

# Statistical physics of disordered polymerized membranes

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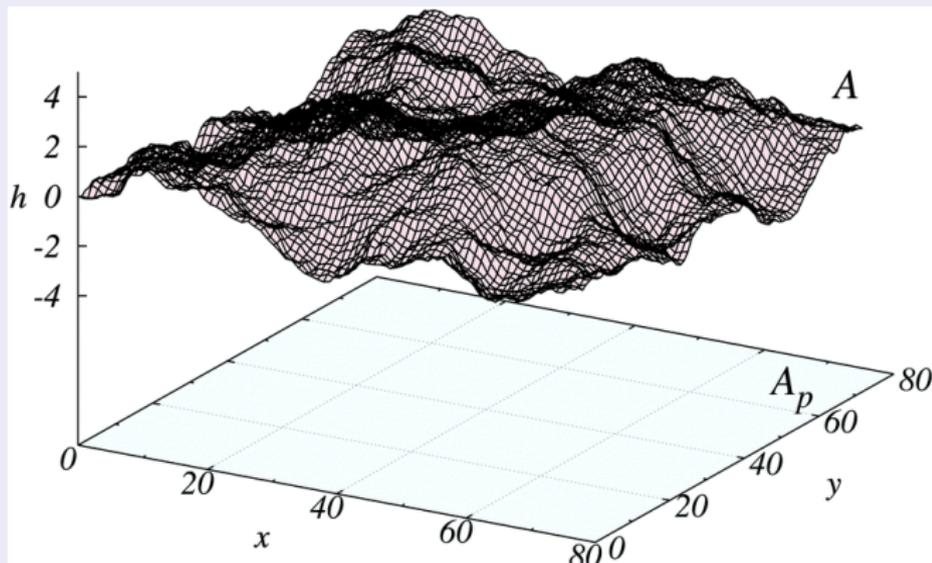
INHOMOGENEOUS RANDOM SYSTEMS (Paris January 27-28 2026)

# Outline

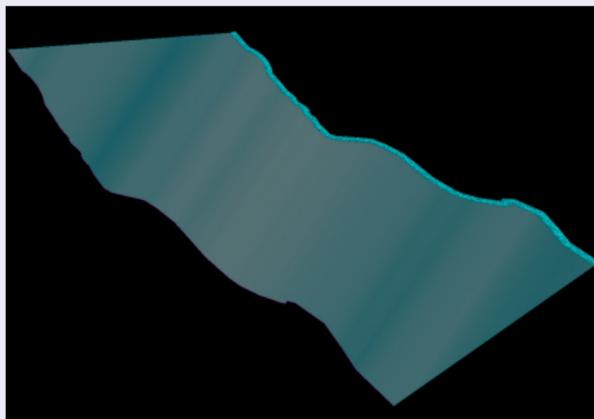
- 1 Introduction
- 2 Fluid membranes and polymerized membranes
- 3 Statistical physics of pure membranes
- 4 Statistical physics of disordered membranes
- 5 Conclusion

# Introduction

- **Membranes:** D-dimensional extended objects embedded in a d-dimensional space subject to quantum and/or thermal and/or disorder fluctuations



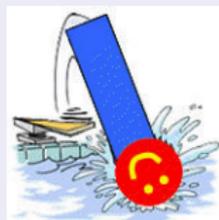
- High energy physics: (sum over) **surfaces** occurs within, e.g.:
  - discretization of quantum gravity
  - string theory (see also branes ... )
    - $\implies$  a string generates a surface (worldsheet)



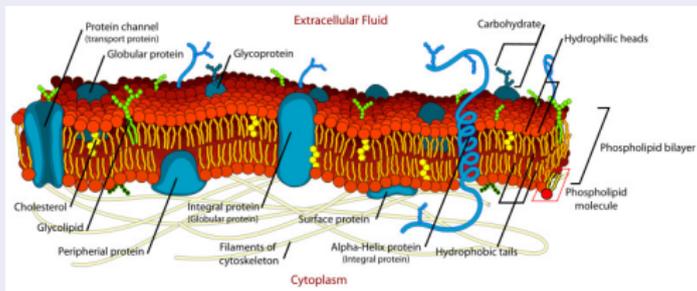
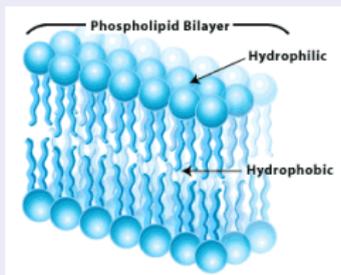
- chemical physics / biology :

⇒ structures made of **amphiphilic molecules** (ex:  
phospholipid)

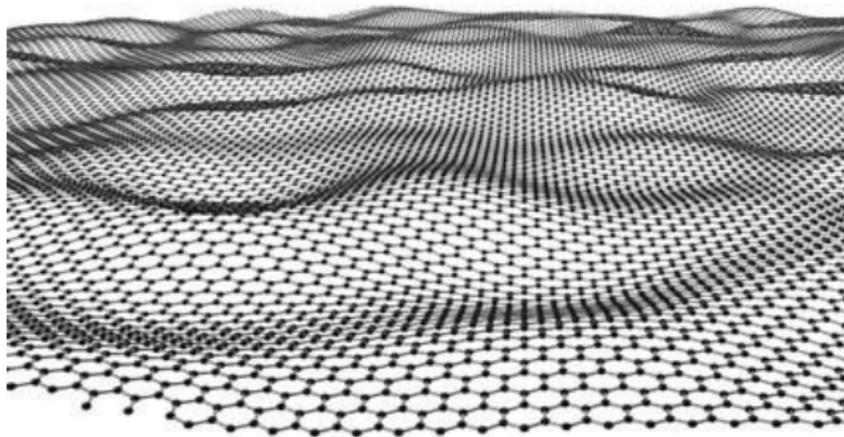
- one hydrophilic head
- hydrophobic tails



