Nonequilibrium critical dynamics: upturns from surface kinetic roughening

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gisc Grupo Interdisciplinar de Sistemas Complejos

Outline of the talk

- Introduction: (non-)equilibrium critical dynamics (Ising)
- Surface kinetic roughening
- KPZ equation
- What defines a universality class?
- KPZ nonlinearity vs: Scaling exponents
  Field statistics
  Dynamic scaling ansatz
- Upturns from surface kinetic roughening (Ising)
- Conclusions

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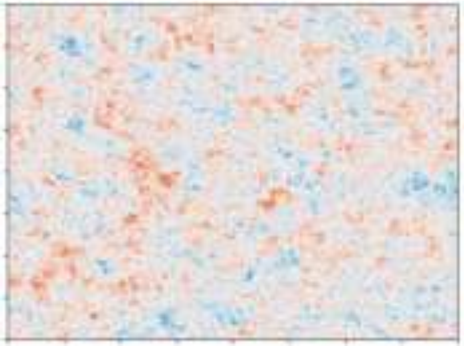
Ising model

$$\mathcal{H}_{\text{Ising}} = -J \sum_{\langle i,j \rangle} s_i \, s_j - H \sum_i s_i, \quad s_i = \pm 1$$

$$Z = \sum_{\{s_i\}} e^{-\mathcal{H}_{\text{Ising}}/k_B T}$$



Local magnetization $\phi(\mathbf{r}) = \dfrac{1}{N(\mathbf{r})} \sum_{i \in \mathbf{r}} s_i$

$$\mathcal{L}[\phi] = \int \mathrm{d}\mathbf{r} \left\{ \frac{r}{2}\phi^2 + \frac{u}{4}\phi^4 + \frac{\nu}{2}(\nabla\phi)^2 \right\}$$

$$Z = \int \mathcal{D}\phi \, e^{-\mathcal{L}/k_B T}$$

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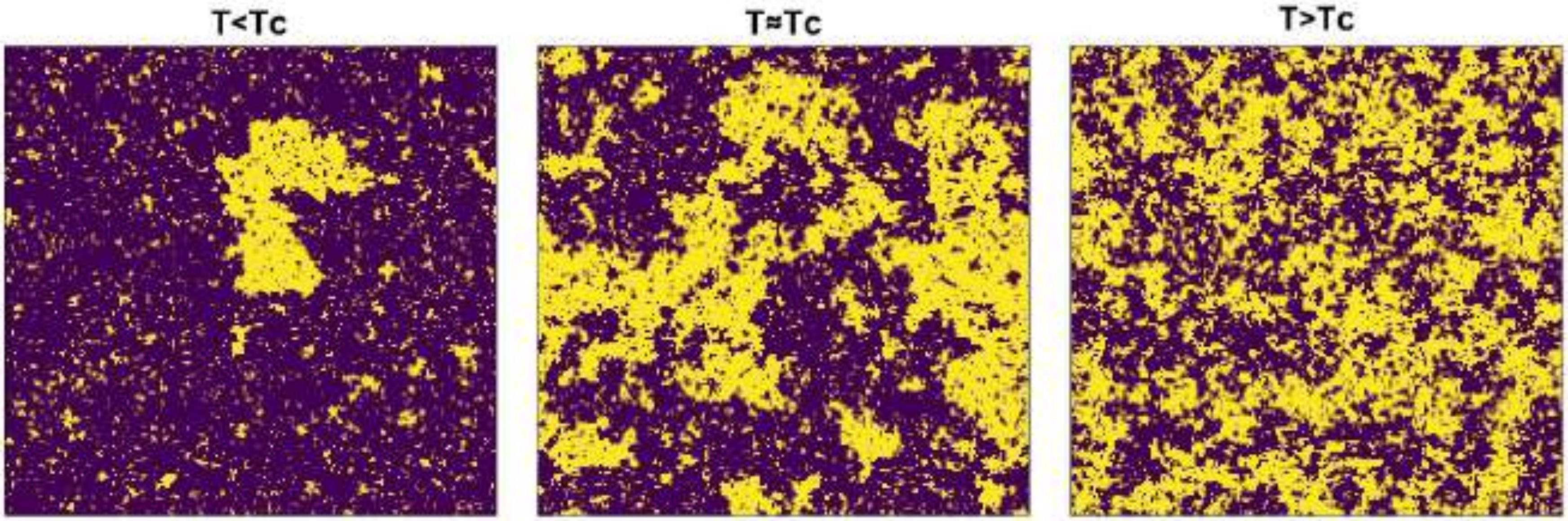
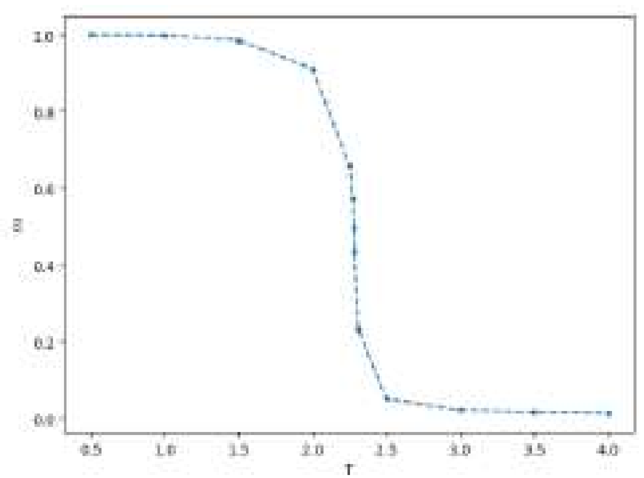
Equilibrium 2nd. order (continuous) transition



$$\langle m \rangle = \frac{1}{N} \langle \sum_i s_i \rangle$$



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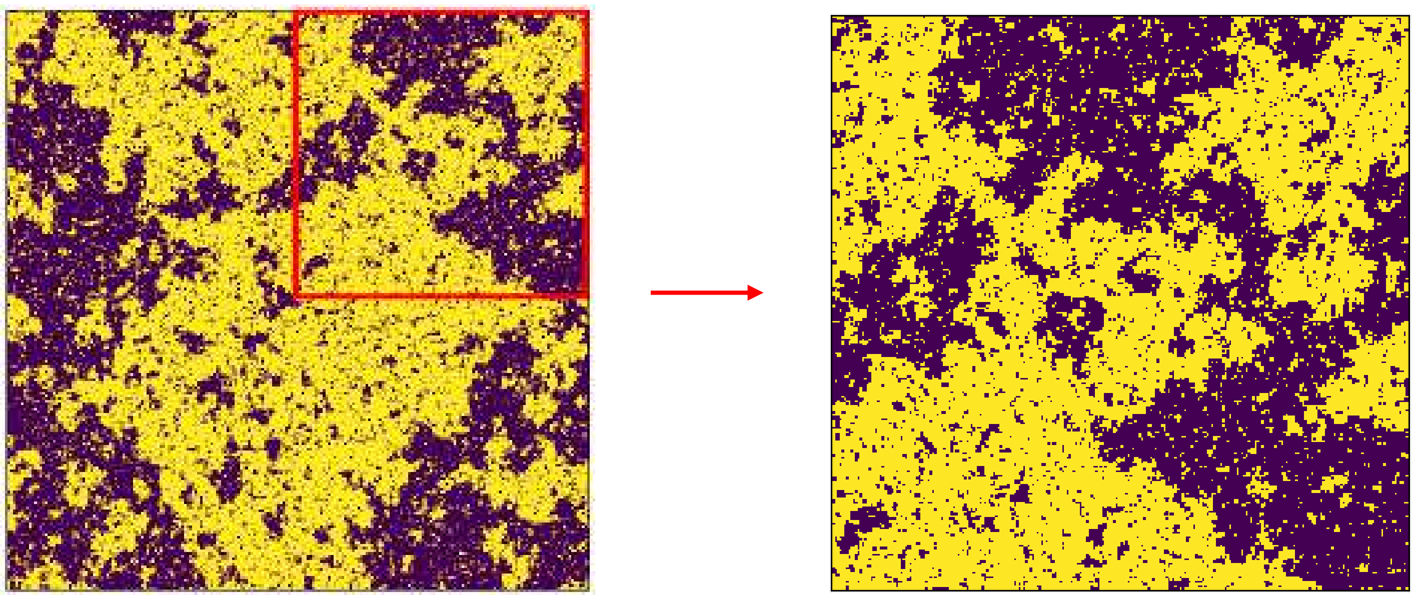
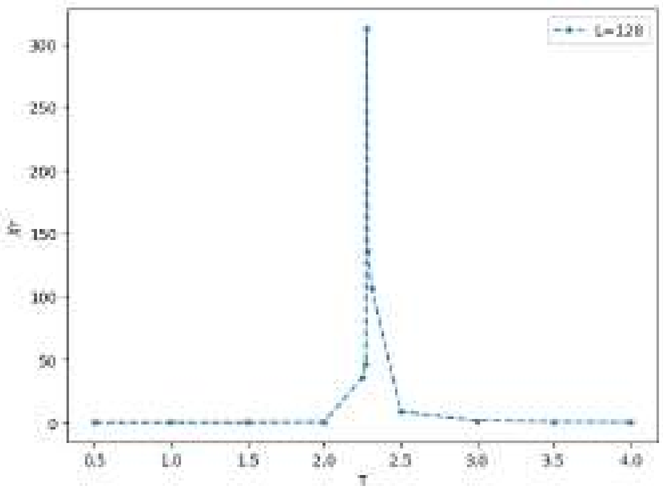
Scale invariance at $T=T_c$

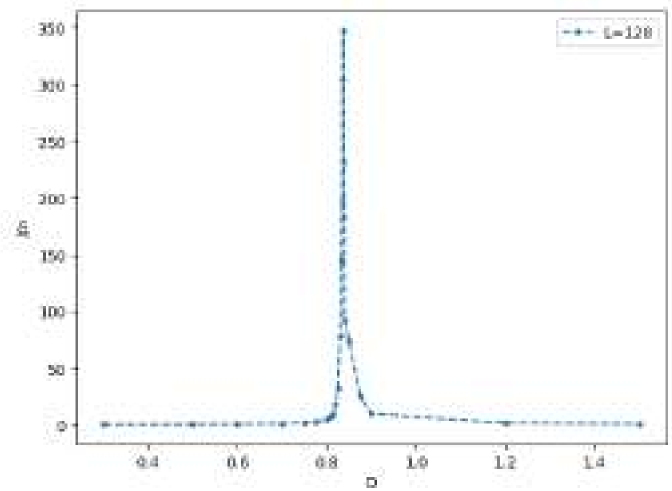


Criticality

$$\chi = \frac{\partial m}{\partial H} \propto \langle m^2 \rangle - \langle m \rangle^2$$



T_c = 2.277



D_c = 0.835

$$\chi \propto \sum_r G_r \qquad G_{ij} = \langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle$$

$$G(r) = \langle \phi(\mathbf{r})\, \phi(\mathbf{0}) \rangle = \int \mathrm{d}\mathbf{k}\, e^{\mathbf{ikr}} \, \langle |\phi(\mathbf{k})|^2 \rangle \approx \int \mathrm{d}\mathbf{k}\, e^{\mathbf{ikr}} \, \frac{1}{\xi^{-2} + k^2}$$

$$T = T_c \Rightarrow \xi \to \infty \Rightarrow \; G(r) \sim \frac{1}{r^{d-2+\eta}} \Rightarrow \chi \to \infty$$

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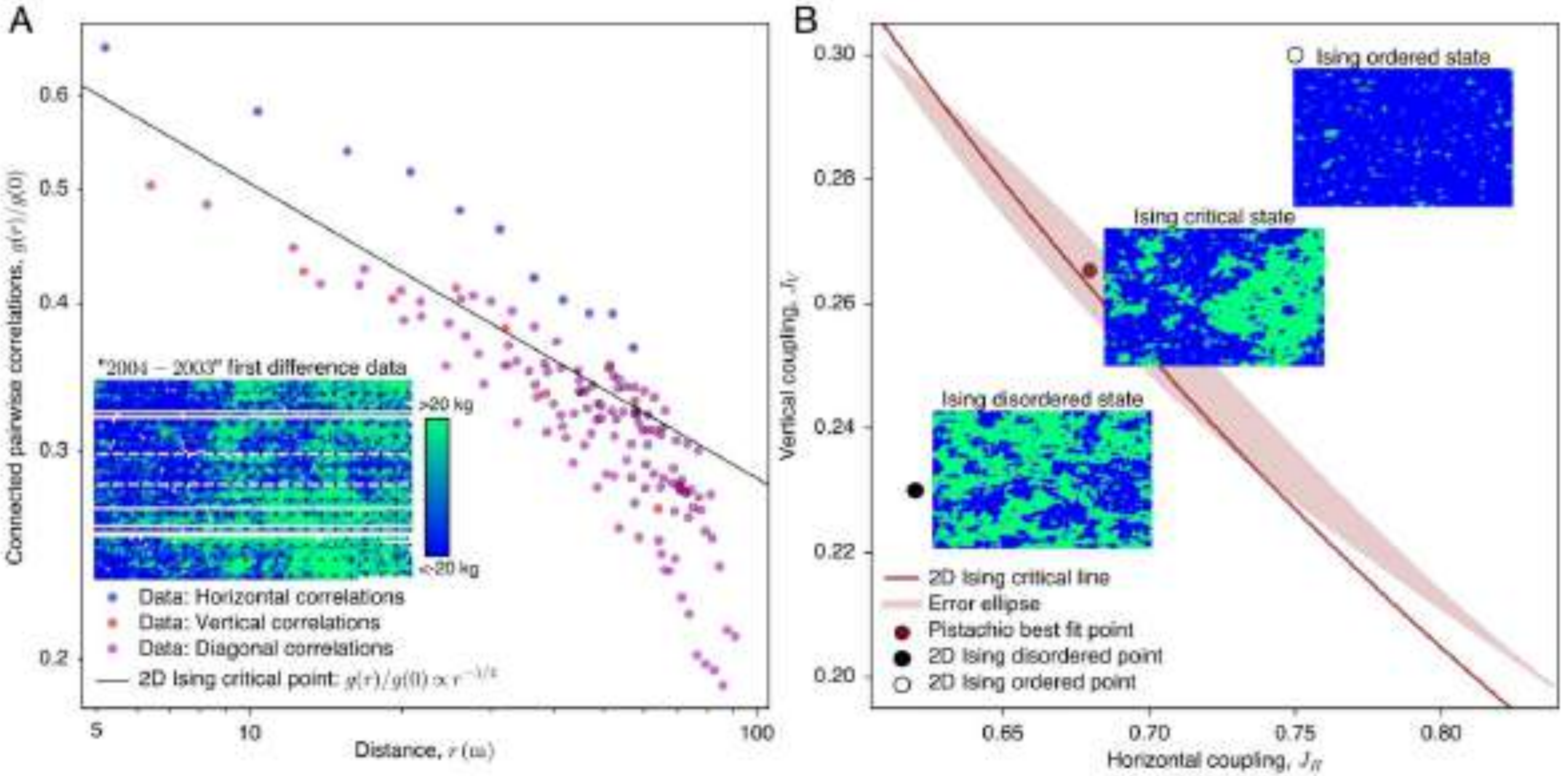
Oscillating populations: Pistachio production



$$\phi_{i,j} \equiv X_{i,j,2004} - X_{i,j,2003} - (X_{2004} - X_{2003})$$

$$g_D(\Delta i, \Delta j) = \frac{1}{2N_{\Delta i, \Delta j}} \sum_i \sum_j \overline{\phi}_{i,j} \overline{\phi}_{i+\Delta i, j+\Delta j}$$

