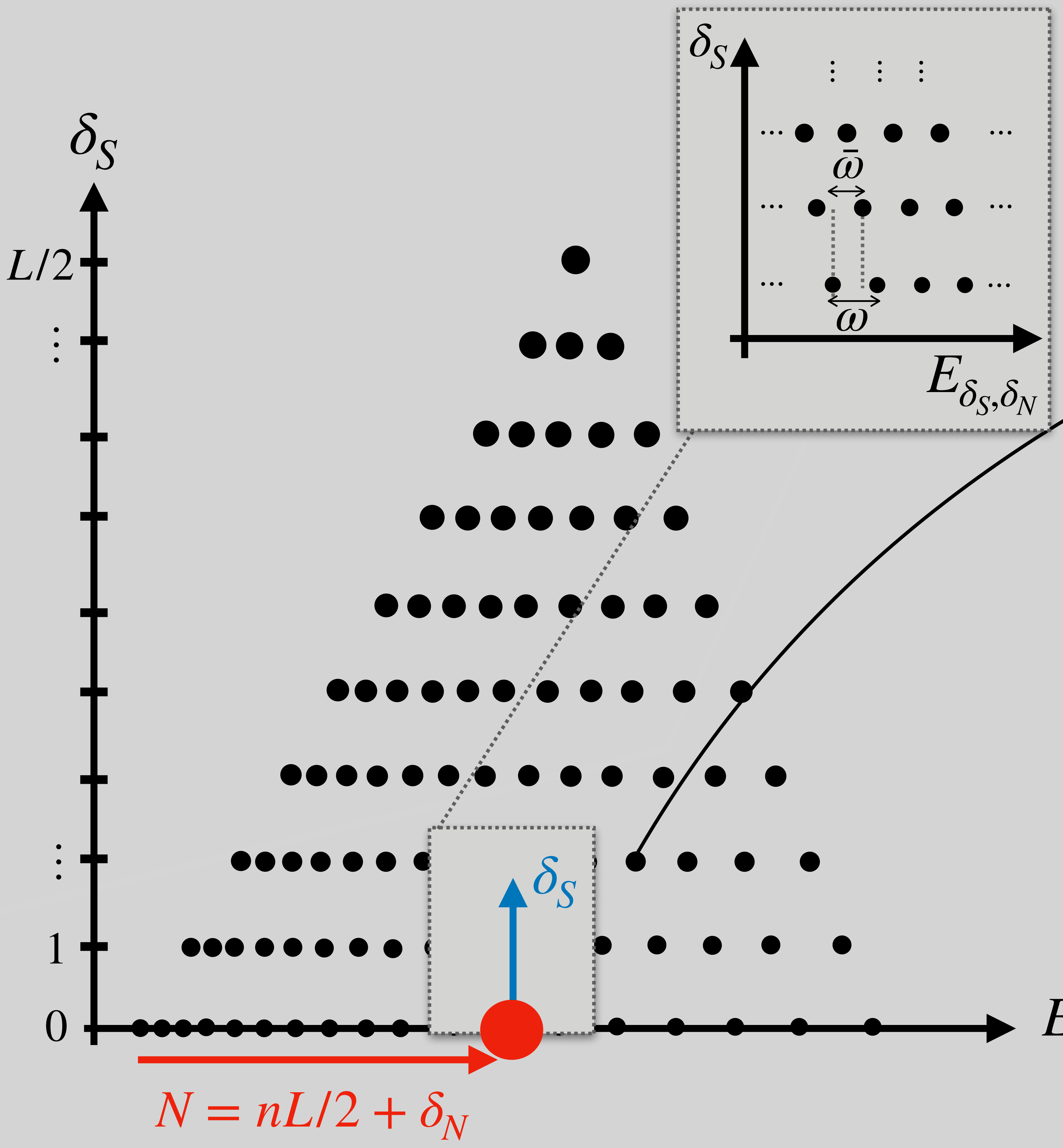
exponential degeneracy in $\delta_S$

$$N = 0,1,\ldots,2S$$

- Large-$L$ semiclassical spectrum:

$$E_{\delta_S,N} \sim L\,\mathscr{E}(n) + \omega(n)\,\delta_N + \bar{\omega}(n)\,\delta_S + \mathcal{O}(L^{-1})$$