⇒ bilayers:



- condensed matter physics: graphene, silicene, phosphorene ...  
uni-layers of atoms located on a honeycomb lattice
- striking properties:
  - high electronic mobility, transmittance, conductivity, ...
  - mechanical properties: both extremely **strong** and **soft**  
⇒ genuine **2D fluctuating membrane**



## Generic questions :

- effects of – thermal – fluctuations ?

⇒ phase transition ?

⇒ structure of the ordered, “flat”, phase at low temperatures ?

- effects of quenched disorder ?

- (effects of quantum fluctuations as  $T \rightarrow 0$  ? )

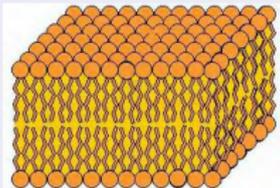
(Coquand and D.M. 16)

- (effects of a dynamics)

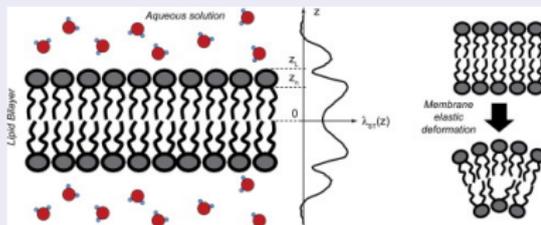
⇒ depends crucially on the nature of the membrane

# Fluid membranes vs polymerized membranes

## Fluid membranes



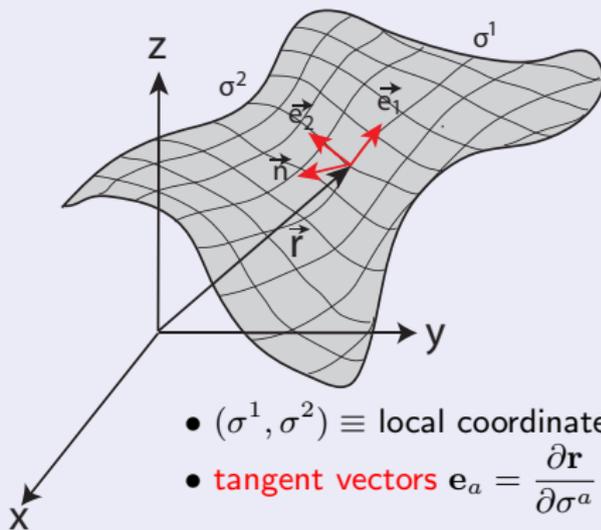
- weakly interacting molecules
  - free diffusion inside the membrane plane  $\implies$  **no shear modulus**
- hydrophobic effect  $\implies$  small elasticity (and compressibility)
  - $\implies$  only **curvature**



## Free energy

- point of the surface described by the embedding:

$$\mathbf{r}: \boldsymbol{\sigma} = (\sigma^1, \sigma^2) \rightarrow \mathbf{r}(\sigma^1, \sigma^2) \in \mathbb{R}^d$$



- $(\sigma^1, \sigma^2) \equiv$  local coordinates on the membrane
- tangent vectors**  $\mathbf{e}_a = \frac{\partial \mathbf{r}}{\partial \sigma^a} = \partial_a \mathbf{r} \quad a = 1, 2$
- unit-norm vector normal** to  $(\mathbf{e}_1, \mathbf{e}_2)$ :  $\hat{\mathbf{n}} = \frac{\mathbf{e}_1 \times \mathbf{e}_2}{|\mathbf{e}_1 \times \mathbf{e}_2|}$

- curvature tensor  $\mathbf{K}$ :  $K_{ab} = -\hat{\mathbf{n}} \cdot \partial_b \mathbf{e}_a = \mathbf{e}_a \cdot \partial_b \hat{\mathbf{n}}$
- $K_{ab}$  can be locally diagonalized with eigenvalues  $K_1$  and  $K_2$ 
  - mean or *extrinsic* curvature:

$$H = \frac{1}{2}(K_1 + K_2) = \frac{1}{2} \text{Tr} \mathbf{K}$$

- Gaussian or *intrinsic* curvature:  $K = K_1 K_2 = \det K_a^b$   
 $\Rightarrow$  no role in fixed topology and open surfaces (Gauss-Bonnet theorem)

$\Rightarrow$  bending energy with rigidity constant  $\kappa$ :

$$F = \frac{\kappa}{2} \int d^2\boldsymbol{\sigma} \sqrt{g} H^2$$

- $g_{\mu\nu} = \partial_\mu \mathbf{r} \cdot \partial_\nu \mathbf{r} \equiv$  **metric induced** by the embedding  $\mathbf{r}(\boldsymbol{\sigma})$
- $\sqrt{g}$  ensures reparametrization invariance of  $F$