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A. E. Noble, J. Machta & A. Hastings Nature Comm. **6**, 6664 (2015)
A. E. Noble *et al.* PNAS **115**, 1825 (2018)

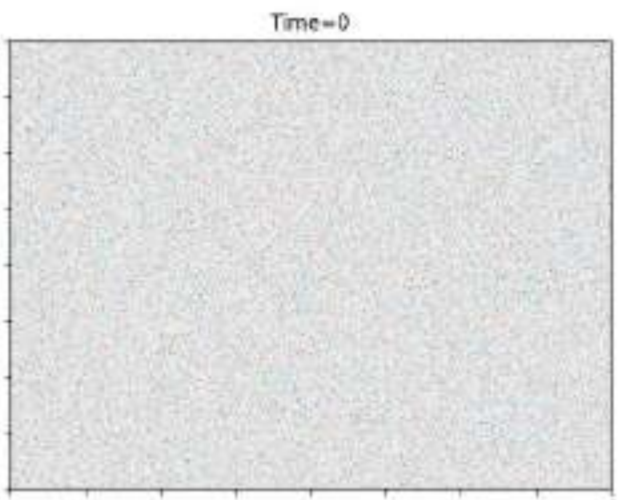
(Critical) dynamics

Stochastic time-dependent
Ginzburg-Landau equation (STDGL)

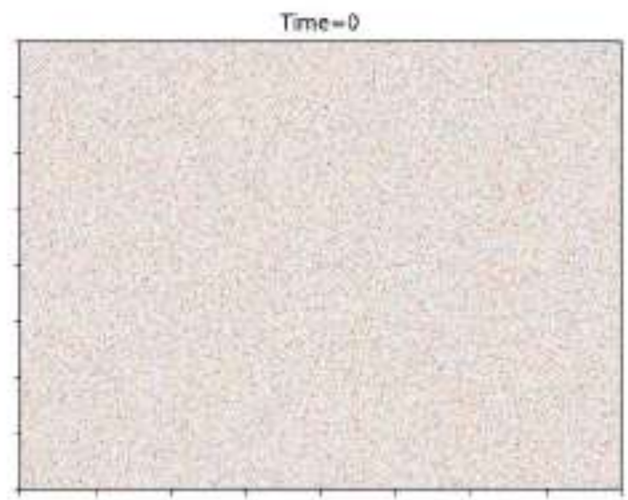
$$\frac{\partial \phi(\mathbf{r},t)}{\partial t} = -\frac{\delta \mathcal{L}[\phi]}{\delta \phi(\mathbf{r})} + \zeta(\mathbf{r},t)$$

$$\frac{\partial \phi(\mathbf{r},t)}{\partial t} = -r\phi(\mathbf{r},t) - u\phi^3(\mathbf{r},t) + \nu \nabla^2 \phi(\mathbf{r},t) + \zeta(\mathbf{r},t)$$

Subcritical (T=0)



Critical ($T=T_c$)



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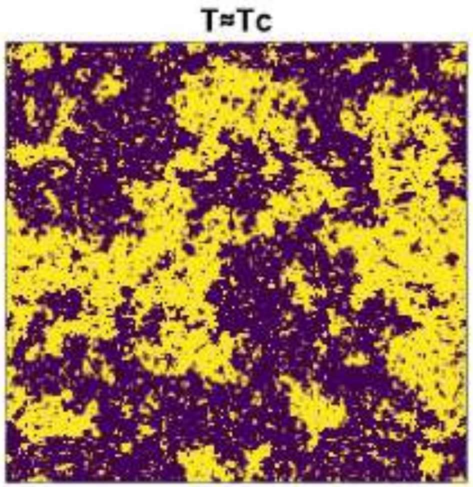
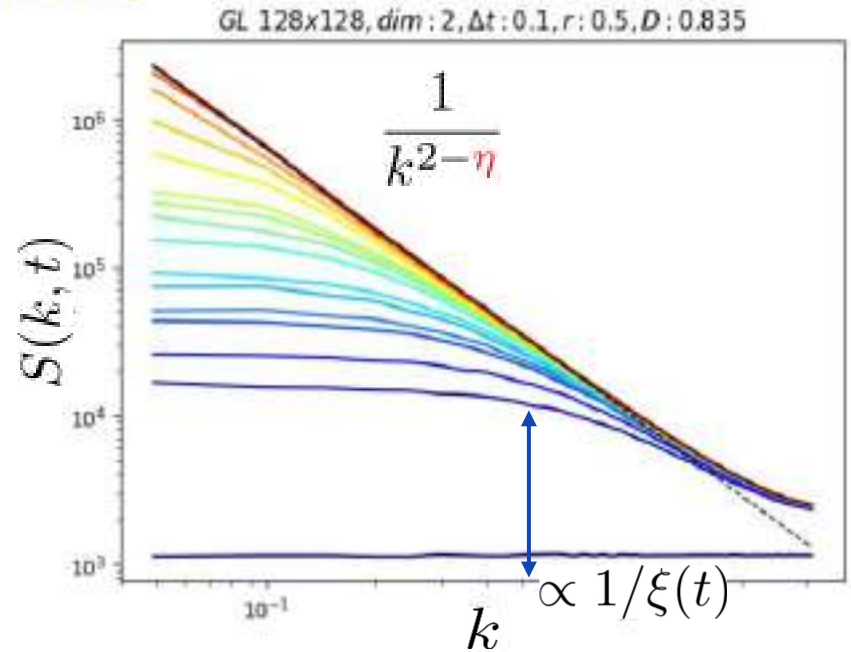
Critical dynamics



$$S(k,t) = \langle |\phi(\mathbf{k},t)|^2 \rangle$$

$$G(r,t) = \int \mathrm{d}\mathbf{k}\, e^{\mathrm{i}\mathbf{k}\mathbf{r}}\, S(k,t) \xrightarrow{t\to\infty} \frac{1}{r^{d-2+\eta}}$$

$$\phi(\mathbf{r}, t=0) = 1$$



$$\xi^{-1}(t) \sim \frac{1}{t^{1/z}}$$

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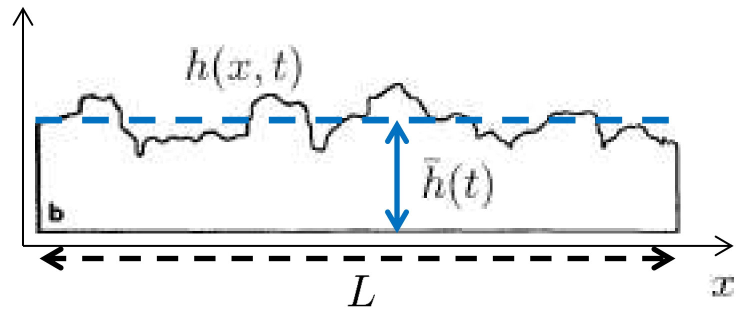
H. Vaquero del Pino & R. C. Phys. Rev. Res. **7**, 043192 (2025)

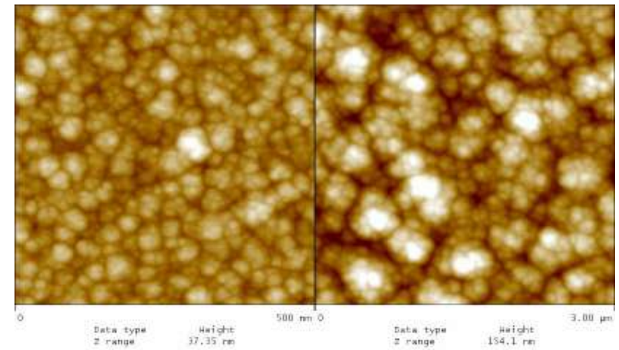
Surface analysis

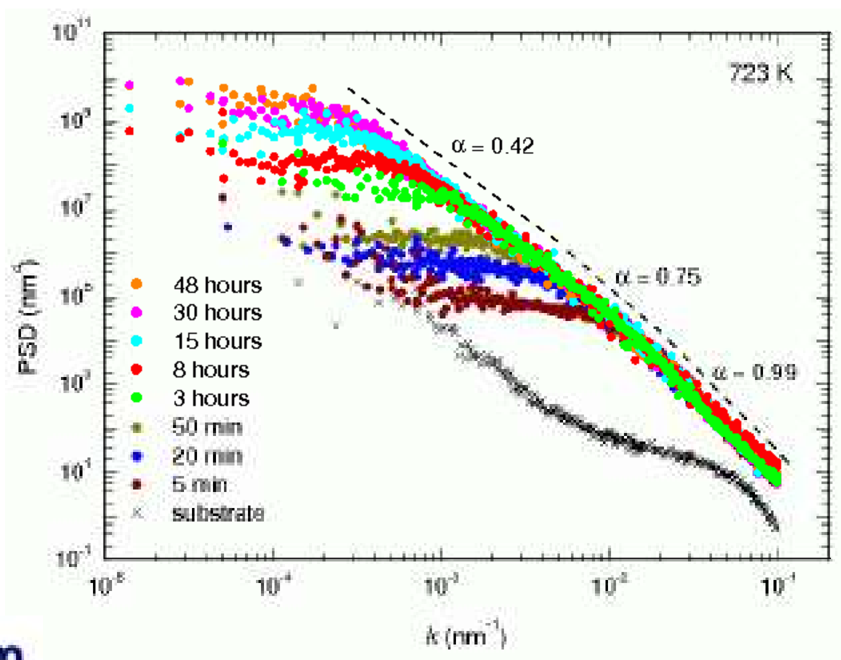
Height **p**ower **s**pectral **d**ensity:

$$h(k,t) = \mathcal{F}(h(x,t) - \bar{h}(t))$$

$$S(k,t) = \langle |h(k,t)|^2 \rangle \rightarrow \frac{1}{k^{2\alpha+2}}$$







$$\xi^{-1}(t) \sim \frac{1}{t^{1/z}}$$

like dynamics of Ising model at $T=T_c$!

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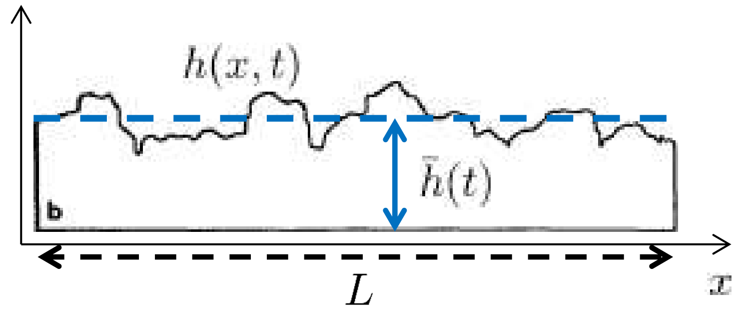
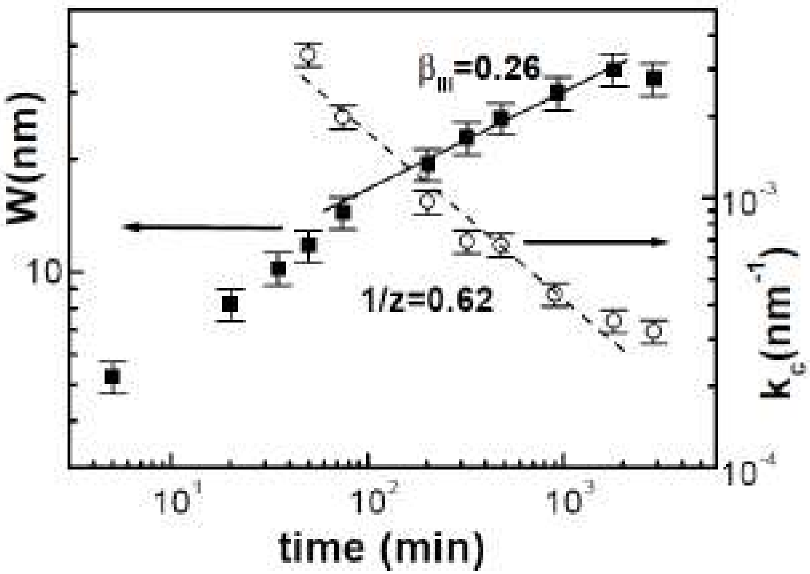
F. Ojeda, R.C., L. Vázquez & R. Salvarezza PRL **84**, 3125 (2000): CVD of silica onto Si(001)

Surface analysis

Surface roughness:

$$W^2(L) = \frac{1}{L^2}\sum_x [h(x,t) - \bar{h}(t)]^2$$
$$= \sum_{k\neq 0} S(k,t)$$





$$W(t) \sim t^{\beta}$$

$$\beta = \frac{\alpha}{z}$$

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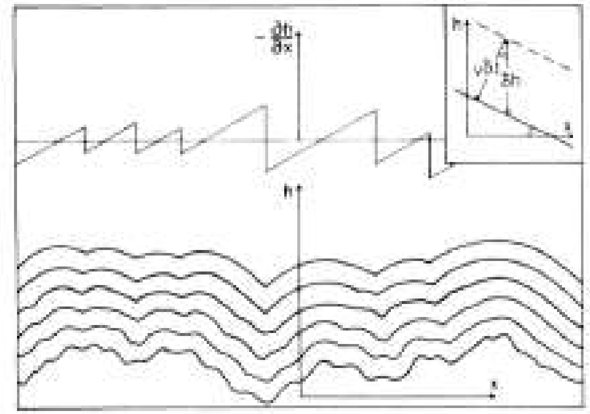
F. Ojeda, R.C., L. Vázquez & R. Salvarezza PRL **84**, 3125 (2000): CVD of silica onto Si(001)

Kardar-Parisi-Zhang equation

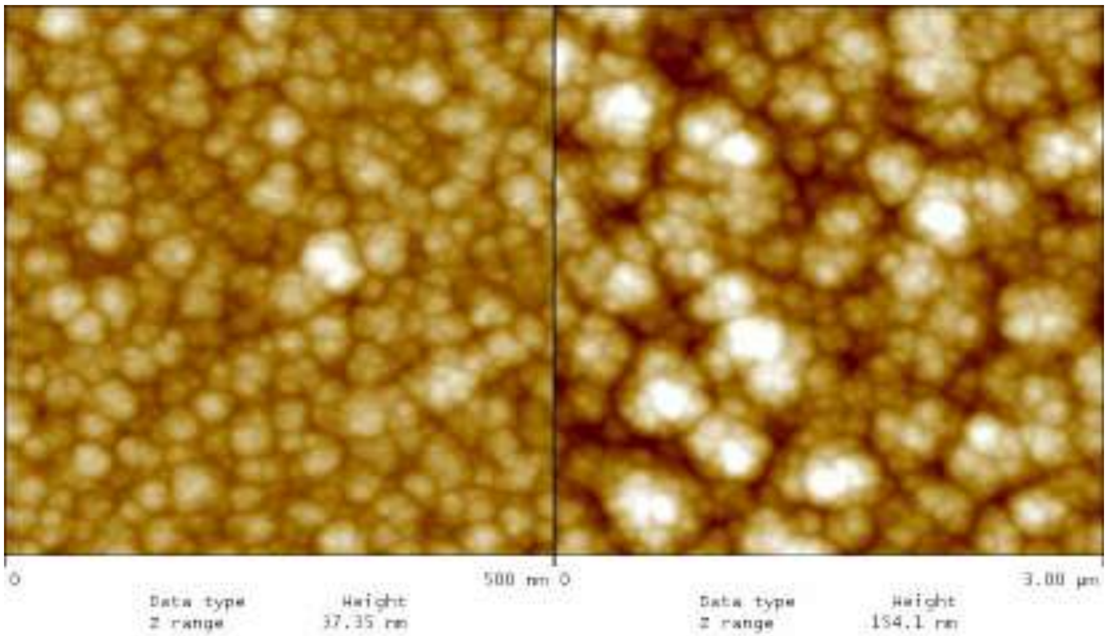
$$\partial_t h = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta$$

$$\langle \eta(\mathbf{r},t)\,\eta(\mathbf{r}',t')\rangle = D\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t')$$