Local regular lattice

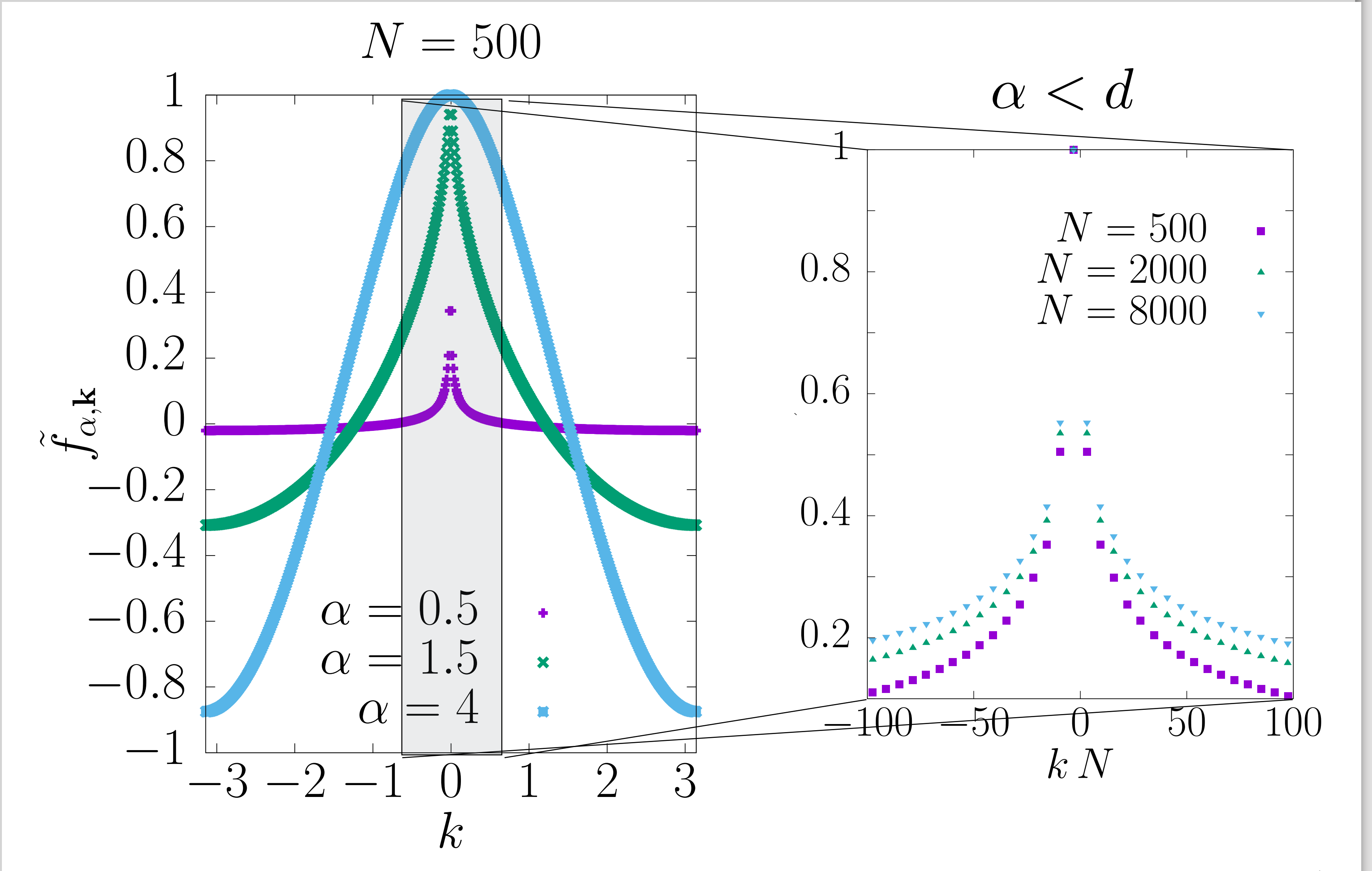



**$k \neq 0$ part:**

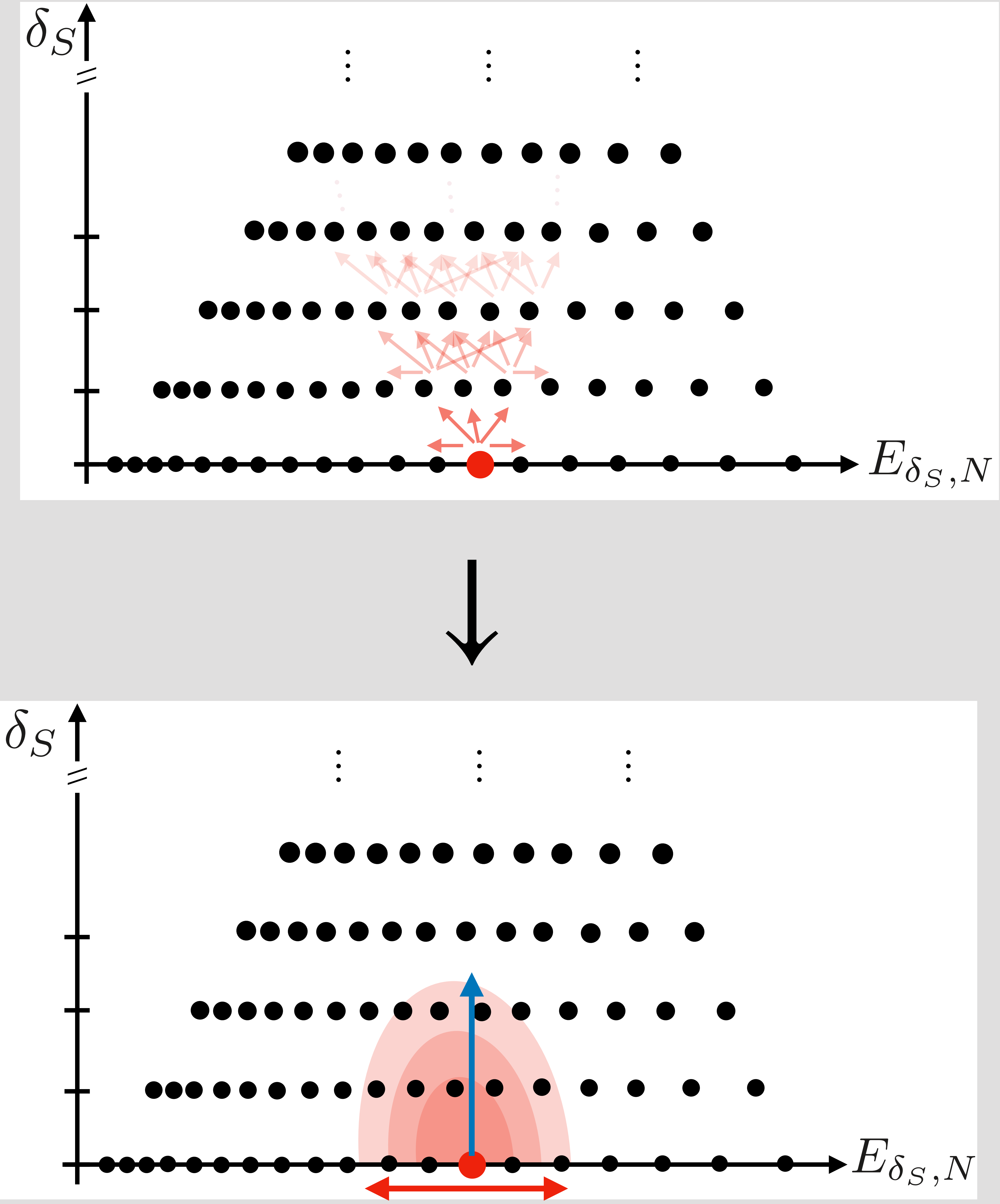
$$V_\alpha = -J_0\sum_{k\neq 0}\tilde{f}_{k,\alpha}\left(\tilde{S}_k^x\tilde{S}_{-k}^x + \tilde{S}_k^y\tilde{S}_{-k}^y\right)$$

$$|\{k_1,\ldots,k_{\delta_S}\}\,;\,N\rangle$$

$\Longrightarrow$ collective eigenstates labelled by magnon occupation



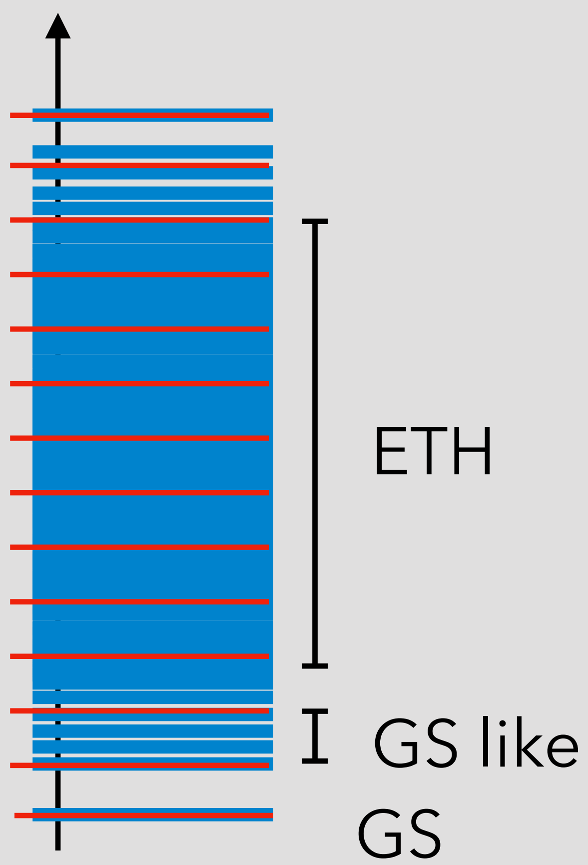
# Intuition for eigenstate localization




$H_{\alpha\neq 0}$ :