- a remark: with  $\partial_a \hat{\mathbf{n}} = K_{ab} \mathbf{e}^b$  one has:

$$F = \frac{\kappa}{2} \int d^2\sigma (\partial_a \hat{\mathbf{n}})^2 \quad \text{or} \quad F = -\frac{\kappa'}{2} \sum_{\langle i,j \rangle} \hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_j$$

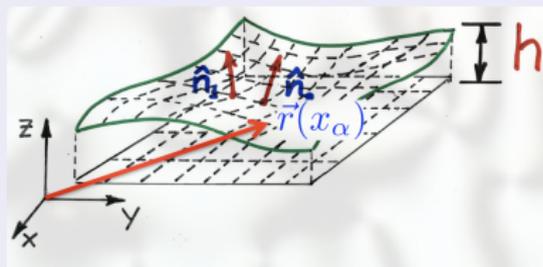
where  $\hat{\mathbf{n}}_i$  is a unit normal vector on the plaquette  $i$

- very close to a  $O(d)$  nonlinear  $\sigma$ -model / Heisenberg spin system:
  - with (rigidity) coupling constant  $\kappa$
  - with 'spins' living on a fluctuating surface
  - with  $d$  playing the role of the number of components  $N$

## Thermal fluctuations ?

- Low temperatures: Monge parametrization

$x = \sigma_1$ ,  $y = \sigma_2$  and  $z = h(x, y)$  with  $h$  height, capillary, mode



- $\mathbf{r}(x, y) = (x, y, h(x, y))$  parametrizes points

- $\hat{\mathbf{n}}(x, y) = \frac{(-\partial_x h, -\partial_y h, 1)}{\sqrt{1 + (\partial_i h)^2}}$
- $\hat{\mathbf{n}}(x, y) \cdot \mathbf{e}_z = \cos \theta(x, y) = \frac{1}{\sqrt{1 + (\partial_i h)^2}}$

- Free energy: Gaussian action + one more derivative

$$F \simeq \frac{\kappa}{2} \int d^2 \mathbf{x} (\Delta h)^2 + \mathcal{O}(h^4)$$

- **Flat phase ?**  $\implies$  harmonic fluctuations of  $\theta(x, y)$ :

$$\begin{aligned} \langle \theta(x, y)^2 \rangle &\simeq k_B T \int d^2 q \langle \partial_i h(\mathbf{q}) \partial_i h(-\mathbf{q}) \rangle \\ &= k_B T \int d^2 q \frac{q^2}{\kappa q^4} \simeq -\frac{k_B T}{\kappa} \log qa \xrightarrow{q \rightarrow 0} \infty \end{aligned}$$

$\implies$  **no long range order** between the normals (Mermin-Wagner)

At next order in  $h$ ,  $\kappa$  is **renormalized** and **decreases** at long distances.:

$$\kappa_R(q) = \kappa - \frac{3k_B T}{2\pi} \left(\frac{d}{2}\right) \ln\left(\frac{1}{qa}\right)$$

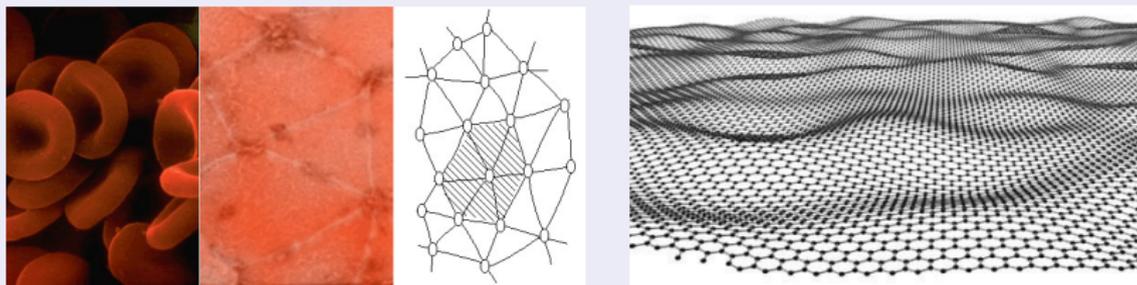
$\implies$  divergence of  $\langle \theta(x, y)^2 \rangle$ : worse  $\implies$  **no long-range order**

$\implies$  strong analogy with 2D-NL $\sigma$  model:

- correlations:  $\langle \hat{\mathbf{n}}(\mathbf{r}) \cdot \hat{\mathbf{n}}(\mathbf{0}) \rangle \sim e^{-r/\xi}$
- correlation length – inverse mass gap:  $\xi \simeq a e^{4\pi\kappa/3k_B T d}$
- $N - 2 \implies d/2$
- nothing really new ...

# Polymerized membranes

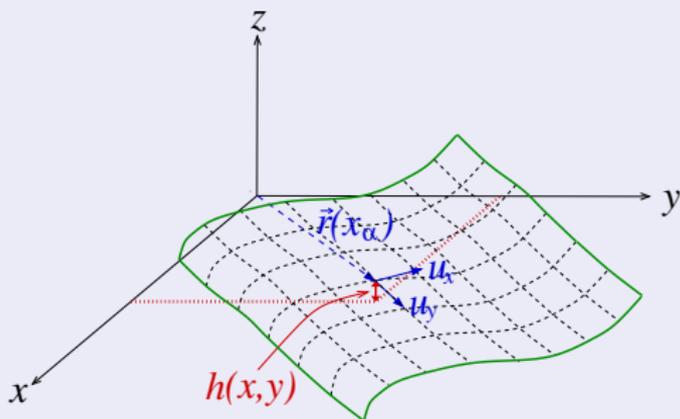
- chemical physics/biology: red blood cell, ...
- condensed matter physics: graphene, phosphorene, ...