M. Kardar, G. Parisi & Y.-C. Zhang Phys. Rev. Lett. **56,** 889 (1986)



SiO$_2$ on Si
AFM top view



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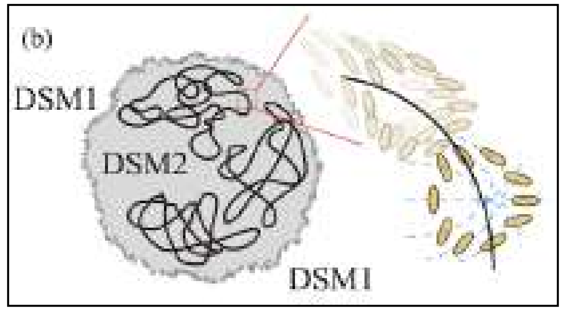
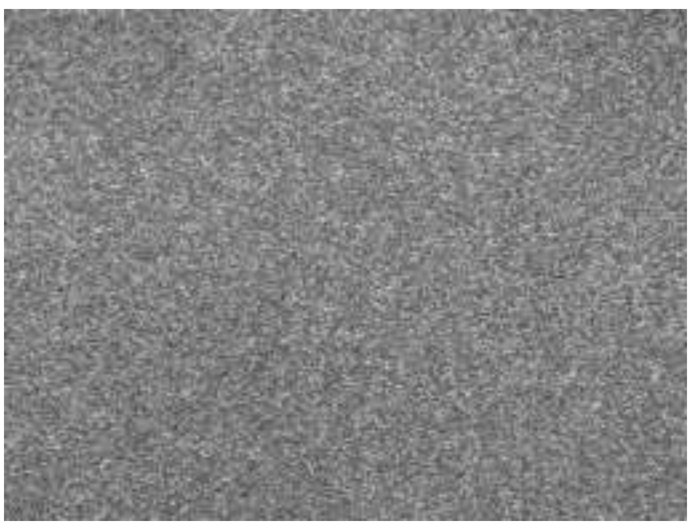
F. Ojeda, R.C., L. Vázquez & R. Salvarezza PRL **84**, 3125 (2000): CVD of silica onto Si(001)

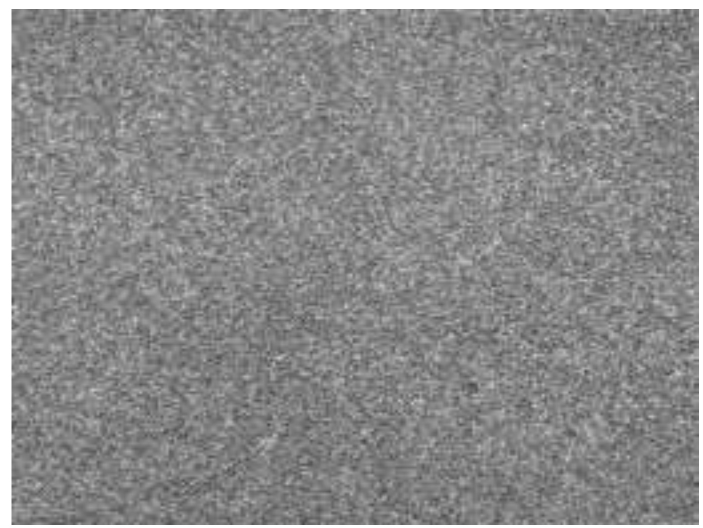
1+1 KPZ universality: experiments + equation

Driven liquid crystals: laser-induced nucleation of favorable turbulent phase



2 mm





K.A. Takeuchi & M. Sano PRL **104**, 230601 (2010), J. Stat. Phys. **147**, 853 (2012)
K.A. Takeuchi, M. Sano, T. Sasamoto, & H. Spohn Sci. Rep. **1**, 34 (2011)

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1+1 KPZ universality: experiments + equation

T. Sasamoto & H. Spohn PRL **104**, 230602 (2010)
P. Calabrese, P. Le Doussal, & A. Rosso EPL **90**, 20002 (2010)
G. Amir G, I. Corwin & J. Quastel Comm. Pure Appl. Math. **64**, 0466 (2011)
P. Calabrese & P. Le Doussal PRL **106**, 250603 (2011)

$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta$$

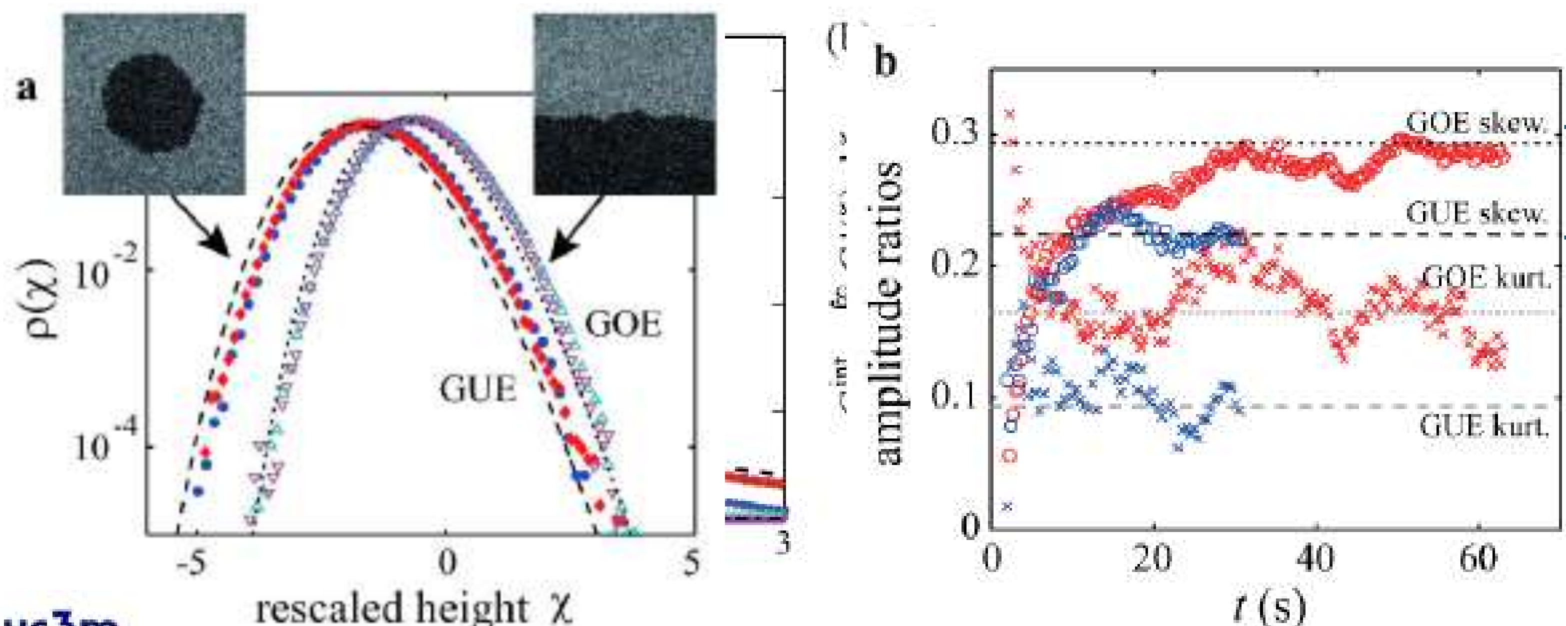
$$h \approx v_\infty t + (\Gamma t)^\beta \chi$$

$$\chi \rightsquigarrow \chi \approx \frac{h - v_\infty t}{(\Gamma t)^\beta}$$

$$C_s(l,t) = \langle h(x+l,t)h(x,t)\rangle - \langle h(x+l,t)\rangle\langle h(x,t)\rangle$$

Universal one-point statistics

Universal two-point correlations



same sign as λ

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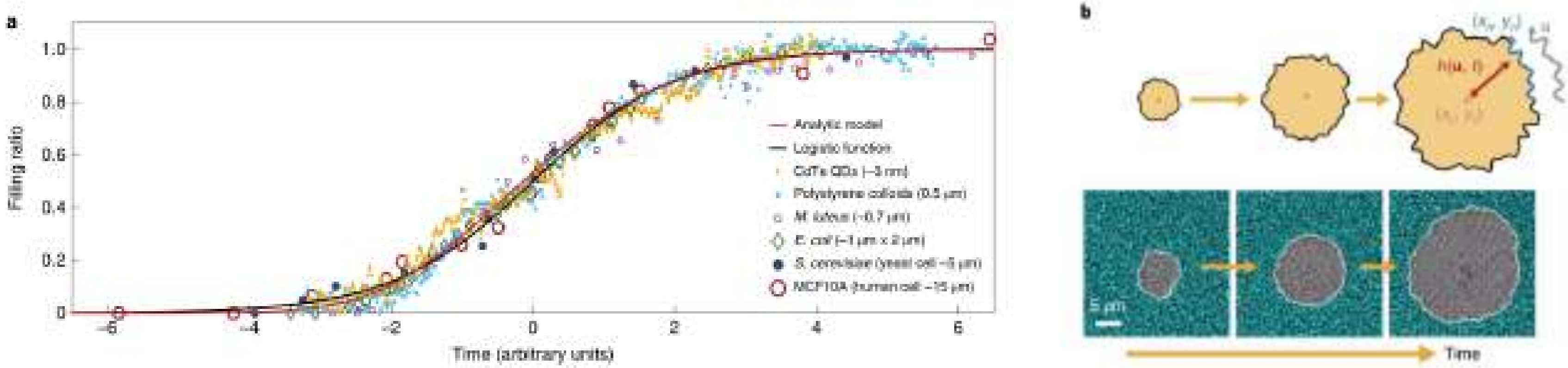
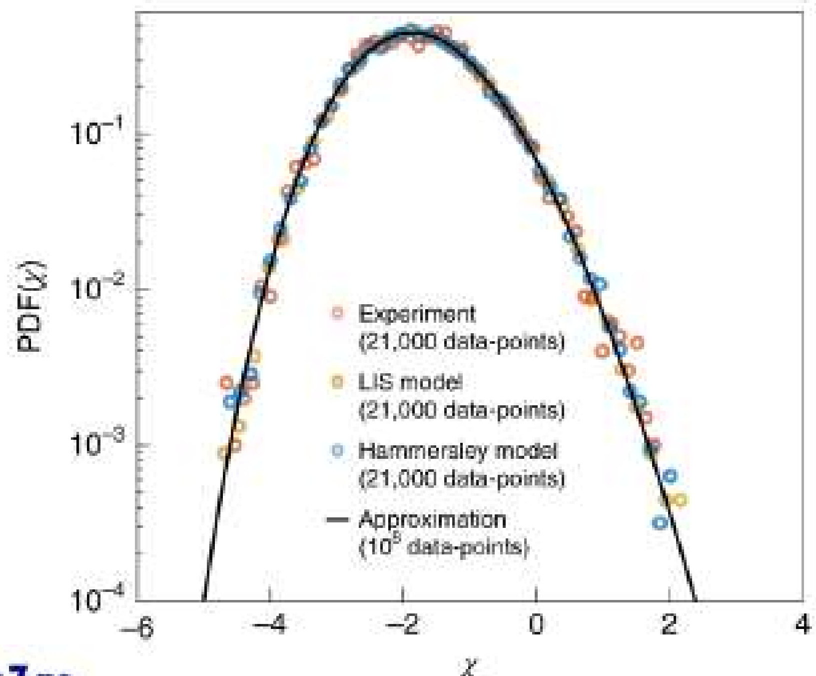
K.A. Takeuchi & M. Sano PRL **104**, 230601 (2010), J. Stat. Phys. **147**, 853 (2012)
K.A. Takeuchi, M. Sano, T. Sasamoto, & H. Spohn Sci. Rep. **1**, 34 (2011)

Universal fluctuations





$$\chi = \frac{\Delta h(\Delta t) - \overline{\Delta h(\Delta t)}}{W(\Delta t)}$$

$$\Delta h(\Delta t) = h(t_0 + \Delta t) - h(t_0)$$

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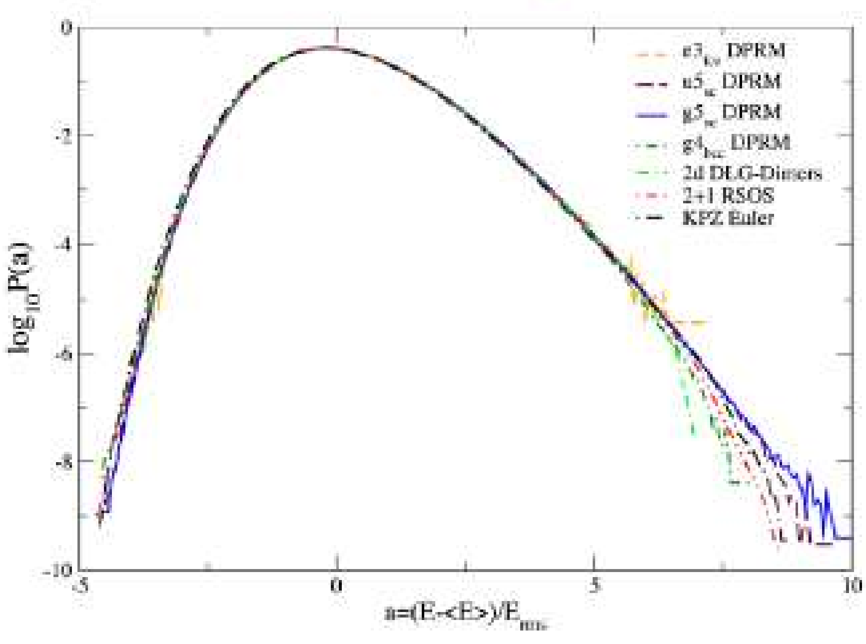
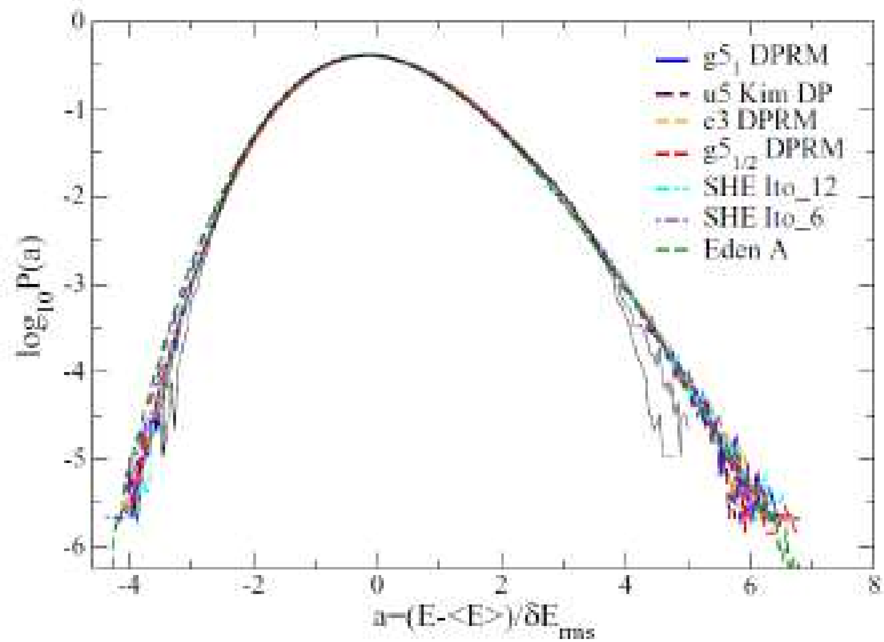
G. Makey *et al.* Nature Phys. **16**, 795 (2020)

2+1 KPZ universality: equation+models

Flat initial condition



Enlarging substrate



T. Halpin-Healy PRL **109**, 170602 (2012)

T. J. Oliveira, S.G. Alves & S.C. Ferreira PRE **87**, 040102 (2013)

T. Halpin-Healy PRE **88**, 042118 (2013)

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A widely encompassing universality class

(1) As long as the rearrangements of particles on the surface can result in holes and vacancies, there is no conservation law.