**off-resonant** creation/destruction of magnon pairs

Different from:




1. small $\alpha$ : $\sum_{k\neq 0} |\tilde{f}_k(\alpha)|^2 \sim L^0$
2. Absence of resonances (integrable mean-field limit)

**eigenstate localization** in subspace

$$\delta_S \ll L/2, \quad |\delta_N| \ll L$$

# Full calculation

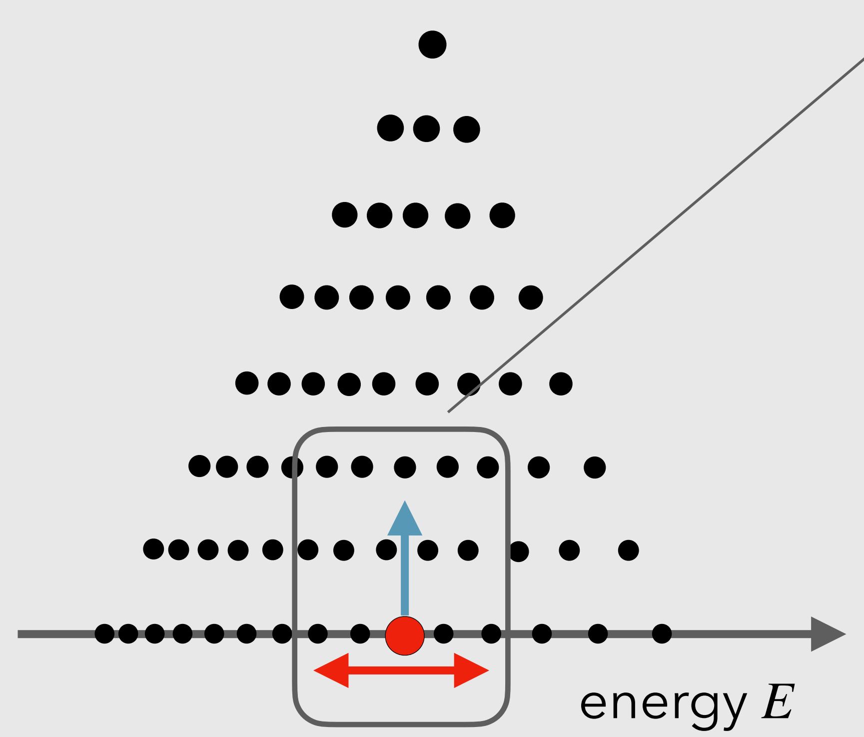
**Self consistent many-body scars for $0 < \alpha \lesssim d$**

1. Write matrix elements around a collective eigenstate

2. Exactly solvable away from resonances $\omega \neq p\bar{\omega}$ (spectrum + eigenvectors)

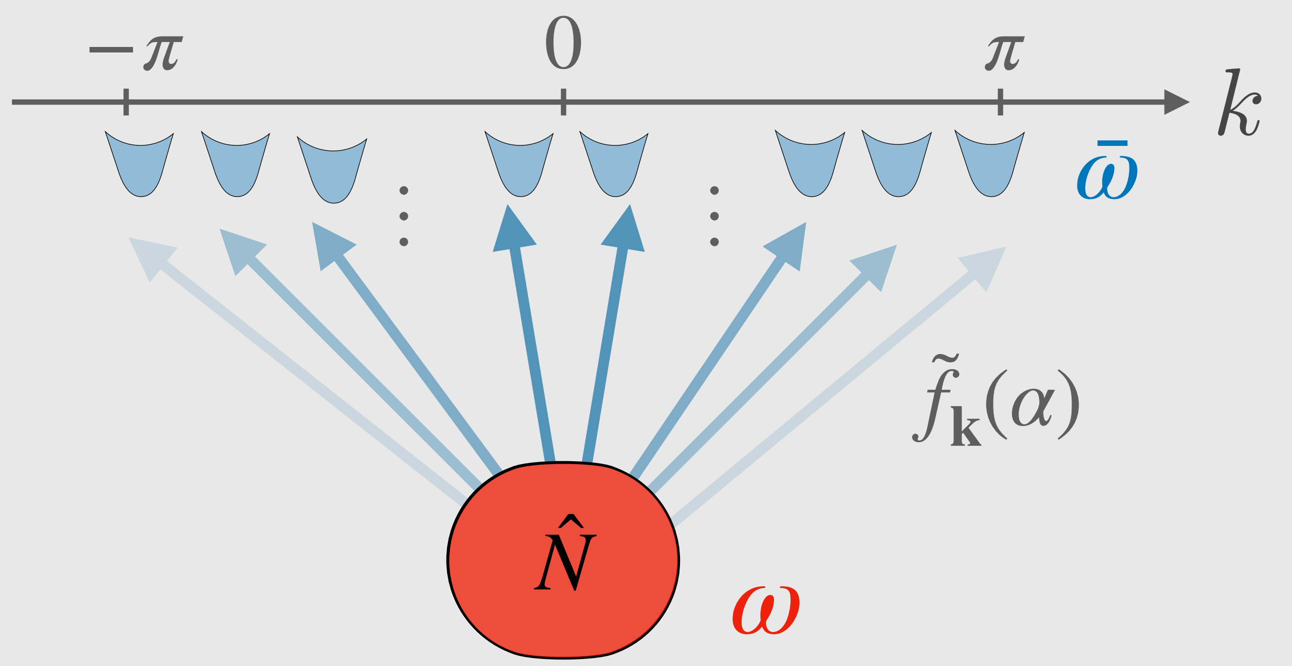
3. Compute $\langle \hat{\delta}_S \rangle \sim \sqrt{\langle \hat{\delta}_N^2 \rangle} \sim \sum_{k \neq 0} |f_k(\alpha)|^2 \sim \begin{cases} \text{finite} & \text{for } 0 < \alpha < 1/2\,, \\ \log L & \text{for } \alpha = 1/2\,, \\ L^{2\alpha-1} & \text{for } 1/2 < \alpha < 1\,. \\ c(\alpha) \cdot L & \text{for } \alpha > 1 \end{cases}$