- strongly interacting molecules by  $V(|\mathbf{r}_i - \mathbf{r}_j|)$   
 $\implies$  **bending** and **elastic** energy contributions

## Free energy

- Flat reference configuration:  $\mathbf{r}_0(x, y) = (x, y, z = 0)$
  - Fluctuations:  $\mathbf{r}(x, y) = \mathbf{r}_0 + u_x(x, y) \mathbf{e}_1 + u_y(x, y) \mathbf{e}_2 + h(x, y) \hat{\mathbf{n}}$
- $h \equiv$  height field and  $u_i \equiv$  phonon fields



- Free energy: **curvature + elasticity/shear**

$$F \simeq \int d^2\mathbf{x} \left[ \frac{\kappa}{2} (\Delta h)^2 + \lambda u_{ab} u_{ab} + \mu u_{aa} u_{aa} \right]$$

$u_{ab} \equiv$  stress tensor  $\sim$  encodes fluctuations with respect to the flat configuration  $\mathbf{r}_0$

$$u_{ab} = \frac{1}{2} (\partial_a \mathbf{r} \cdot \partial_b \mathbf{r} - \partial_a \mathbf{r}_0 \cdot \partial_b \mathbf{r}_0) = \frac{1}{2} [\partial_a u_b + \partial_b u_a + \partial_a \mathbf{u} \cdot \partial_b \mathbf{u} + \partial_a h \partial_b h]$$

$\lambda, \mu$ : Lamé coefficients

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$\lambda, \mu$ : Lamé coefficients



- coupling between height and phonon fluctuations

$\implies$  **frustration** of height fluctuations:

$\implies$  **long range order** between normals

- Self-consistent screening approximation (SCSA)  $\sim$  resummed  $1/d$  expansion (Nelson and Peliti 87)

$$\kappa_{eff}(\mathbf{q}) = \kappa + k_B T \mathcal{K} \int d^2 k \frac{[\hat{q}_a P_{ab}^T \hat{q}_b]^2}{\kappa_{eff}(\mathbf{q} + \mathbf{k}) |\mathbf{q} + \mathbf{k}|^4}$$

$$\implies \kappa_{eff}(\mathbf{q} \rightarrow \mathbf{0}) \sim \frac{\sqrt{k_B T \mathcal{K}}}{|\mathbf{q}|}$$

*i.e.* effective rigidity increased by fluctuations !

- fluctuations of normal vector  $\hat{\mathbf{n}}$ :

$$\langle \theta(x, y)^2 \rangle = k_B T \int d^2 q \frac{1}{\kappa_{eff}(\mathbf{q}) q^2} < \infty!$$

$\implies$  Long-range order between normals in  $D = 2$  and less !

- no trouble with **Mermin-Wagner Theorem**

$F_{eff}$  can be rewritten as an interaction between Gaussian curvatures:

$$F_{eff} = \frac{\kappa}{2} \int d^2\mathbf{x} (\Delta h)^2 + \mathcal{K} \int K(\mathbf{x}) G(\mathbf{x} - \mathbf{y}) K(\mathbf{y})$$

$K \equiv$  Gaussian curvature:

$$K(\mathbf{x}) = -\Delta(\partial_a h \partial_b h) + \partial_a \partial_b (\partial_a h \partial_b h)$$

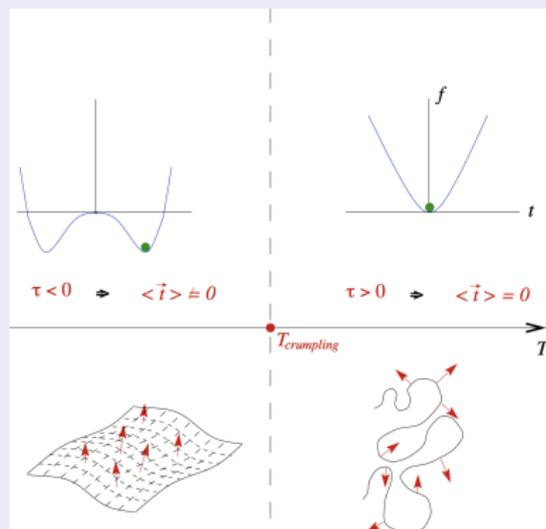
with a non-decreasing kernel:

$$G(\mathbf{x} - \mathbf{y}) \propto |\mathbf{x} - \mathbf{y}|^2 \ln |\mathbf{x} - \mathbf{y}|$$

$\implies$  **evades Mermin-Wagner Theorem !**

## Phase transition in polymerized membranes

- spontaneous symmetry breaking in  $D = 2$  and even in  $D < 2$   
 $\implies$  crumpled-to-flat transition



## Flat phase of polymerized membranes

$\implies$  low-temperature, ordered, flat, phase with **non-trivial** correlations in the I.R.

$$\begin{cases} G_{hh}(\mathbf{q}) \sim q^{-(4-\eta)} \\ G_{uu}(\mathbf{q}) \sim q^{-(4-2\eta)} \end{cases}$$

with  $\eta \neq 0 \implies$  associated e.g. correlations of stable membrane (e.g. graphene monolayer)