(2) There is a *translation symmetry*, $v[h(\mathbf{x})+c] = v[h(\mathbf{x})]$, implying that v depends only on gradients of $h(\mathbf{x})$.

(3) For simplicity, we shall focus on *isotropic* surfaces, in which all directions in $\mathbf{x}$ are equivalent [See, however, D. Wolf, Phys. Rev. Lett. **67**, 1783 (1991)].

(4) There is no up–down symmetry, i.e. $v[h(\mathbf{x})] \neq -v[-h(\mathbf{x})]$. The absence of such symmetry allows addition of terms of both parities.

With these conditions, the lowest order terms in the equation of motion give [M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986); E. Medina, T. Hwa, M. Kardar, and Y.-C. Zhang, Phys. Rev. A **39**, 3053 (1989)],

$$\frac{\partial h(\mathbf{x},t)}{\partial t} = u + \nu\nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \cdots + \eta(\mathbf{x},t), \tag{9.81}$$

with the non-conservative noise satisfying the correlations in Eq. (9.74).

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M. Kardar Statistical Physics of Fields (CUP, 2007)

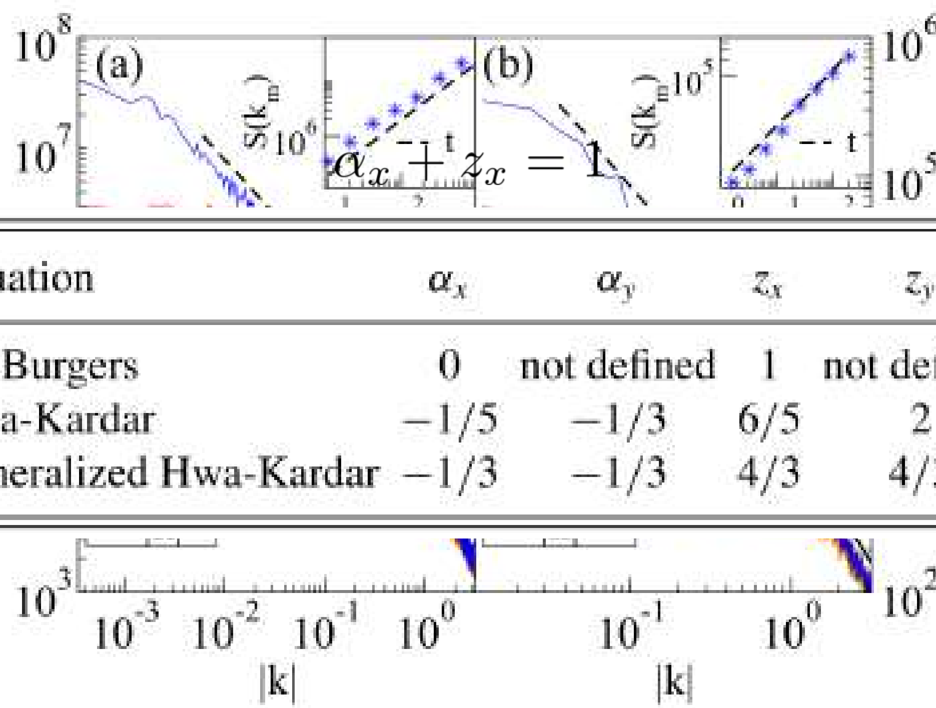
Lack of up-down symmetry ⇏ TW statistics: Burgers equation with non-conserved noise

J. Krug Adv. Phys **46**, 139 (1997)

The skewness is a measure of th[…]bution, to be expected beca[…] while 𝒬 gauges the wei[…]ussian: for a

1+1D $$\partial_t \phi = \nu \partial_x^2 \phi + \lambda \phi \partial_x \phi + \eta$$

2+1D $$\partial_t \phi = \nu_x \, \partial_x^2 \phi + \nu_y \, \partial_y^2 \phi + \lambda_x \phi \partial_x \phi + \lambda_y \phi \partial_y \phi + \eta$$



$$\alpha_x + z_x = 1$$

Equation	α_x	α_y	z_x	z_y
1D Burgers	0	not defined	1	not defined
Hwa-Kardar	$-1/5$	$-1/3$	$6/5$	2
Generalized Hwa-Kardar	$-1/3$	$-1/3$	$4/3$	$4/3$

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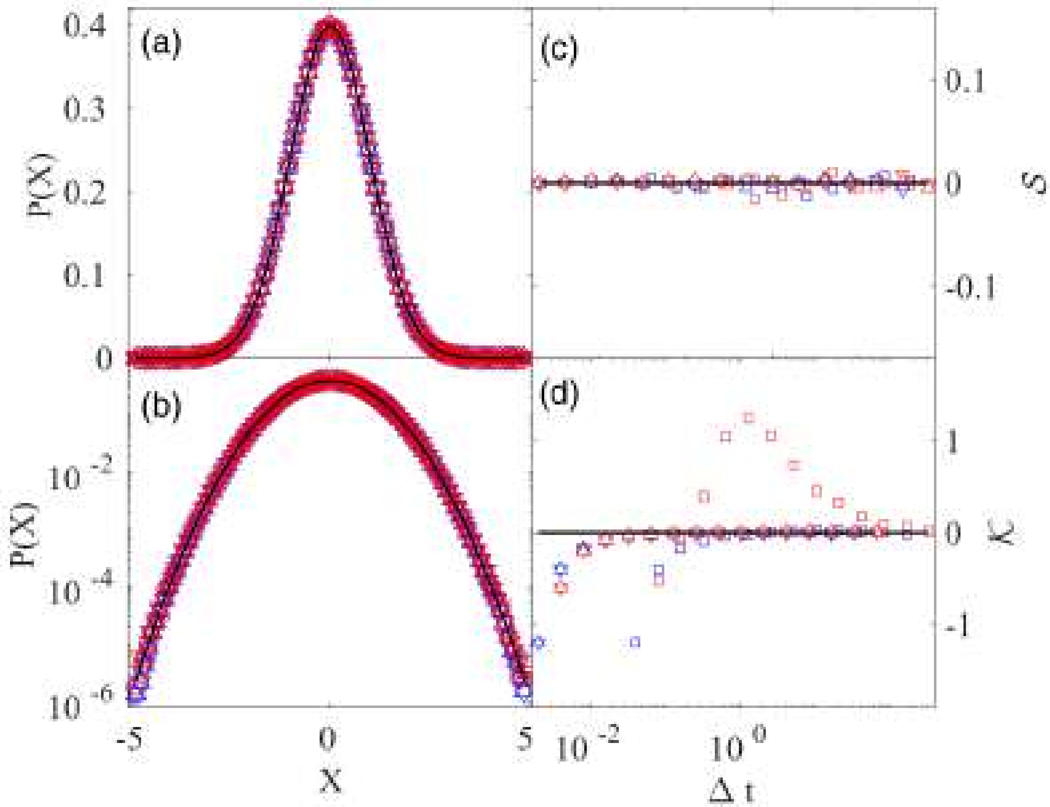
E. Rodríguez-Fernández & R. C. Phys. Rev. E **99**, 042108 (2019)

Lack of up-down symmetry $\not\Rightarrow$ TW statistics: Burgers equation with non-conserved noise



$$X(x, \Delta t, t_0) = \frac{\Delta\phi - \overline{\Delta\phi}}{(\Gamma \Delta t)^{\beta}}$$

$$\Delta\phi(x, \Delta t, t_0) = \phi(x, t_0 + \Delta t) - \phi(x, t_0)$$

- □ Burgers
- △ HK
- ▽ gHK

Growth regime: $t_0=0$, $\Delta t < t_{sat}$

Saturation: $t_0 > t_{sat}$

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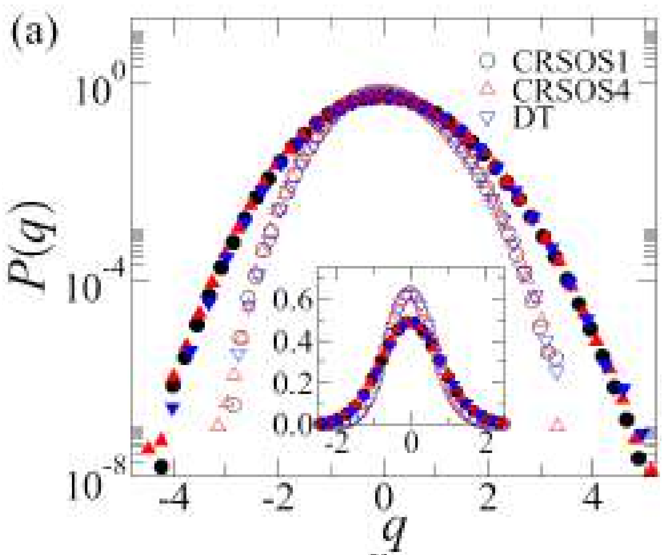
E. Rodríguez-Fernández & R. C. Phys. Rev. E **99**, 042108 (2019)

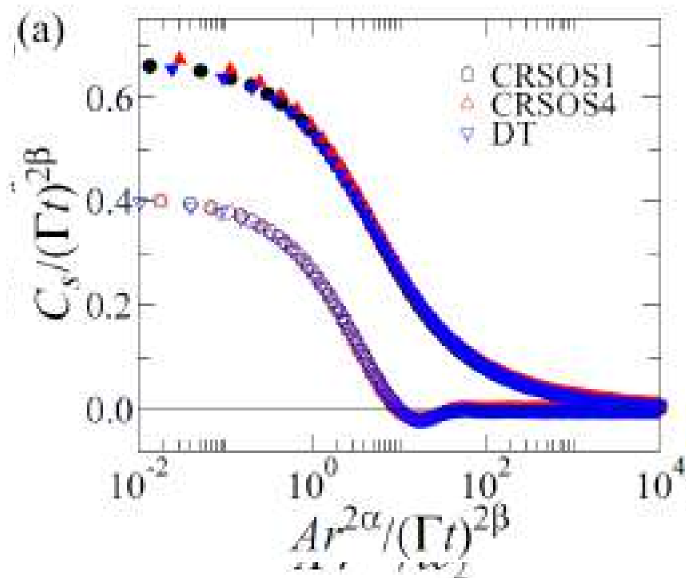
Analogous universal traits: Non-linear MBE class

$$\partial_t h = F - \nu_4 \nabla^4 h + \lambda_4 \nabla^2 (\nabla h)^2 + \eta$$

$$h = Ft + (\Gamma t)^\beta \chi$$ Prähofer-Spohn ansatz

2+1 growth





Open symbols: Flat init. cond.

Closed symbols: Enlarging substr.

I.S.S. Carrasco & T. J. Oliveira Phys. Rev. E **94**, 050801(R) (2016)

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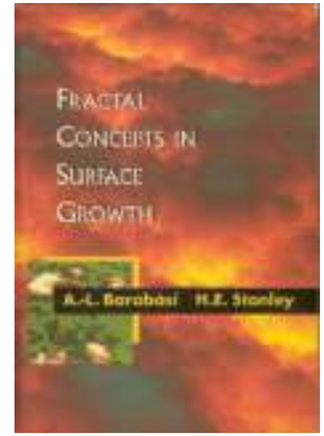
Universality class = exponent values?

There are a number of questions that we must answer if we wish to classify the growth equations in terms of universality classes. For example, what exactly is a universality class? We shall use here the standard working definition, that two systems belong to the same universality class if they share the same set of scaling exponents. This



A.-L. Barabási & H.E. Stanley Fractal Concepts in Surface Growth (CUP, 1995)

E.g. 1+1 dimensional nonlinear MBE equation H. K. Janssen PRL **78**, 1082 (1997)

$$\partial_t h = -\nu_4 \partial_x^4 h + \lambda_4 \partial_x^2 (\partial_x h)^2 + \eta$$

$$\alpha_\delta = 1 - \delta$$
$$z_\delta = 3 - 2\delta$$
$$\delta \simeq 0.03$$

Non-Gaussian PDF

$$2\alpha_\delta + 1 = z_\delta$$

Non-renormalization of noise

Same exponents as

$$\partial_t h_k = -\nu_4 |k|^{z_\delta} h_k + \eta_k$$

$$2\alpha_\delta + 1 = z_\delta$$

Linear equation

But Gaussian PDF!