4. Self-consistency



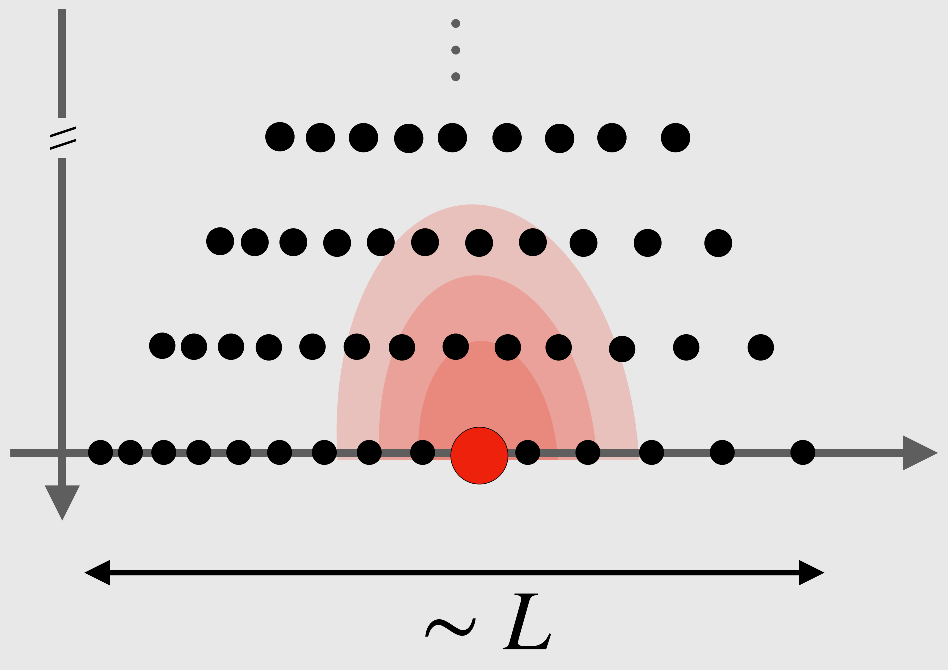

1. $\alpha \neq 0$ : Rotor-magnon Hamiltonian

interacting quantum impurity model

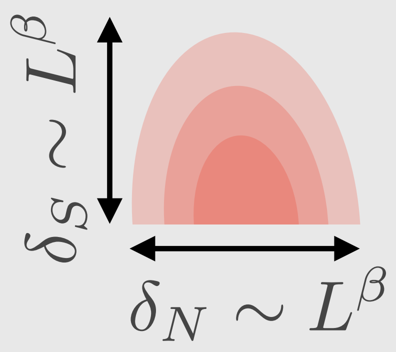



4. $0 < \alpha < d$ : Self-consistent eigenstate localization

2. $e^{i\hat{F}} e^{i\hat{S}}\, \hat{H}_{\alpha,\text{eff}}\, e^{-i\hat{S}} e^{-i\hat{F}} = \omega \hat{N} + \sum_{k \neq 0} \omega_k(\alpha)\, \hat{n}_k$




3.




$\beta = \begin{cases} 0 & \text{for } 0 < \alpha < d/2 \\ 2\alpha - d & \text{for } d/2 < \alpha < d \end{cases}$

# Prediction: Instability from mean-field chaos

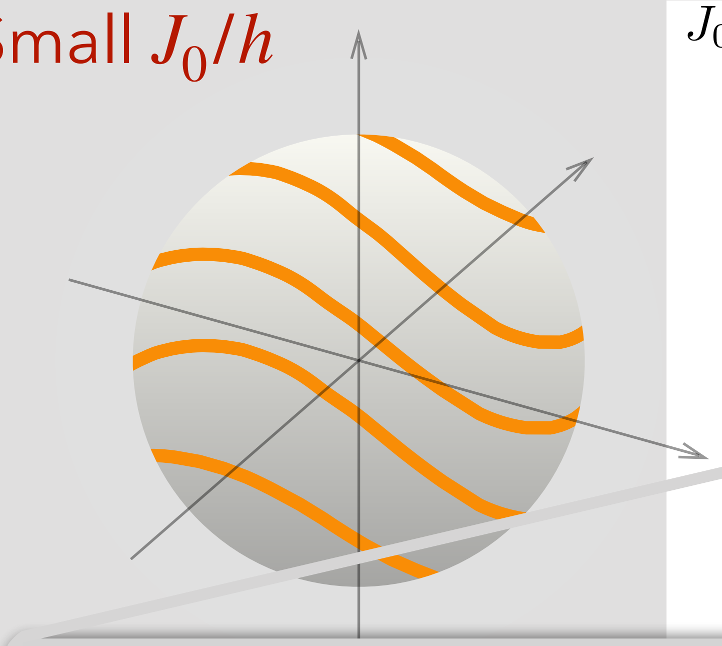
Quantum many-body kicked top:

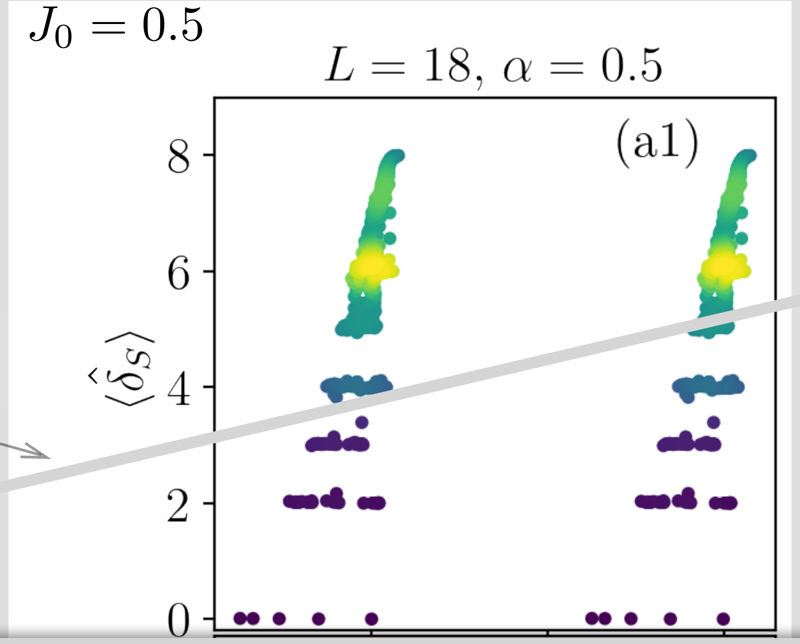
$$\hat{H}_\alpha(t) = \begin{cases} -\dfrac{J_0}{\mathcal{N}_{\alpha,L}} \displaystyle\sum_{j=1}^{L} \sum_{r=1}^{L/2} \dfrac{\hat{\sigma}_j^x \hat{\sigma}_{j+r}^x}{r^\alpha} & t \in \left[-\dfrac{T}{4}, \dfrac{T}{4}\right) \mod T \\ -h \displaystyle\sum_{j=1}^{L} \hat{\sigma}_j^z & t \in \left[\dfrac{T}{4}, \dfrac{3}{4}T\right) \mod T \end{cases}$$