- **Challenges:**

- i) understanding the structure of the low temperature phase
- ii) computing  $\eta$

# Statistical physics of pure membranes

One-loop perturbative approach of the flat phase

(Aronovitz, Golubović and Lubensky 88; Gitter, David, Leibler and Peliti 89)

$$F \simeq \int d^D \mathbf{x} \left[ \frac{\kappa}{2} (\Delta h)^2 + \lambda g_{ab}^2 + \mu g_{aa}^2 \right]$$

$\implies$  perturbative expansion in  $\lambda$ ,  $\mu$  and  $\epsilon = 4 - D$

- **non-trivial fixed point**  $P_4$  governs the flat phase
- **increasing** rigidity  $\kappa_{eff}(\mathbf{q}) \underset{q \rightarrow 0}{\sim} q^{-\eta}$
- **increasing** elasticity  $\mathcal{K}_{eff}(\mathbf{q}) \underset{q \rightarrow 0}{\sim} q^\eta$

- Flat phase properties poorly determined in  $D = 2$  (far from  $D = 4$ ):  $\eta = 0.96$  far from MC predictions:  $\eta = 0.85$
- SCSA: nice result  $\eta = 0.821$  (Le Doussal and Radzihovsky 92-18)
- SCSA or weak-coupling expansion extremely tedious beyond leading order due to :
  - multiplicity of fields:  $h, u$  and the coupling constants  $\lambda, \mu$
  - derivative nature/complexity of the interaction

⇒ use of a non perturbative RG approach  
(J.-P. Kownacki and D.M. 09)

# Non perturbative renormalization group

Wilson program: (K.G. Wilson, L.P. Kadanoff, J.B. Kogut ... 70's)

- gradual integration over high momentum fluctuations

$$\mathcal{Z} = \int \mathcal{D}\zeta e^{-\mathbf{H}[\zeta]}$$

splitting:  $\zeta(\mathbf{q}) = \zeta_{>}(\mathbf{q}) + \zeta_{<}(\mathbf{q})$

with: 
$$\begin{cases} \zeta_{>}(\mathbf{q}) = \zeta(\mathbf{q}) & \text{for } k \leq q \leq \Lambda = a^{-1} \\ \zeta_{<}(\mathbf{q}) = \zeta(\mathbf{q}) & \text{for } 0 \leq q \leq k \end{cases}$$

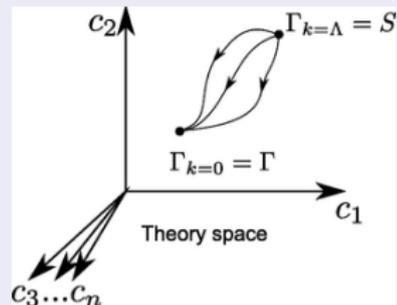
$$\mathcal{Z} = \int \mathcal{D}\zeta_{<} e^{-\mathbf{H}_k[\zeta_{<}]}$$

Wilson-Polchinski equation:

$$\partial_k \mathbf{H}_k = \frac{1}{2} \int_q \partial_k C_{>}(\mathbf{q}) \cdot \left( \frac{\delta^2 \mathbf{H}_k}{\delta \zeta_{<}(\mathbf{q}) \delta \zeta_{<}(-\mathbf{q})} - \frac{\delta \mathbf{H}_k}{\delta \zeta_{<}(\mathbf{q})} \frac{\delta \mathbf{H}_k}{\delta \zeta_{<}(-\mathbf{q})} \right)$$

- best formulation in terms of **running effective action** (or running Gibbs free energy)  $\Gamma_k$  in which high-momentum fluctuations –  $k \leq q \leq \Lambda$  – have been integrated out (Wetterich 90's)

- $\Gamma_{k=\Lambda} = H[\zeta] \equiv$  microscopic scale
- $\Gamma_k = \Gamma_k[\phi] \equiv$  running scale  $k$
- $\Gamma_{k=0} = \Gamma[\phi] \equiv$  macroscopic scale



$\Gamma_k$  follows an **exact** equation (Wetterich 93):

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \int d^d \mathbf{q} \partial_k R_k(\mathbf{q}^2) \frac{1}{\Gamma_k^{(2)}[\phi] + R_k(\mathbf{q}^2)}$$

with  $R_k(\mathbf{q}^2) \equiv$  cut-off function

- **one-loop** structure close to that of perturbative field theory:

$$\partial_k \Gamma_k[\phi] = \frac{1}{2} \text{---} \bigcirc \text{---}$$

but with the **full**, *i.e.* **field-dependent**, propagator:

$$\Gamma_k^{(2)}[\phi]_{ij} = \frac{\delta^2 \Gamma_k[\phi]}{\delta \phi_i(\mathbf{q}) \delta \phi_j(-\mathbf{q})}$$

$\implies$  **nonperturbative** in the coupling constants,  $d$ ,  $D$ , etc !