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H.Vaquero del Pino & R. C. Phys. Rev. Res. **7**, 043192 (2025)

A family of non-local equations

Competition between (nonlocal) instabilities and KPZ nonlinearity

$$\partial_t h_{\mathbf{k}} = (\nu |k|^{\mu} - \mathcal{K} k^{m}) h_{\mathbf{k}} + \frac{\lambda}{2} \mathcal{F}[(\nabla h)^2] + \eta_{\mathbf{k}}$$

$$0 < \mu < 2, \; m \geq 2$$

Many celebrated particular cases (μ, m):

Saffman-Taylor = Mullins-Sekerka = (1,3)

Michelson-Sivashinsky = (1,2)

Kuramoto-Sivashinsky = (2,4)

…

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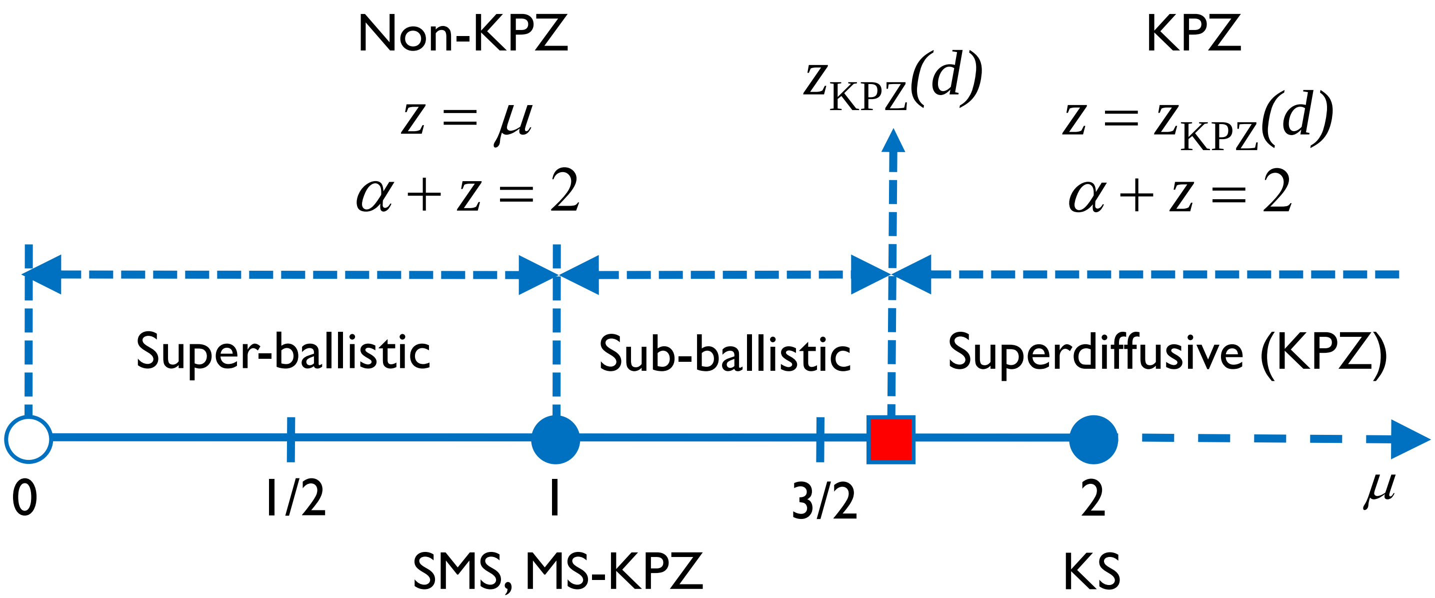
M. Nicoli, R. C. & M. Castro PRL **102**, 256102 (2009); JSTAT P10030 (2011)

DRG and numerics



Examples:

μ	d=1 z_{KPZ}=3/2	d=2 z_{KPZ}=1.61
1	NON-KPZ	NON-KPZ
3/2	KPZ	NON-KPZ
7/4	KPZ	KPZ

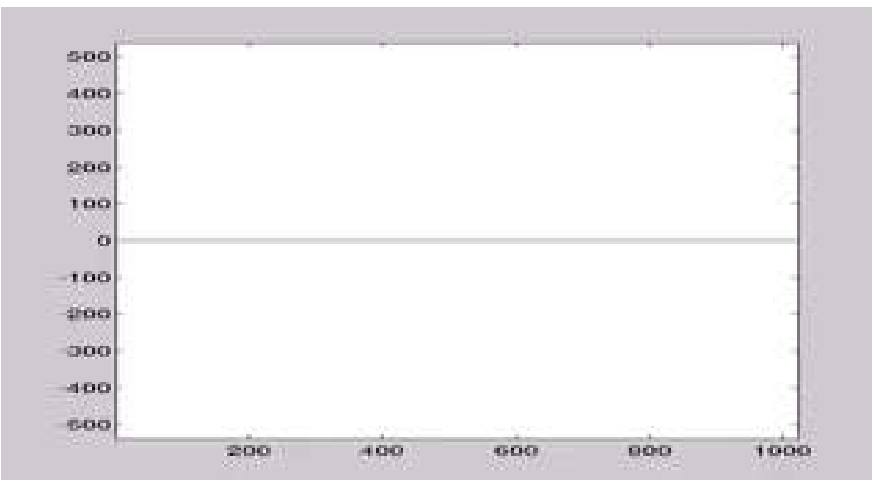
M. Nicoli, R. C. & M. Castro PRL **102**, 256102 (2009); JSTAT P10030 (2011)

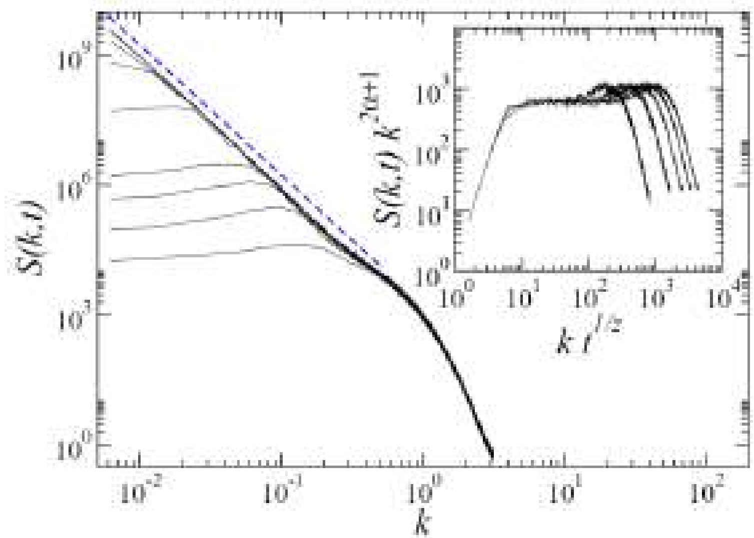
Diffusion-limited growth (1+1 growth)

Highly efficient growth events $\rightarrow$ non-local instability

$$\frac{\partial h_k}{\partial t} = \left[a_1|k| - a_3|k|^3\right] h_k + \frac{\nu}{2}\mathcal{F}\left[(\partial_x h)^2\right] + \eta_k$$

- Mullins-Sekerka
- Cusp dynamics
- Disordered asymptotics





NON-KPZ universality class: $\alpha = z = 1, \quad \alpha + z = 2$

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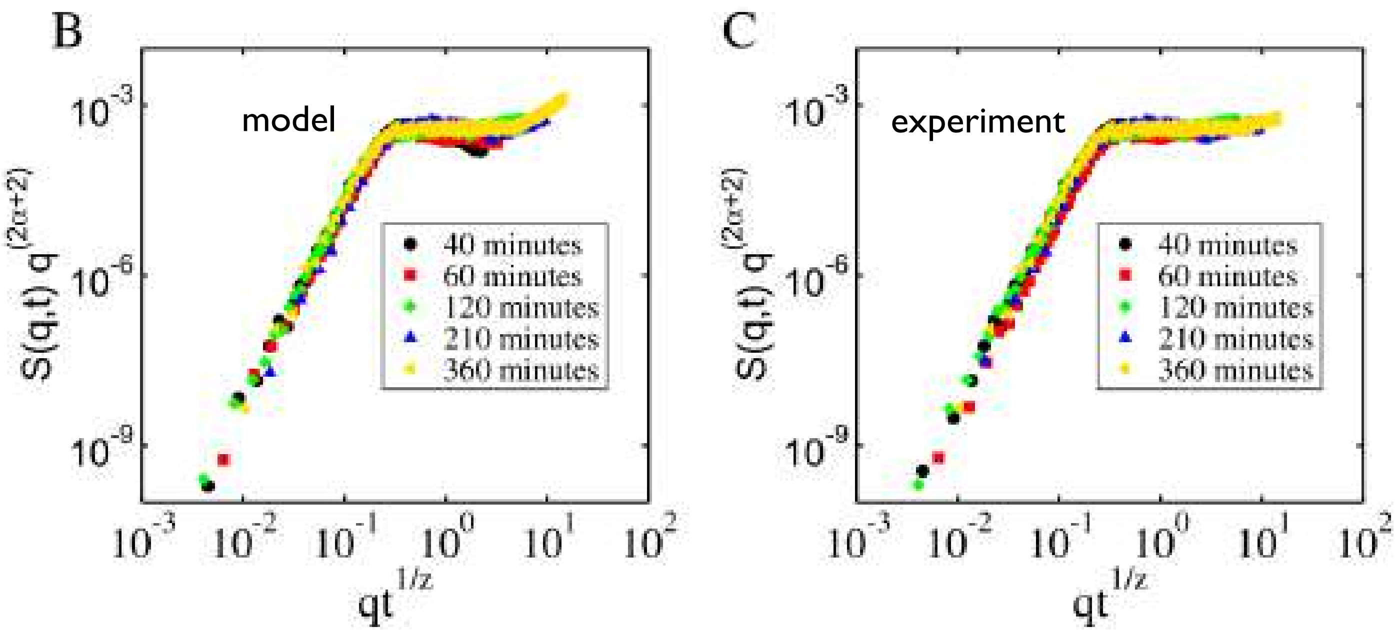
R.C. & M. Castro PRL 87, 236103 (2001); M. Nicoli, M. Castro & R. C. PRE **78**, 021601 (2008)

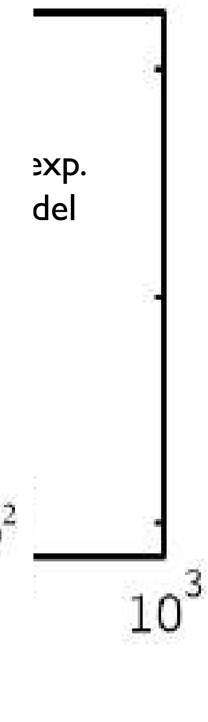
CVD growth with fast interface kinetics (2+1 growth)

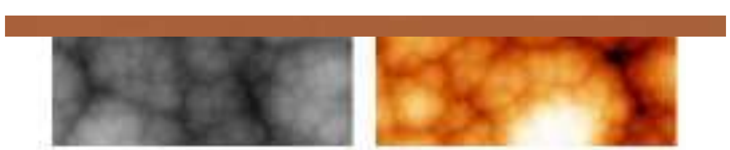
MS+KPZ AFM 1 μm^2

NON-KPZ universality class (dimension-independent exponents):

$$\alpha = z = 1, \quad \alpha + z = 2$$







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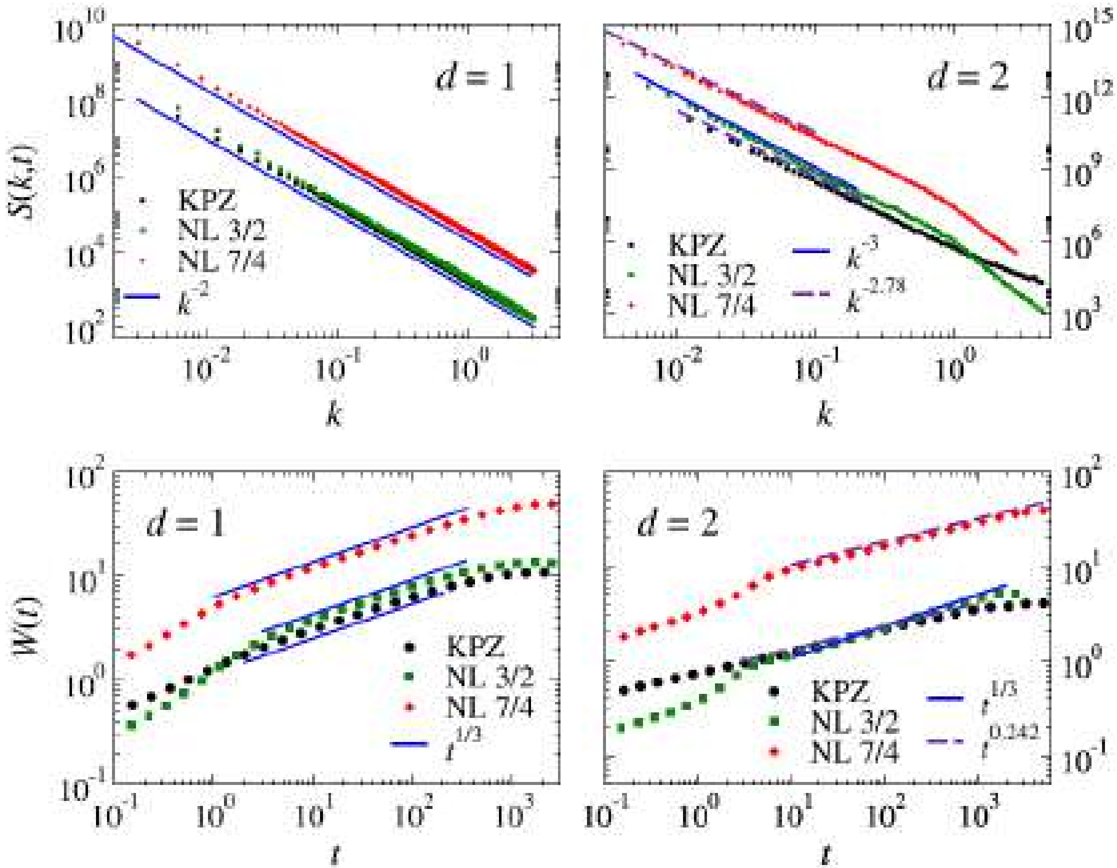
M. Castro *et al.* New J. Phys. **14**, 103039 (2012)

Departure from KPZ scaling in 2+1 dimensions



μ	d=1 z_{KPZ}=3/2	d=2 z_{KPZ}=1.61
3/2	KPZ	NON-KPZ
7/4	KPZ	KPZ

The μ=3/2 system changes universality class with dimension

S. Das Sarma, P. Punyindu & Z. Toroczkai PRE **65**, 036144 (2002) for MBE SOS models

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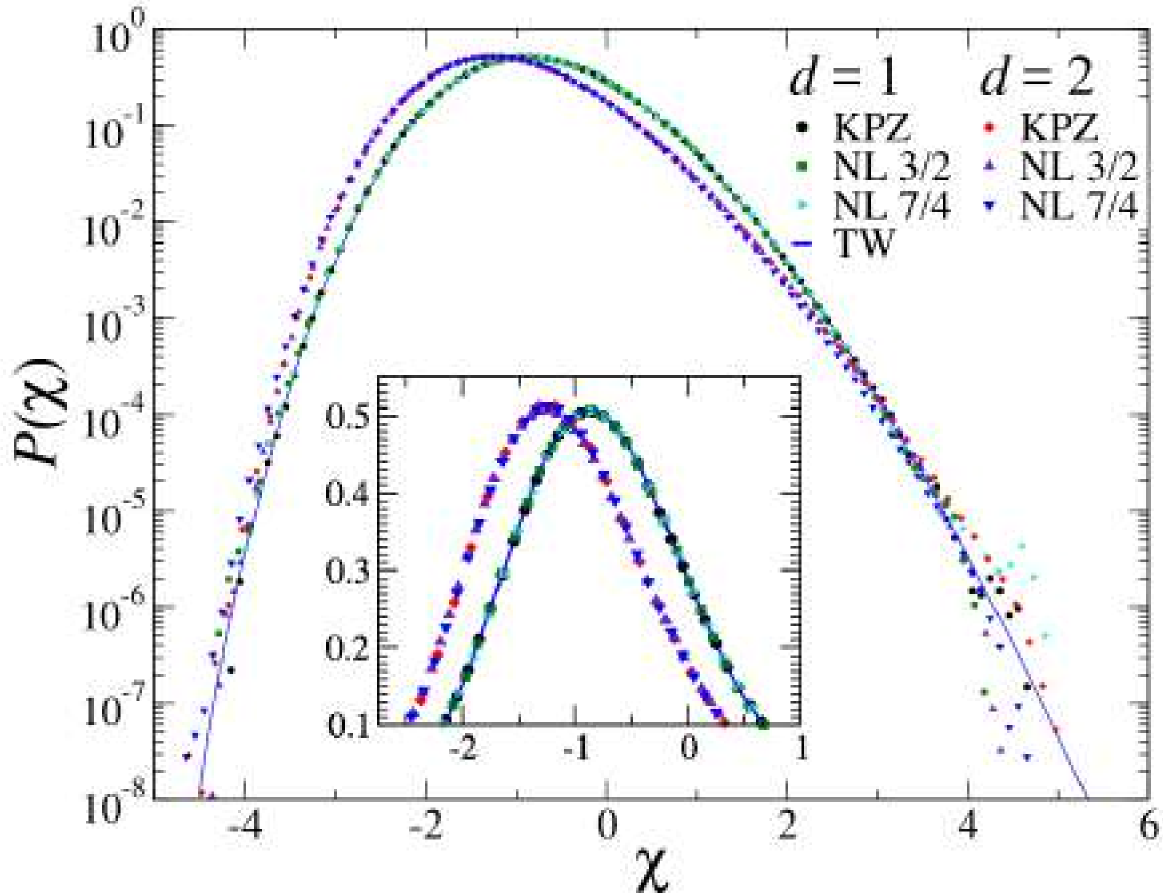
M. Nicoli, R. C. & M. Castro JSTAT P11001 (2013)