$\alpha = 0$ : semiclassical Haake, …

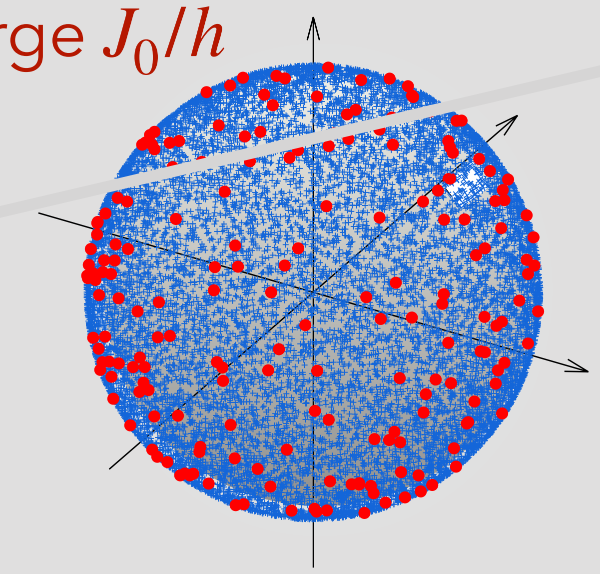
integrability-chaos crossover

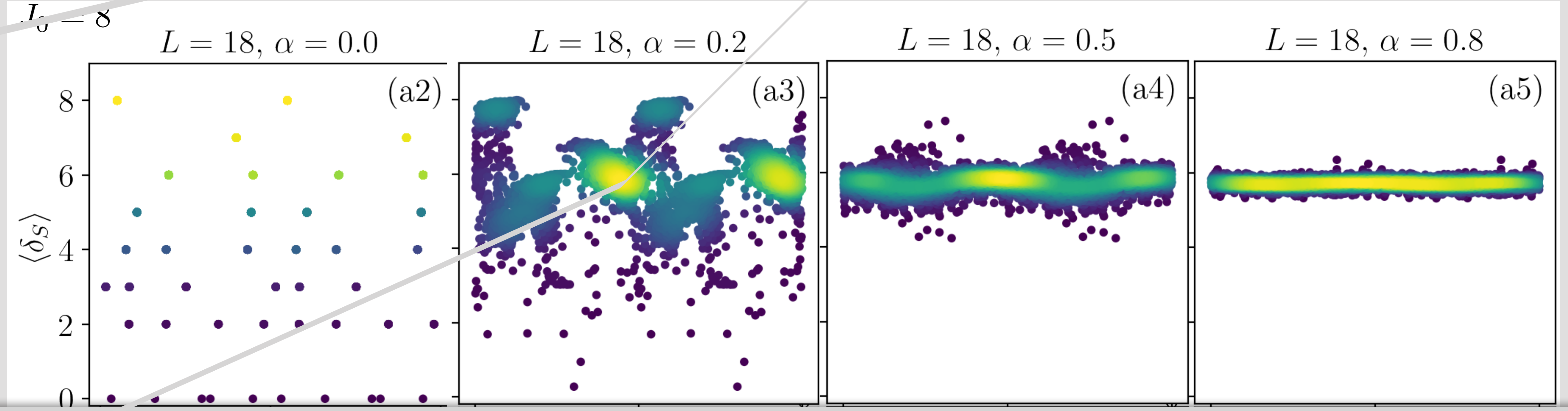
Small $J_0/h$





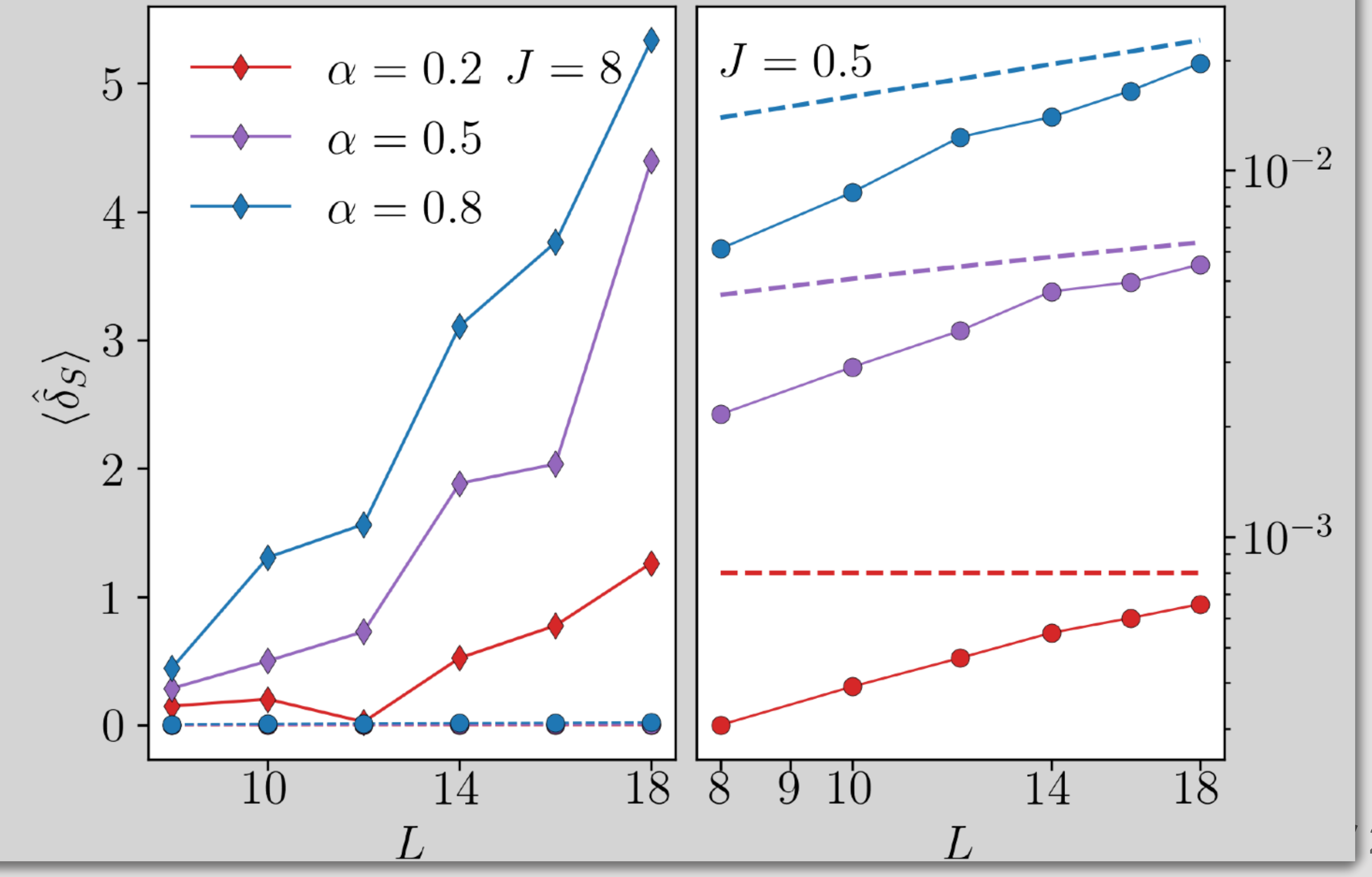

Large $J_0/h$






Scaling with system size

- Absence of heating for $0 < \alpha < d$ in classical KAM regime
- Semiclassical chaos destroyes quantum many-body scarring