- Effective action  $\Gamma_k[\partial_\mu \mathbf{r}]$  for membranes:

(Kownacki and D.M. 09)

- ansatz for  $\Gamma_k[\partial_\mu \mathbf{r}]$ : **bending** and **elastic** terms

$$\Gamma_k[\partial_\mu \mathbf{r}] = \int d^D \mathbf{x} \frac{\kappa}{2} (\Delta \mathbf{r})^2 + \lambda (\partial_a \mathbf{r} \cdot \partial_b \mathbf{r} - \delta_{ab})^2 + \mu (\partial_a \mathbf{r} \cdot \partial_a \mathbf{r} - \delta_{aa})^2$$

+ power-counting **non renormalizable** terms

$\Rightarrow$  compute  $\Gamma_k^{(2)}[\partial_\mu \mathbf{r}]$

$\Rightarrow$  plug it into the Wetterich's equation

$\Rightarrow$  compute the (non perturbative) RG equations

- Flat phase:  $\eta = 0.849$

MC computation with a interatomic potential for graphene:

$\eta = 0.850$  (Los, Katsnelson, Yazyev, Zakharchenko and Fasolino 09)

- key point: graphene very well described by the ordered phase of a derivative " $\phi^4$ -like" theory at leading order !

Striking facts : in the flat phase (only):

- no corrections at orders  $\varphi^4 \sim (\partial\mathbf{r})^4$  !  
(Essafi, Kownacki and D.M. 14)
- no quantitative corrections at all orders in  $\partial^{2p}$  !  
 $\eta = 0.849$  (Braghin and Hasselmann 10) compared to  $\eta = 0.85$

$\implies$  **extreme stability** of the approach

Question: structure and properties of the **perturbative** theory at higher orders in  $\lambda$  and  $\mu$  ?

## Polymerized membranes at two-loop and three loop order

(Coquand, D.M., Metayer and Teber 20-22)

$$\begin{aligned}
 S[\vec{u}, \vec{h}] = & \frac{1}{2} \int d^D x \left[ \kappa (\Delta \vec{h})^2 \right. \\
 & + \lambda \left( (\partial_i u_i)^2 + \partial_i u_i (\partial_j \vec{h} \cdot \partial_j \vec{h}) + \frac{1}{4} (\partial_j \vec{h} \cdot \partial_i \vec{h})^2 \right) \\
 & \left. + \mu \left( (\partial_i u_i)^2 + \partial_i u_j \partial_j u_j + \partial_i u_j (\partial_i \vec{h} \cdot \partial_j \vec{h}) + i \leftrightarrow j + \frac{1}{2} (\partial_j \vec{h} \cdot \partial_i \vec{h})^2 \right) \right]
 \end{aligned}$$

## Properties

- derivative field theory  $\implies$  **momentum dependent** vertices
- theory "living" in **space-time**: no internal degrees of freedom  
 $h_\alpha$  with  $\alpha = 1 \dots d - D$  and  $u_i$  with  $i = 1 \dots D$

## Results

36 years after Aronovitz et al.:

- the non-trivial stable fixed point  $P_4$  controls the flat phase with remarkable, rapidly decreasing, series

$$\begin{cases} \eta_{3L} & = 0.4800 \epsilon - 0.01152 \epsilon^2 - 0.00334 \epsilon^3 \\ \eta_{\text{NPRG re-expanded}} & = 0.4800 \epsilon - 0.00918 \epsilon^2 - 0.00333 \epsilon^3 \end{cases}$$

- a rapidly converging exponent  $\eta$ : in  $D = 2$  (i.e.  $\epsilon=2$ ) and  $d = 3$ :

– 1L :  $\eta = 0.96$

– 2L :  $\eta = 0.9139$

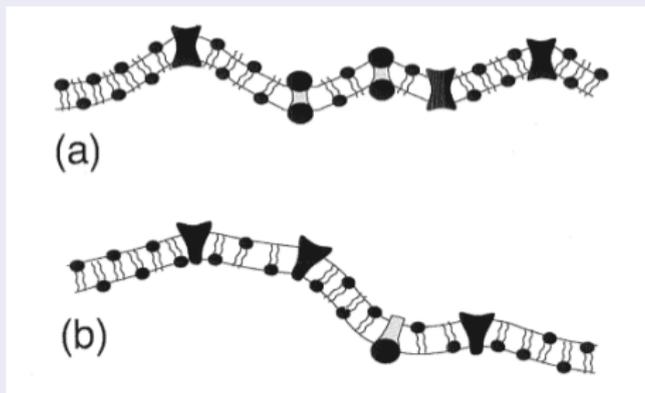
– 3L :  $\eta = 0.8872$

– 4L :  $\eta = 0.8760$  (Pikelner 22) to be compared to NPRG  $\eta = 0.85$

# Statistical physics of disordered membranes

Disorder in membranes: imperfect polymerization, vacancies, impurities etc  $\Rightarrow$  "defects"

- isotropic defects  $\Rightarrow$  elastic disorder (a)
- anisotropic defect  $\Rightarrow$  curvature disorder (b)



## Free energy:

$$\Gamma[\mathbf{r}] = \int d^D x \left\{ \frac{\kappa}{2} \left( \Delta \mathbf{r} - \frac{\mathbf{c}(\mathbf{x})}{\kappa} \right)^2 + \lambda \left( \partial_a \mathbf{r} \cdot \partial_b \mathbf{r} - \delta_{ab} (1 + 2m(\mathbf{x})) \right)^2 + \mu \left( \partial_a \mathbf{r} \cdot \partial_a \cdot \mathbf{r} - \delta_{aa} (1 + 2m(\mathbf{x})) \right)^2 \right\}$$

with  $\mathbf{c}(\mathbf{x})$  and  $m(\mathbf{x})$  Gaussian random fields coupled to **curvature** and **metric**