Departure from KPZ scaling in 2+1 dimensions

Consistent KPZ PDF behavior in d=1 and 2 with non-KPZ exponents (d=2)



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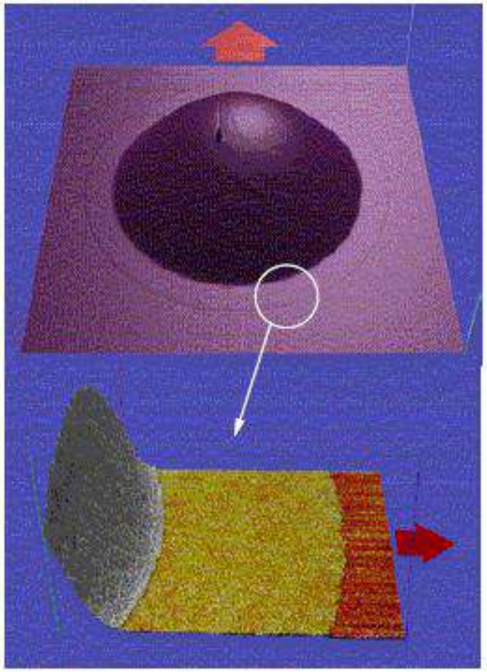
M. Nicoli, R. C. & M. Castro JSTAT P11001 (2013)

TW with "very" non-KPZ exponents: precursor spreading

Precursor film dynamics:

Under complete wetting conditions, a layer of molecular thickness forms that precedes the macroscopic wetting front and grows as $t^{1/2}$
(cf. Tanner's laws $t^{1/10}$ or $t^{1/7}$)



AFM
Polyestirene on Si
Droplet height: 80 nm
Droplet diameter: 12 μm

Height precursor film: 0.7 nm
NanoScience Group
University North Carolina

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P.-G. de Gennes. F. Brochard-Wyart & D. Quéré Capillarity and Wetting Phenomena (Springer, 2004)

TW with "very" non-KPZ exponents: precursor spreading

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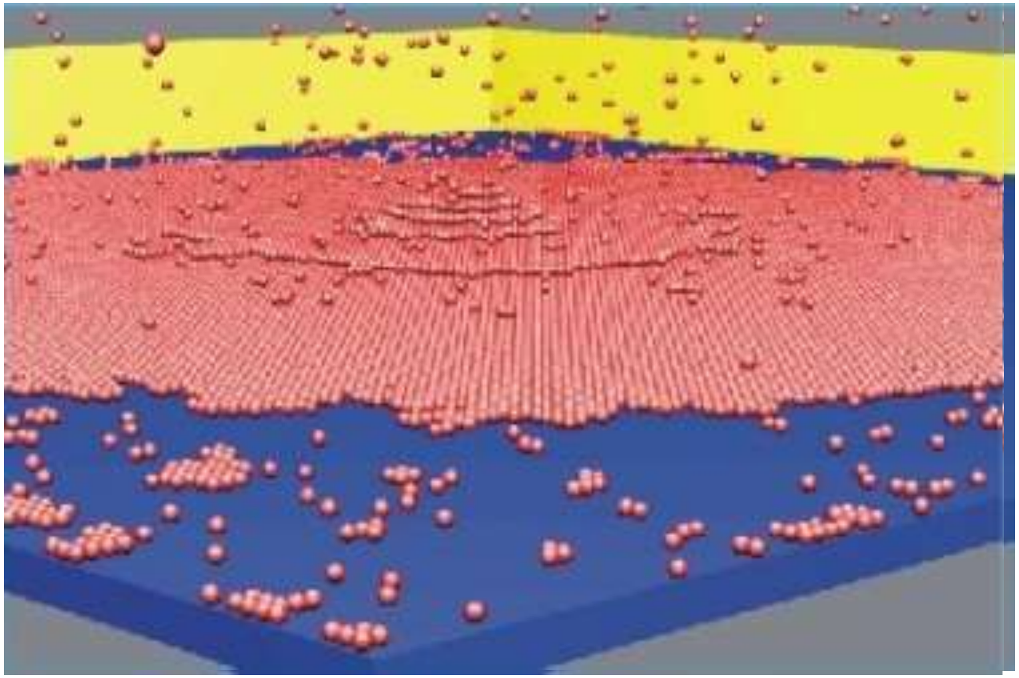

NanoScience Group U. North Carolina

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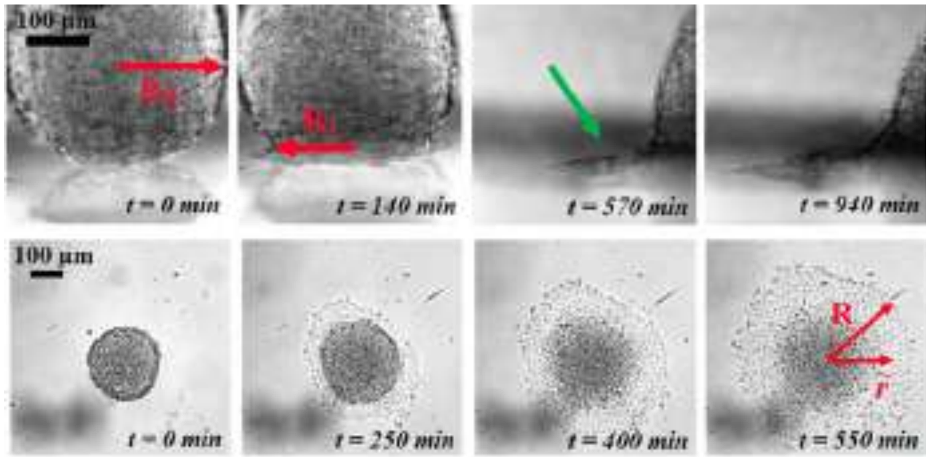
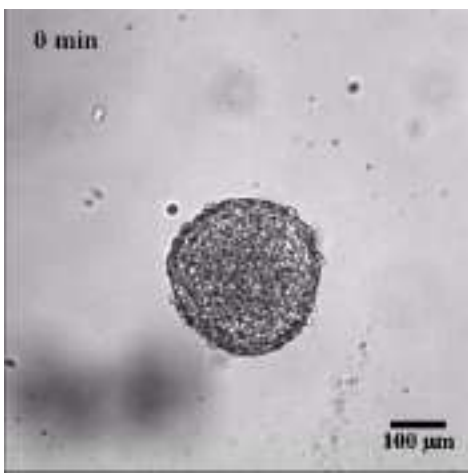
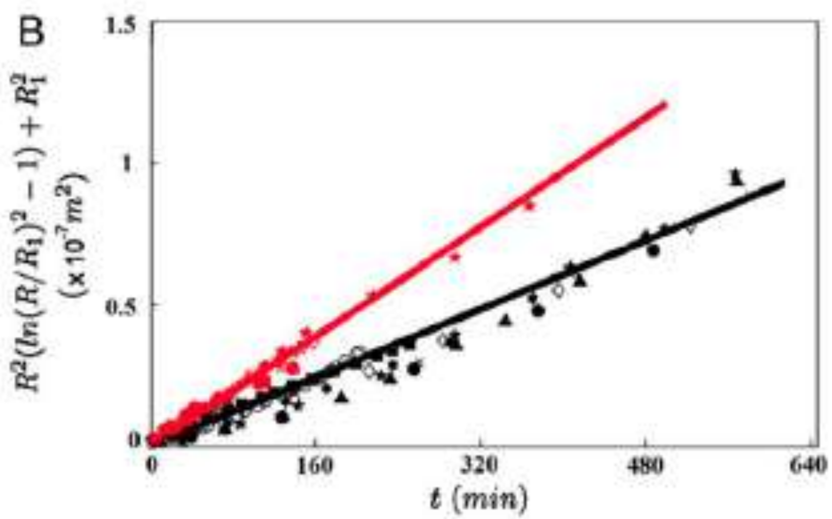
Precursor spreading







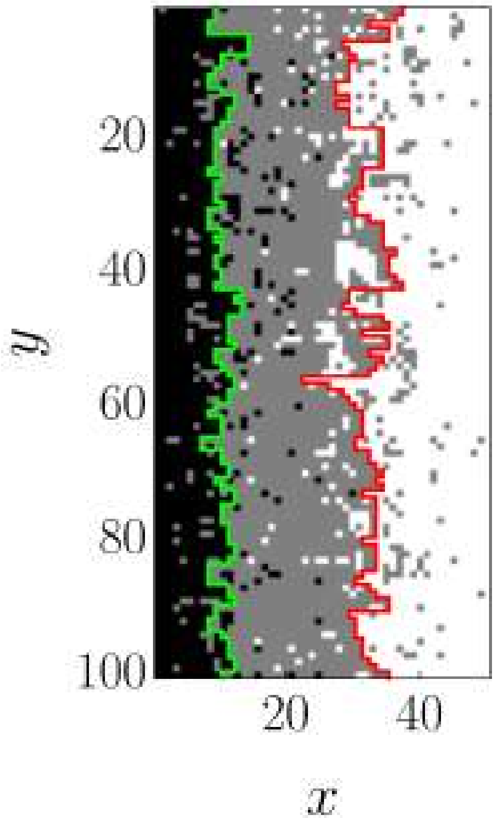
$$R(t) \sim t^{1/2}$$

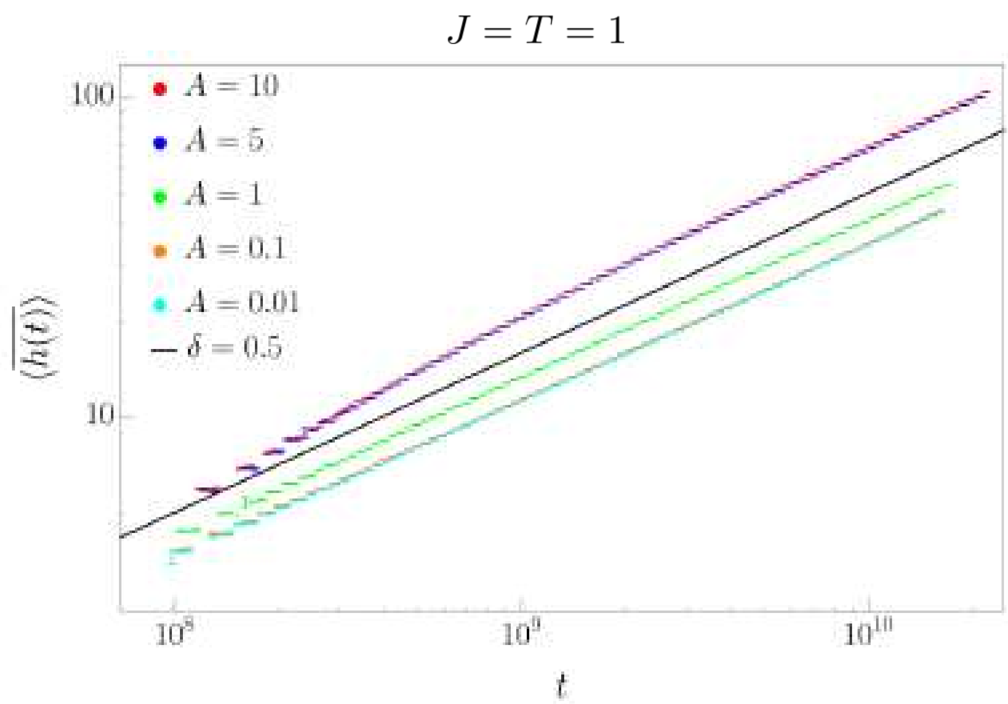
S. Douezan, K.Guevorkian, R.Naouar, S.Dufour, D.Cuvelier & F.Brochard-Wyart PNAS **108**, 7315 (2011)

Precursor spreading: kMC model

A. Lukkarinen, K. Kaski & D. B. Abraham PRE **51**, 2199 (1995)

$$\mathcal{H} = -J\sum_{\langle r,s\rangle} n(\boldsymbol{r},t)n(\boldsymbol{s},t) - A\sum_{\boldsymbol{r}} \frac{n(\boldsymbol{r},t)}{Z^3}$$