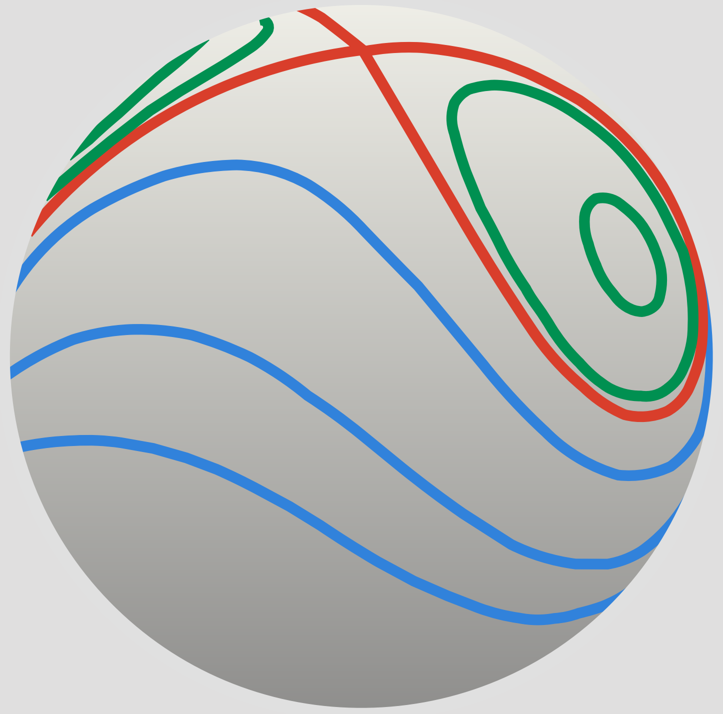


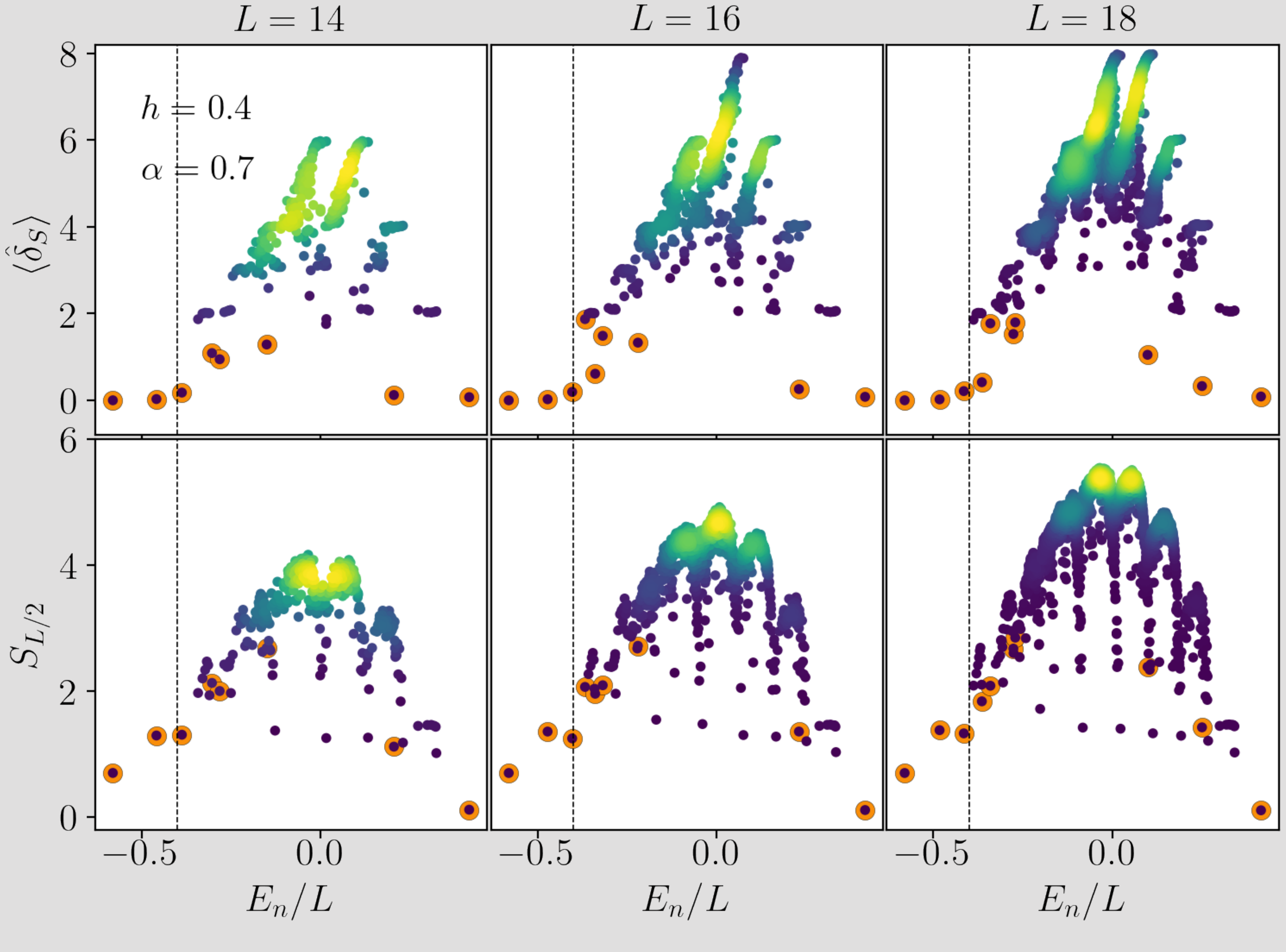
# Prediction: instability at the Excited State Phase Transition