- average over (quenched) disorder using replica trick:

$$\overline{F} = \overline{\log Z} = \lim_{n \rightarrow 0} \frac{\overline{Z^n} - 1}{n}$$

⇒ effective action with **interacting replica** : A,B

$$\Gamma[\mathbf{r}] = \int d^d x \sum_A \left\{ \frac{\bar{\kappa}}{2} (\Delta \mathbf{r}^A)^2 + \bar{\lambda} \left( \partial_a \mathbf{r}^A \cdot \partial_b \mathbf{r}^A - \delta_{ab} \right)^2 + \bar{\mu} \left( \partial_a \mathbf{r}^A \cdot \partial_a \mathbf{r}^A - \delta_{aa} \right)^2 \right\} \\ - \frac{\bar{\Delta}_\kappa}{2} \sum_{A,B} \Delta \mathbf{r}^A \cdot \Delta \mathbf{r}^B \\ - \bar{\Delta}_\lambda \sum_{A,B} \left( \partial_a \mathbf{r}^A \cdot \partial_b \mathbf{r}^A - \delta_{ab} \right) \left( \partial_a \mathbf{r}^B \cdot \partial_b \mathbf{r}^B - \delta_{ab} \right) \\ - \bar{\Delta}_\mu \sum_{A,B} \left( \partial_a \mathbf{r}^A \cdot \partial_a \mathbf{r}^A - \delta_{aa} \right) \left( \partial_b \mathbf{r}^B \cdot \partial_b \mathbf{r}^B - \delta_{bb} \right)$$

with  $\bar{\Delta}_\kappa, \bar{\Delta}_\lambda, \bar{\Delta}_\mu$  disorder variances

\*

## A long story :

1) pure metric, elastic disorder (Radzihovsky and Nelson 91)

$$\begin{aligned} \kappa_{eff}^D(\mathbf{q}) &= \kappa_{eff}(\mathbf{q}) + k_B T \mathcal{K} \int d^2 k \frac{[\hat{q}_a P_{ab}^T \hat{q}_b]^2}{\kappa_{eff}(\mathbf{q} + \mathbf{k}) |\mathbf{q} + \mathbf{k}|^4} \\ &\quad - (\Delta_\lambda + \Delta_\mu) \mathcal{K}^2 \int d^2 k \frac{[\hat{q}_a P_{ab}^T \hat{q}_b]^2}{\kappa_{eff}(\mathbf{q} + \mathbf{k}) |\mathbf{q} + \mathbf{k}|^4} \end{aligned}$$

⇒ stability of the **disorder-free** fixed point  $P_4$

2) elastic and curvature disorder

$$\begin{aligned} \kappa_{eff}^D(\mathbf{q}) &= \kappa_{eff}(\mathbf{q}) + \Delta_\kappa \mathcal{K} \int d^2 k \frac{[\hat{q}_a P_{ab}^T \hat{q}_b]^2}{\kappa_{eff}^2(\mathbf{q} + \mathbf{k}) |\mathbf{q} + \mathbf{k}|^4} \\ &\quad - (\Delta_\lambda + \Delta_\mu) \mathcal{K}^2 \int d^2 k \frac{[\hat{q}_a P_{ab}^T \hat{q}_b]^2}{\kappa_{eff}(\mathbf{q} + \mathbf{k}) |\mathbf{q} + \mathbf{k}|^4} \end{aligned}$$

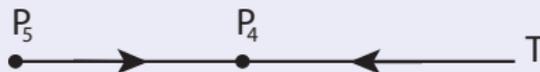
- curvature disorder  $\Delta_\kappa$  : stabilizes the membrane
- metric disorder  $\Delta_\mu, \Delta_\lambda$  destabilizes the membrane

## Weak coupling approach of the flat phase

- one-loop, weak coupling, analysis in  $D = 4 - \epsilon$   
(Morse, Lubensky and Grest,91, Morse and Lubensky 92)

$\implies$  confirms stability of the **disorder-free** fixed point  $P_4$

$\implies$  new **zero- $T$ , disordered** fixed point  $P_5$  but unstable



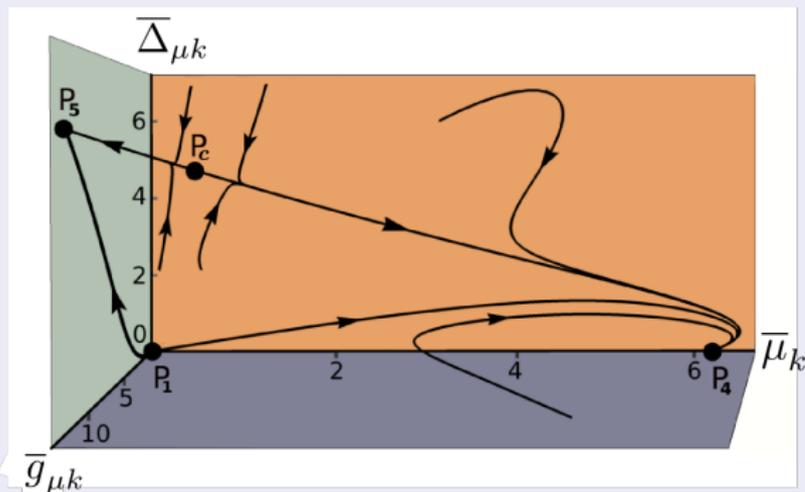
- Same result via SCSA (Le Doussal and Radzihovsky 92-18)

# NPRG approach of the flat phase

- Functional RG approach (Coquand, Essafi, Kownacki and D.M. 17)

⇒ new *critical fixed point*  $P_c$  between  $P_4$  and  $P_5$

↑ Disorder



→ Temperature

- phase transition with  $\eta_c = 0.490 \implies \kappa_c \underset{q \rightarrow 0}{\sim} q^{-\eta_c} \ll \kappa_4$
- stable disordered phase  
with  $\eta_5 = 0.448 \implies \kappa_5 \underset{q \rightarrow 0}{\sim} q^{-\eta_5} \ll \kappa_4$

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### Question:

- could  $P_c$  be just an artifact of the NPRG ?