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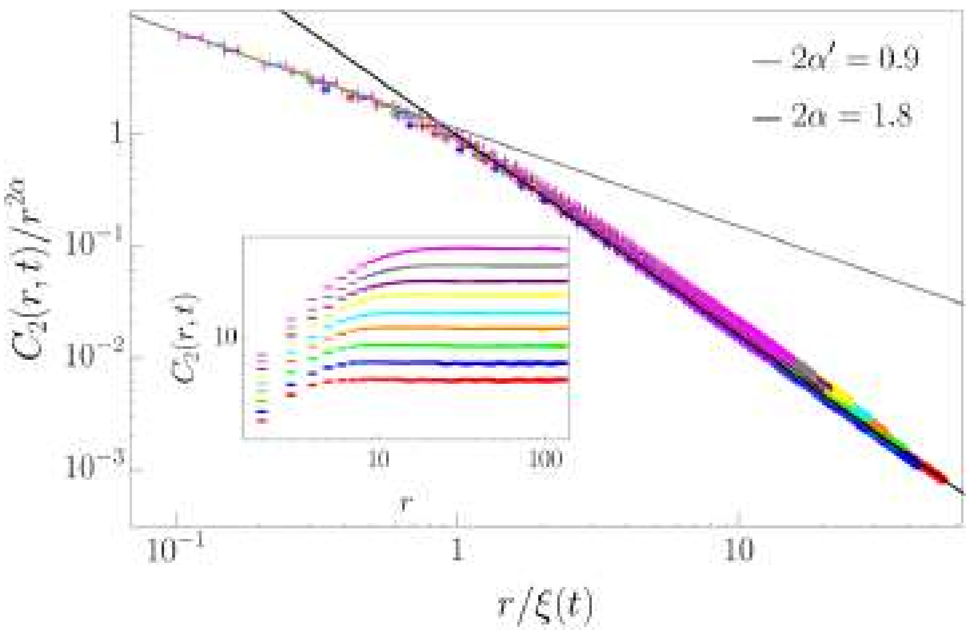
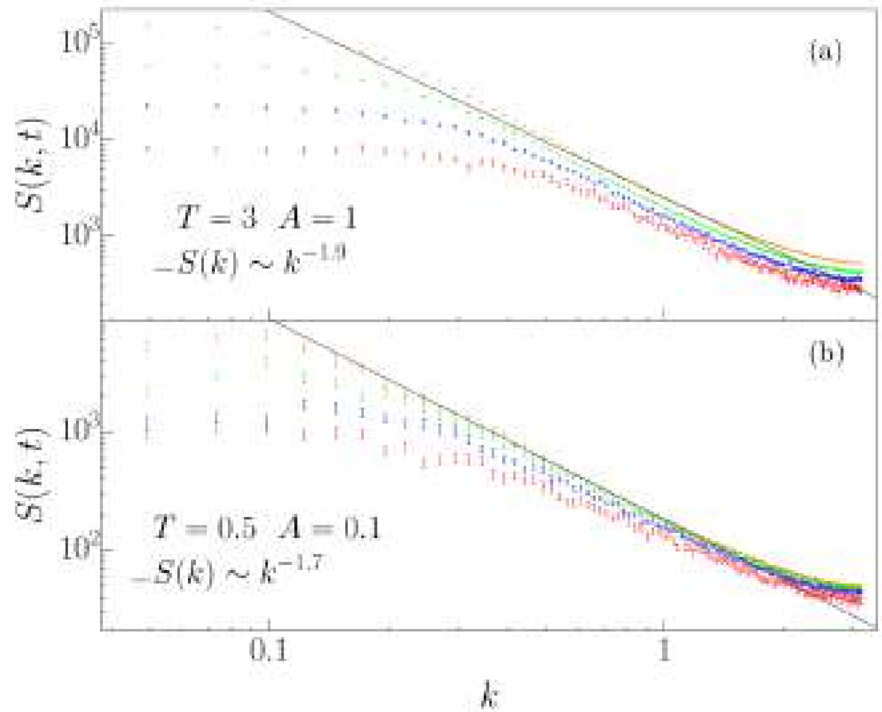
J. M. Marcos, P. Rodríguez-López, J. J. Meléndez, R. C. & J. J. Ruiz-Lorenzo PRE **105**, 054801(2022)

Precursor spreading: intrinsic anomalous scaling

$$C_2(r,t) = \frac{1}{L_y}\sum_y \langle [h(y+r,t) - h(y,t)]^2 \rangle$$

$$J = T = A = 1$$





$$\alpha = 0.89 \quad z = 3.3$$

$$\alpha_{\rm loc} = 0.46$$

Non-KPZ exponents: $\alpha + z \neq 2$

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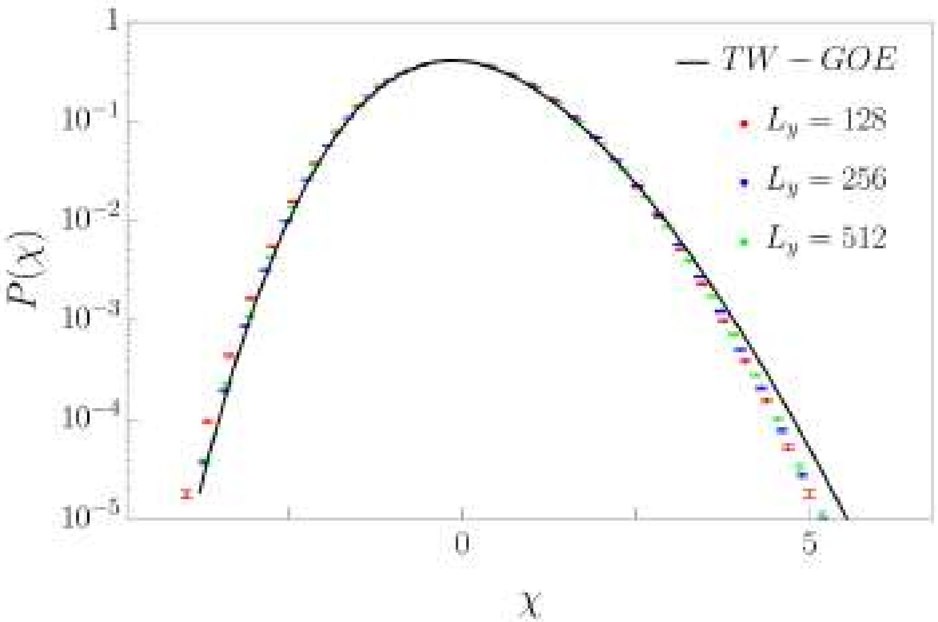
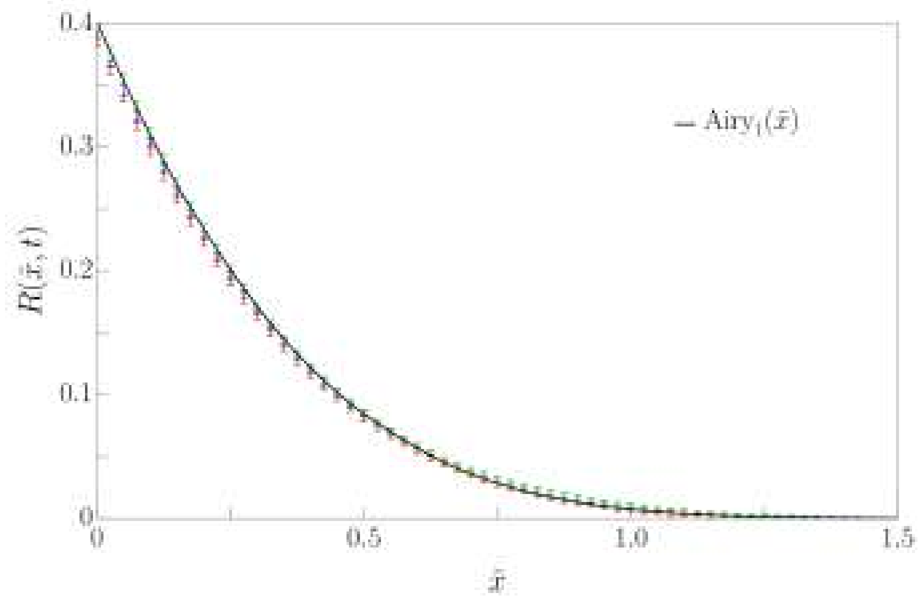
J. M. Marcos, P. Rodríguez-López, J. J. Meléndez, R. C. & J. J. Ruiz-Lorenzo PRE **105**, 054801(2022)

Precursor spreading: fluctuation statistics





$$\alpha = 0.89 \quad z = 3.3$$

$$\alpha_{\mathrm{loc}} = 0.46$$

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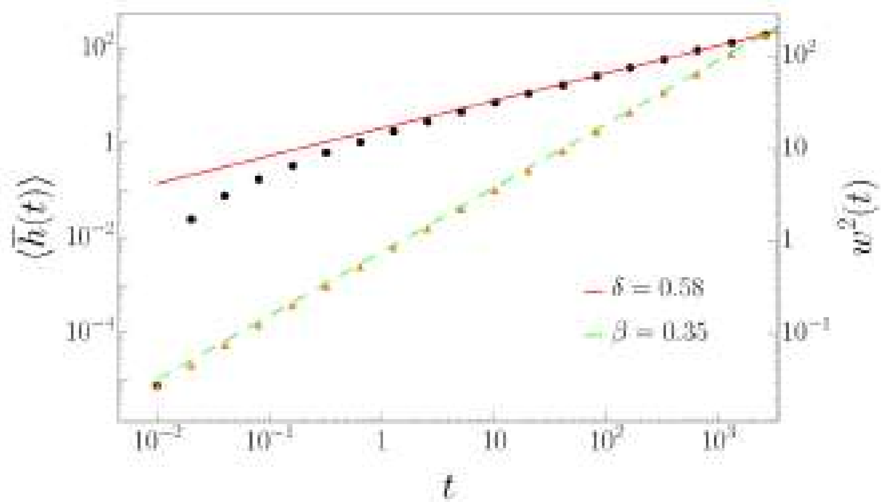
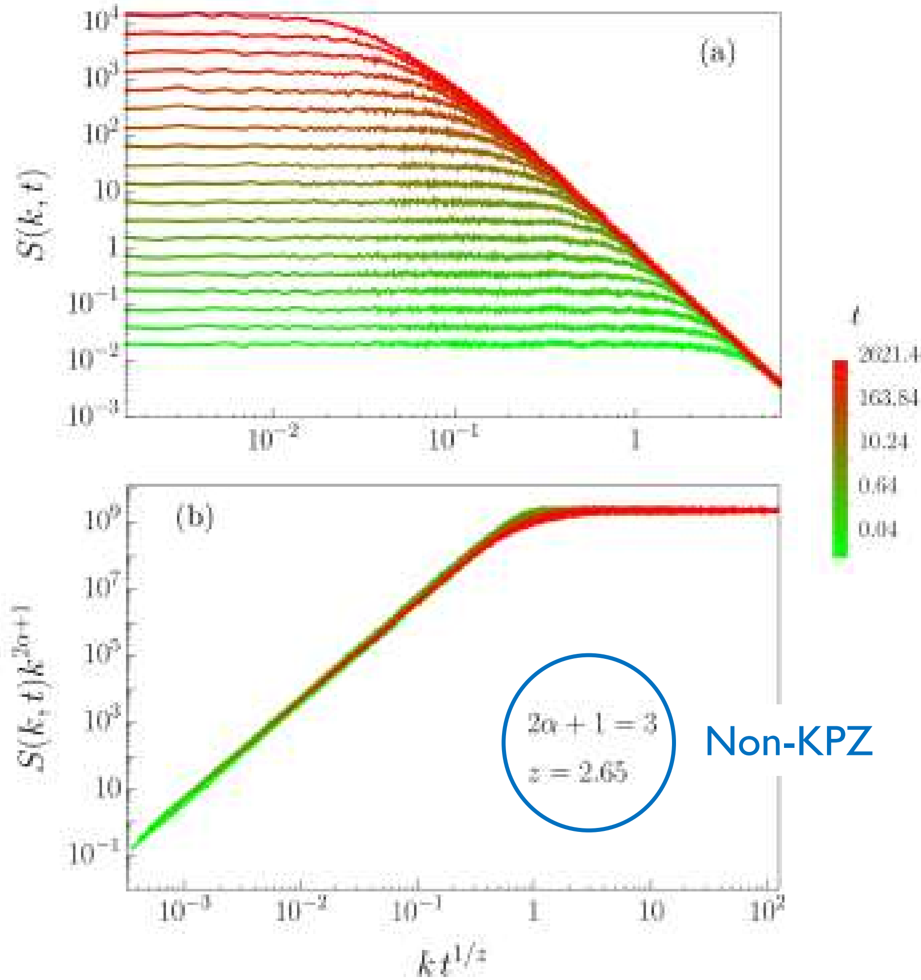
J. M. Marcos, P. Rodríguez-López, J. J. Meléndez, R. C. & J. J. Ruiz-Lorenzo PRE **105**, 054801(2022)

D. B. Abraham, R. C. & E. Moro PRL **88**, 206101 (2002)

Precursor spreading: continuum model

$$\partial_t \hat{h}_k(t) = -\nu |k|^3 \hat{h}_k(t) + \frac{\lambda}{\sqrt{t}} \hat{\mathcal{F}}_k[(\partial_y h)^2] + \hat{\eta}_k(t)$$





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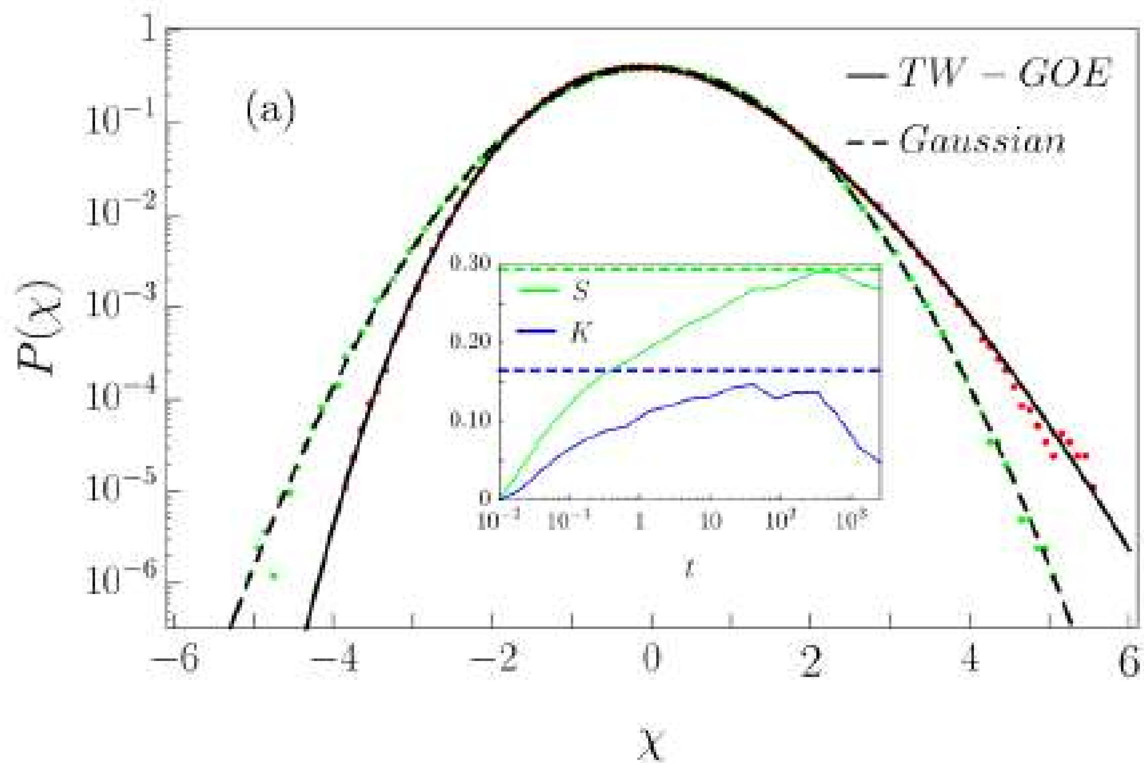
J. M. Marcos, P. Rodríguez-López, J. J. Meléndez, R. C. & J. J. Ruiz-Lorenzo PRE **105**, 054801(2022)

Precursor spreading: continuum model

$$\partial_t \hat{h}_k(t) = -\nu |k|^3 \hat{h}_k(t) + \frac{\lambda}{\sqrt{t}} \hat{\mathcal{F}}_k[(\partial_y h)^2] + \hat{\eta}_k(t)$$