## Mean-field criticality at finite energy densities

Rev: Cejnar, Strànsky Macek Kloc, J.phys.A 2021

For $\alpha = 0$ associated to separatrix trajectories





# In summary…

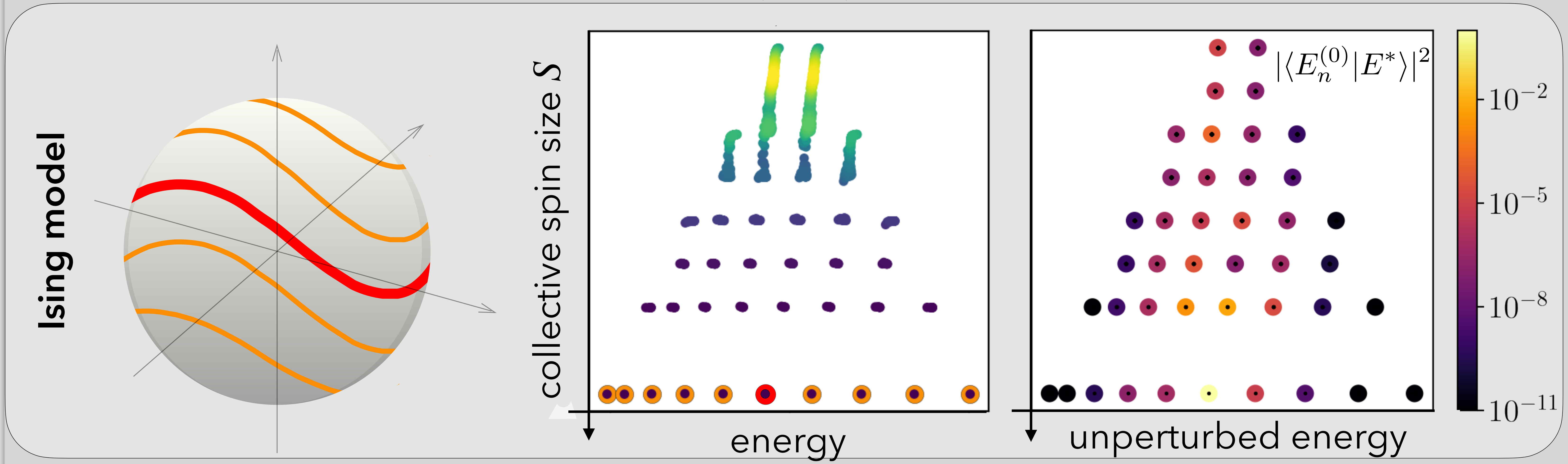
Lerose, Parolini, Fazio, Abanin and **SP**, Arxiv 2309.12504

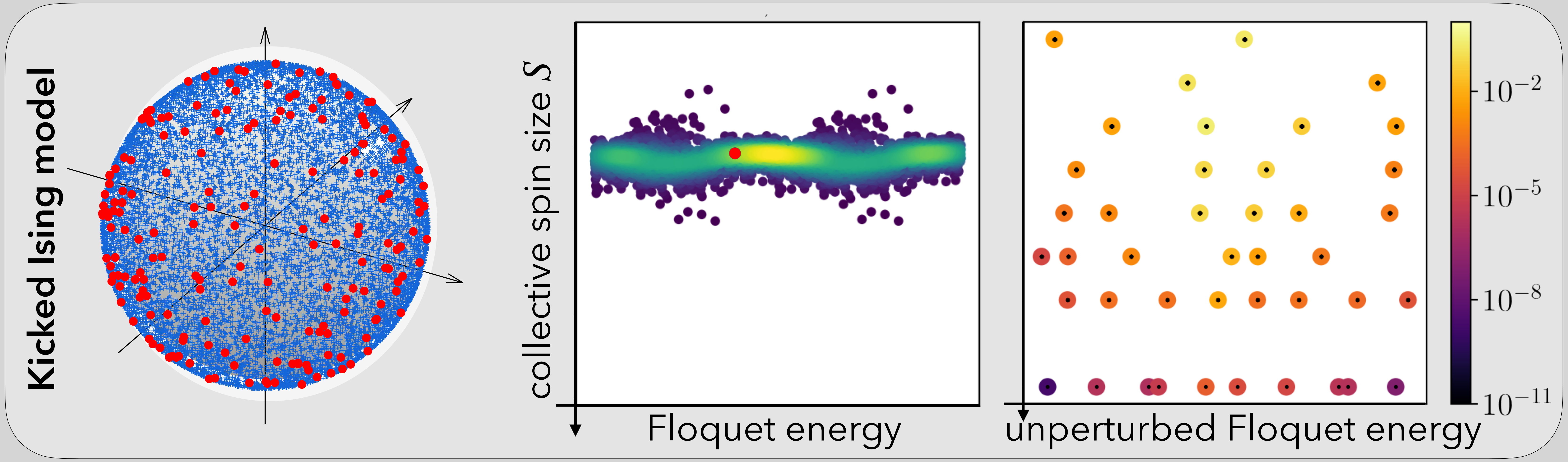
## 1) Integrability of the underlying classical limit