# Disordered membranes at two and three loop order

(Metayer and D.M. 22)

$$S = \int d^D x \left\{ \frac{\tilde{\kappa}^{AB}}{2} \Delta \mathbf{h}^A(\mathbf{x}) \Delta \mathbf{h}^B(\mathbf{x}) + \tilde{\lambda}_{AB} g_{ab}^A(\mathbf{x}) g_{ab}^B(\mathbf{x}) \right. \\ \left. + \tilde{\mu}_{AB} g_{aa}^A(\mathbf{x}) g_{bb}^B(\mathbf{x}) \right\}$$

with:  $g_{ij}^A \simeq \frac{1}{2} \left[ \partial_i u_j^A + \partial_i u_j^A + \partial_i \mathbf{h}^A \cdot \partial_j \mathbf{h}^A \right]$  and with generalized coupling constants

$$\begin{cases} \tilde{\kappa}^{AB} &= \tilde{\kappa} \delta^{AB} - \tilde{\Delta}_{\kappa} J^{AB} \\ \tilde{\mu}^{AB} &= \tilde{\mu} \delta^{AB} - \tilde{\Delta}_{\mu} J^{AB} \\ \tilde{\lambda}^{AB} &= \tilde{\lambda} \delta^{AB} - \tilde{\Delta}_{\lambda} J^{AB} \end{cases}$$

where  $J^{AB} \equiv 1 \forall A, B$ .

- a fixed point  $P_c$  of order  $\epsilon^2$  found !
- **Very proximity** between three-loop and NPRG !

$$\left\{ \begin{array}{l} \eta_{3L} \\ \eta_{\text{NPRG re-expanded}} \end{array} \right. = \begin{array}{l} 0.42857 \epsilon - 0.03695 \epsilon^2 - 0.01191 \epsilon^3 \\ 0.42857 \epsilon - 0.03621 \epsilon^2 - 0.01318 \epsilon^3 \end{array}$$

- a rapidly converging exponent  $\eta_c$ : in  $D = 2$  (i.e.  $\epsilon=2$ ) and  $d = 3$ :

– 1L :  $\eta_c = 0.8571$

– 2L :  $\eta_c = 0.7093$

– 3L :  $\eta_c = 0.6140$

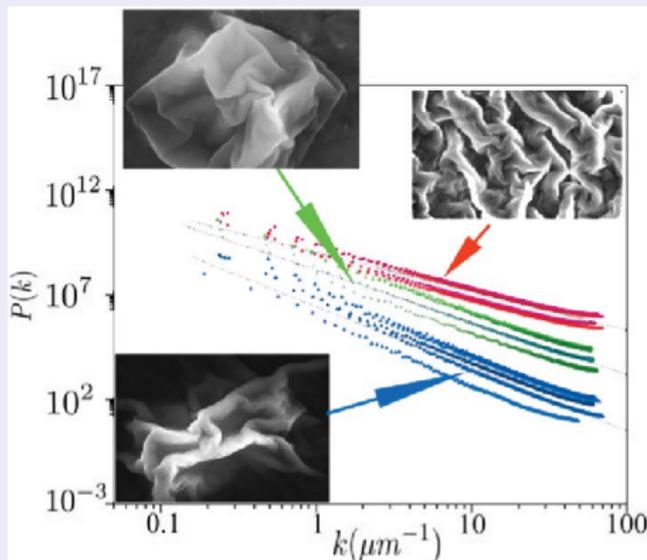
to be compared to NPRG  $\eta_c = 0.490$  and experiment  $\eta_c = 0.492$

Three scaling behaviours associated with  $P_5$ ,  $P_4$  and  $P_c$  observed in partially polymerized fluid membranes

(S. Chaieb, V.K. Natrajan and A.A. El-rahman 06)

Power spectrum  $P(k) \sim k^{3-\eta}$  (F.T. of  $\langle (h(\mathbf{x}) - h(\mathbf{0}))^2 \rangle$ )

$\implies$  same  $\eta_4$ ,  $\eta_c$  and  $\eta_5$





One expects a phase transition and a disorder-controlled - glassy - phase at low temperature in graphene and graphene-like materials

To be confirmed by

- numerical investigations
- investigations of graphene-like materials displaying both elastic and curvature disorders

# Conclusion

- membranes display a very rich physics:
  - pure systems due to (hidden) long range interactions
  - disordered systems: new fixed points, new phases
- in the low temperature phase :  $\implies$  **glassy phase**  
(methodologically: thanks to NPRG approach !)  
 $\implies$  is it observable in graphene or graphene-like materials where impurities give rise to curvature disorder ?
- consequences for electronic, thermodynamical, optical, etc properties ?  
 $\implies$  coupling between phonons and electrons (work in progress)