Classical limit $\alpha = 0$

Exact spectrum $0 < \alpha < d$

Eigenstate (de)localization $0 < \alpha < d$

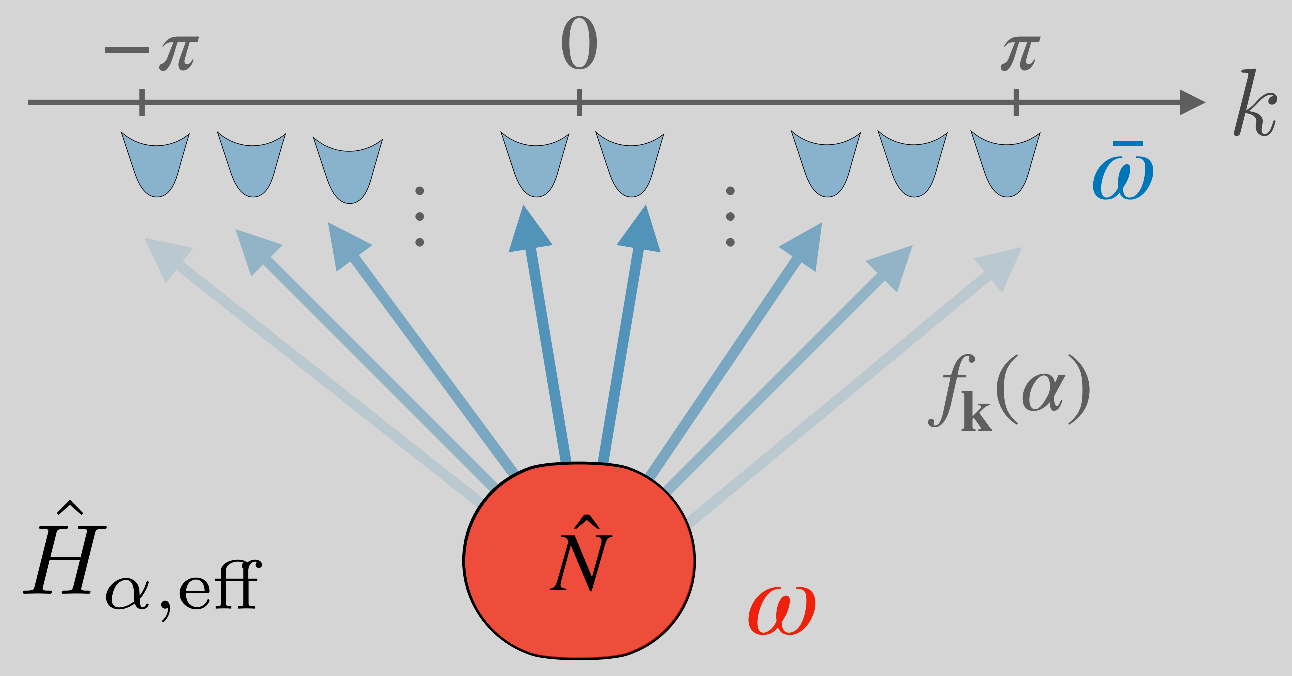



## 2) Long-range interactions

$0 < \alpha < d$

Shown by a self-consistent magnon/rotor projection

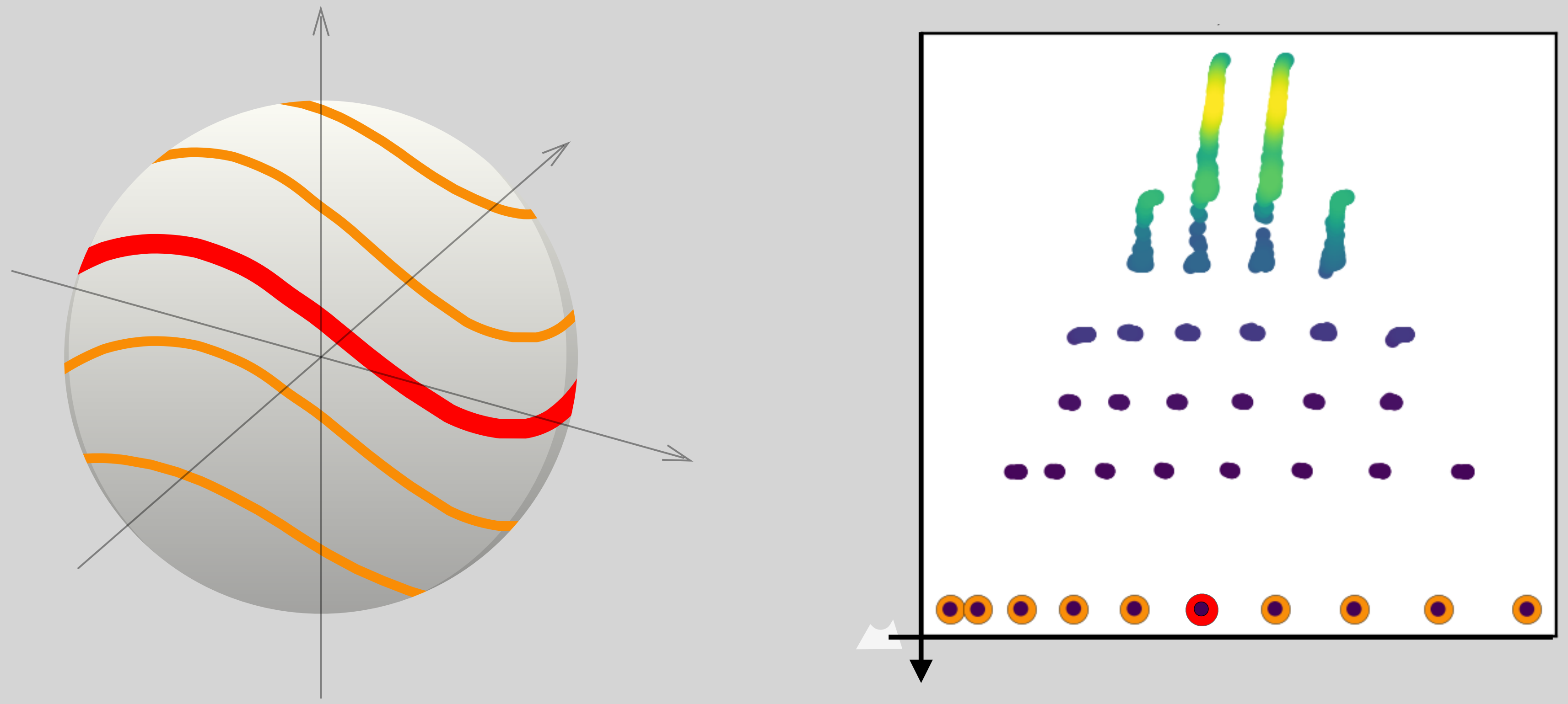



these systems effectively interpolates between few body and many-body physics

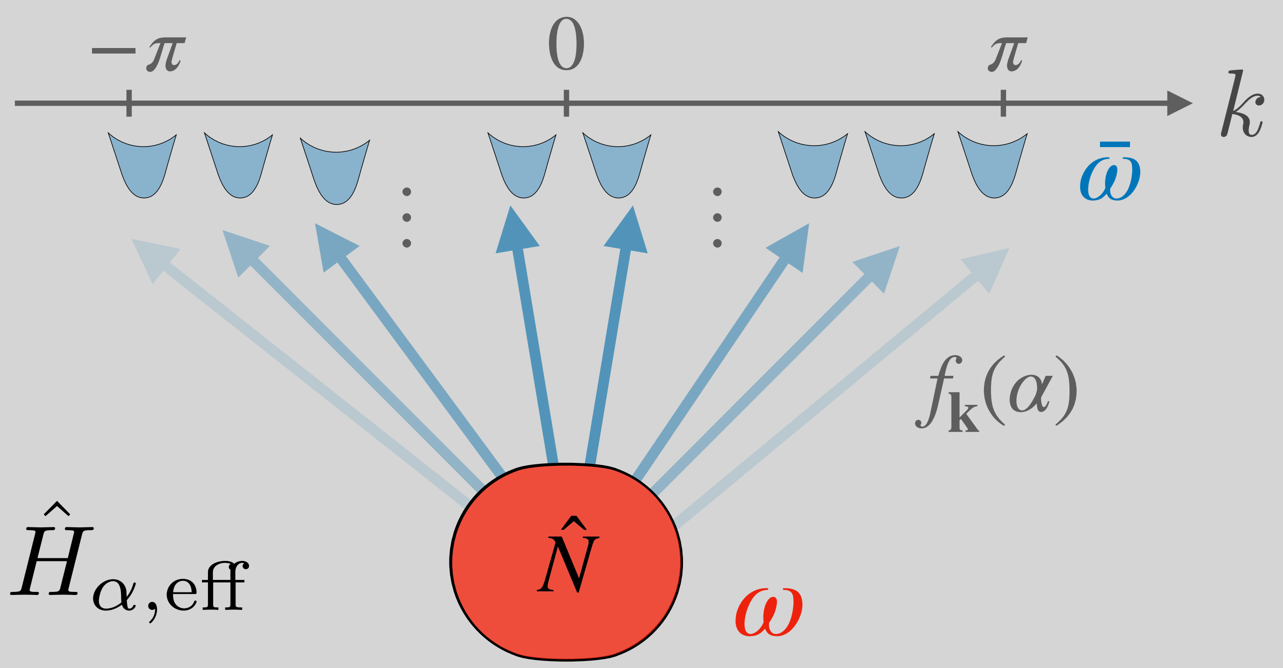
# … many open perspectives

Lerose, Parolini, Fazio, Abanin and **SP**, Arxiv 2309.12504

**1) Integrability of the underlying classical limit**



**2) Long-range interactions**




- Quantum KAM? Instability to many-body perturbations?
- suppressed heating for $0 < \alpha < d$: time-crystal?
- *Robust* Dicke-like states with $\alpha \neq 0$ $\rightarrow$ Useful metrological applications?
- generalize mechanism to be applied to other systems (PXP)?
- novel analytical method → new numerical approach?

Thanks!