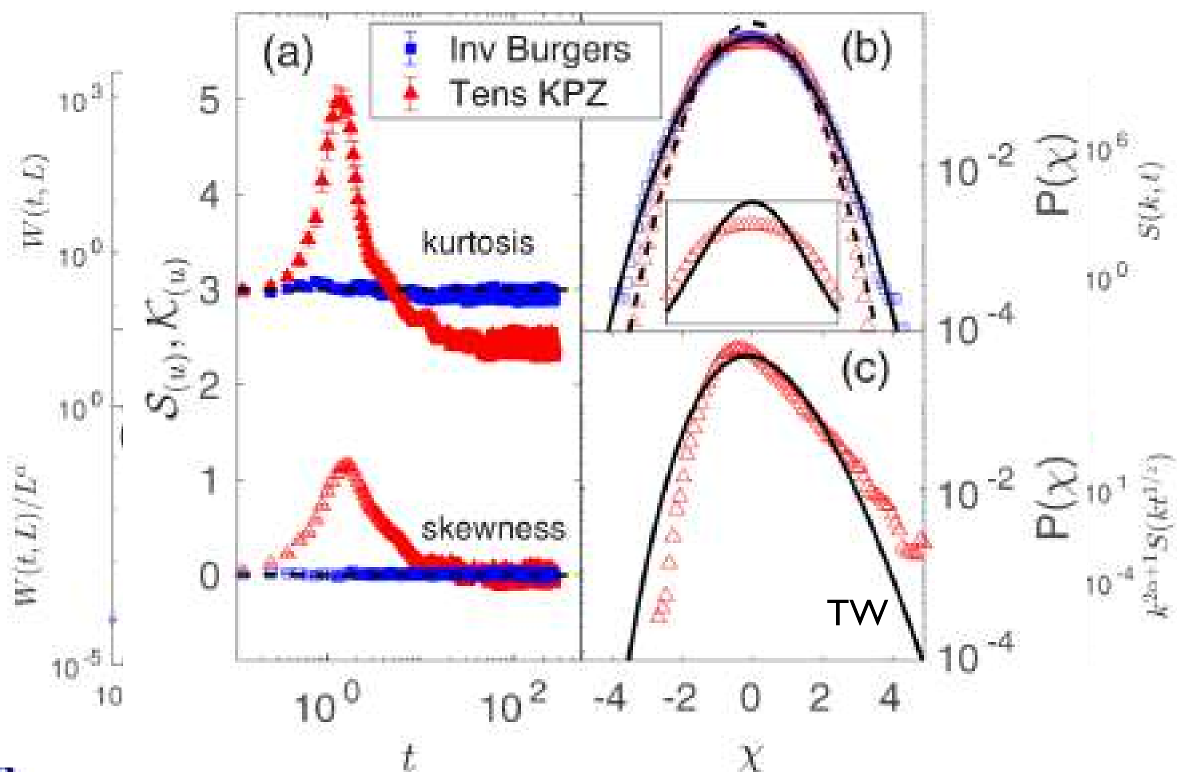


J. M. Marcos, P. Rodríguez-López, J. J. Meléndez, R. C. & J. J. Ruiz-Lorenzo PRE **105**, 054801(2022)

KPZ nonlinearity with non-KPZ exponents & fluctuations + anomalous scaling: tensionless KPZ equation

C. Cartes, E. Tirapegui, R. Pandit & M. Brachet PTRS A **380**, 2021009 (2021)
C. Fontaine, F. Vercesi, M. Brachet & L. Canet PRL **131**, 247101 (2023)

$$\partial_t h = \frac{\lambda}{2}(\partial_x h)^2 + \eta(x,t)$$



$\alpha = 1$
(b) Steady state $\alpha + z = 2$
$z = 1$
(c) Growth regime
$\alpha_s = 1/2$

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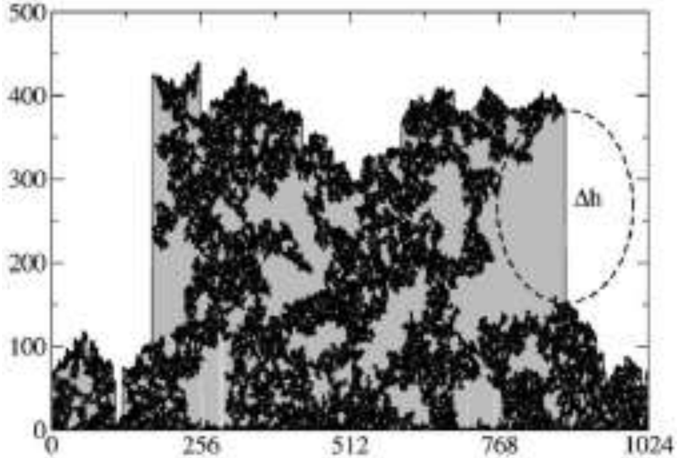
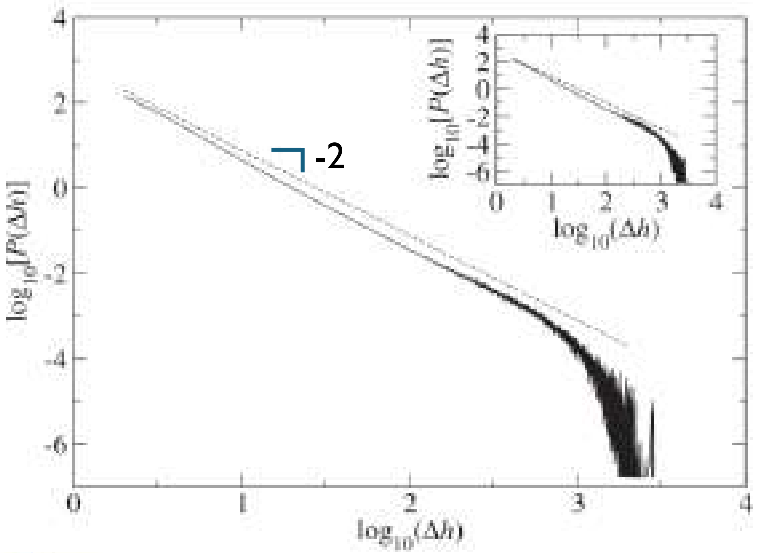
E. Rodriguez-Fernandez, S. N. Santalla, M. Castro & R. C. PRE **106**, 024802 (2022)

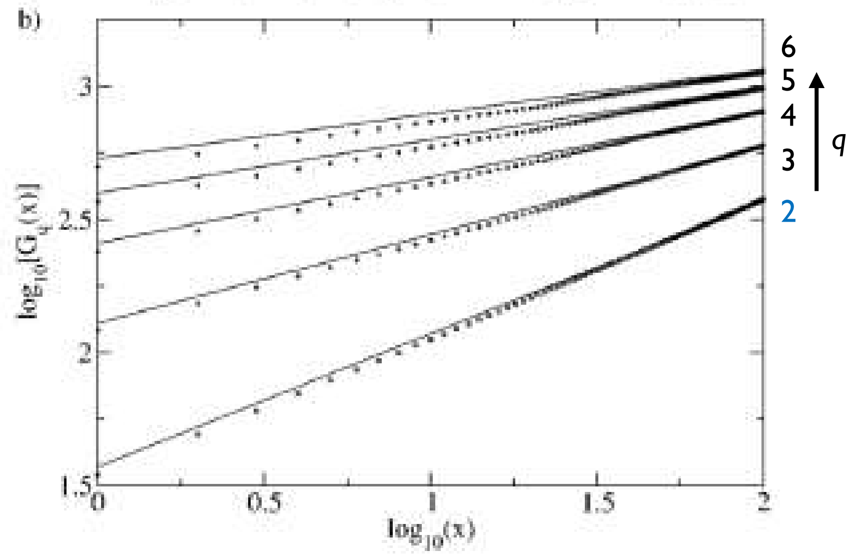
TKPZ universality class? Invasion percolation w/o trapping



$$\alpha = 1 \qquad \alpha + z = 2$$
$$z = 1$$
$$\alpha_s = 1/2$$



$$G_q(x,t) = \langle \overline{|h(x_0,t) - h(x_0+x,t)|^q} \rangle^{1/q}$$



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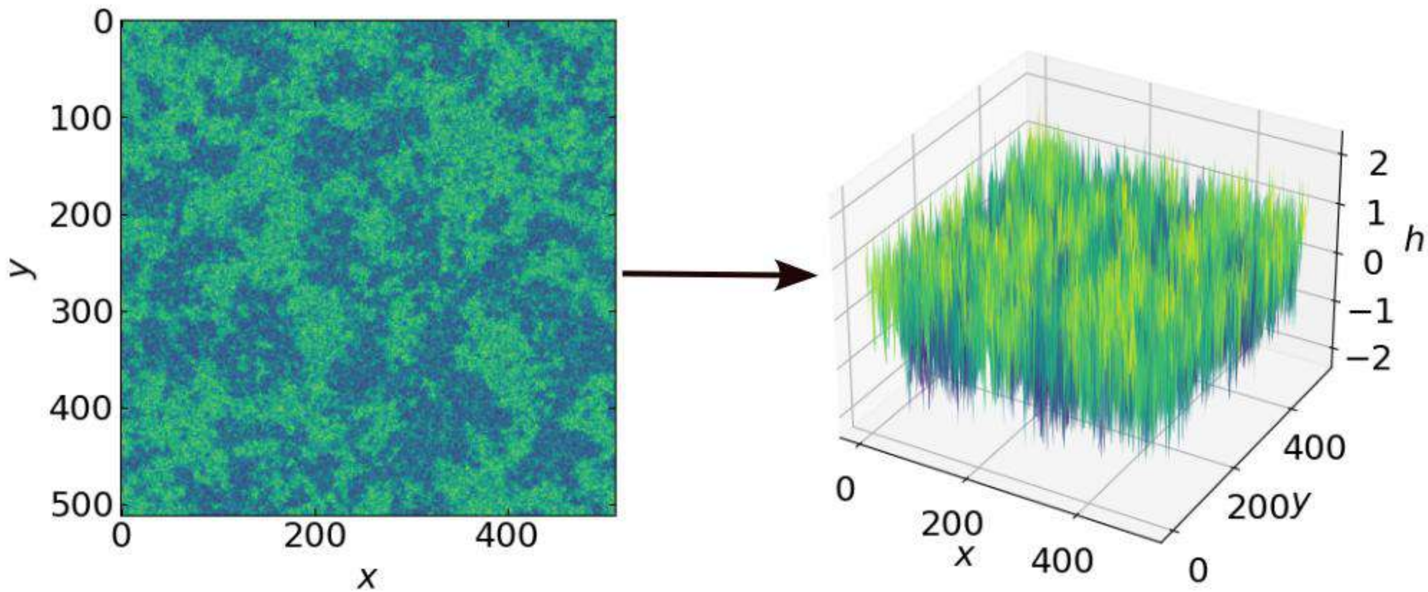
J. Asikainen, S. Majaniemi, M. Dubé & T. Ala-Nissila PRE **65**, 052104 (2002)
J. Asikainen, S. Majaniemi, M. Dubé, J. Heinonen & T. Ala-Nissila Eur. Phys. J. B **30**, 253 (2002)

Critical ($T=T_c$) Ising model as a rough surface



$$S(k) \sim \frac{1}{k^{2-\eta}} = \frac{1}{k^{2\alpha+d}} \Rightarrow \alpha = \frac{2-d-\eta}{2}$$

$$\eta_{\text{Ising2D}} = \frac{1}{4} \Rightarrow \alpha_{\text{Ising2D}} = -\frac{1}{8} = -0.125$$

$$z_{\text{Ising2D}} \approx 2 - 2\alpha_{\text{Ising2D}}(6\ln(4/3) - 1) \simeq 2.18$$

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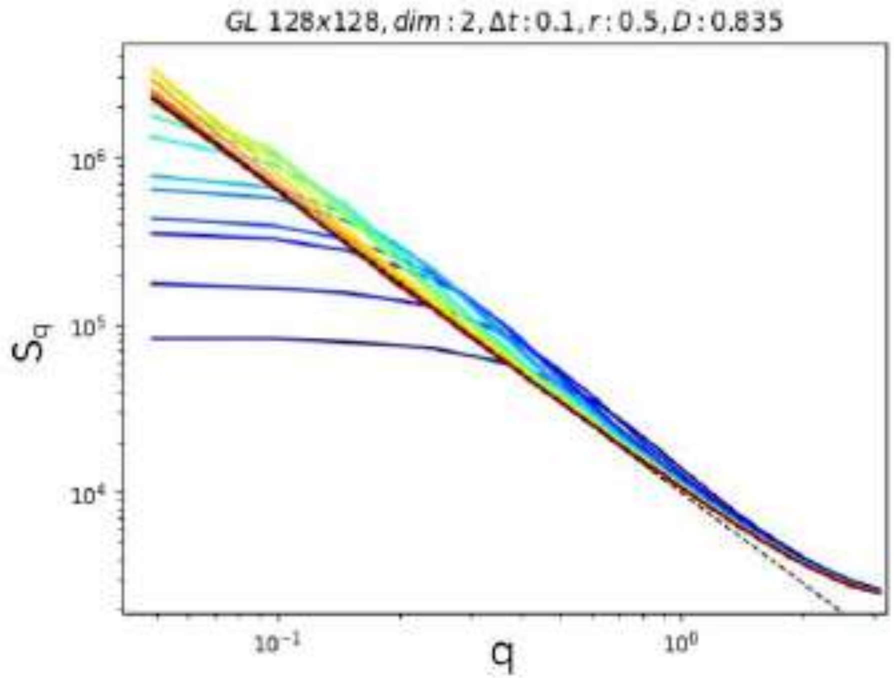
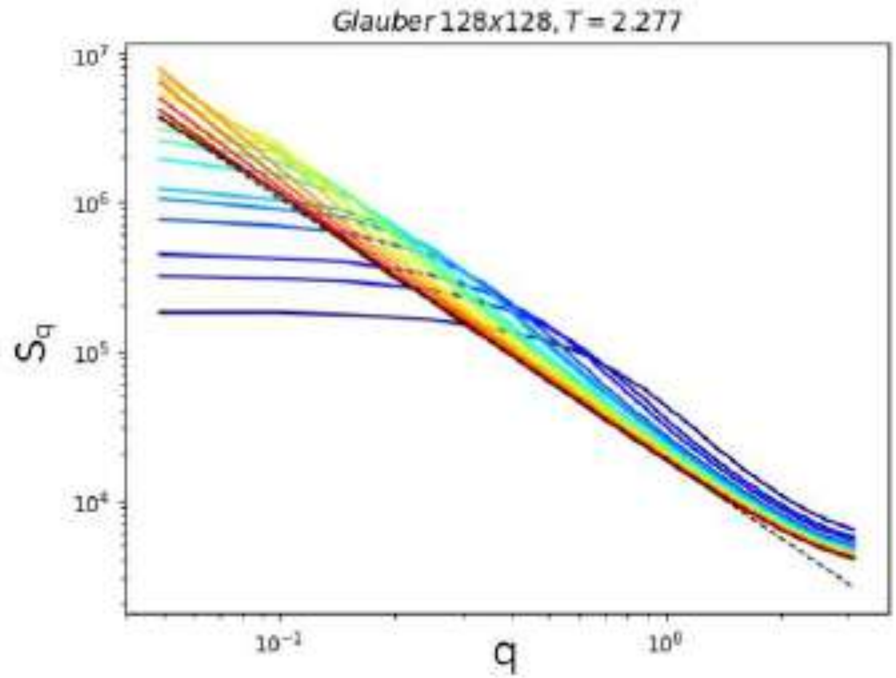
H. Vaquero del Pino & R. C. Phys. Rev. Res. **7**, 043192 (2025)

Anomalous scaling in critical Ising dynamics

Critical quench from T=∞ → T_c $\qquad \phi(\mathbf{r}, t=0) = 0$



Continuous
STDGL equation



Discrete
Glauber dynamics

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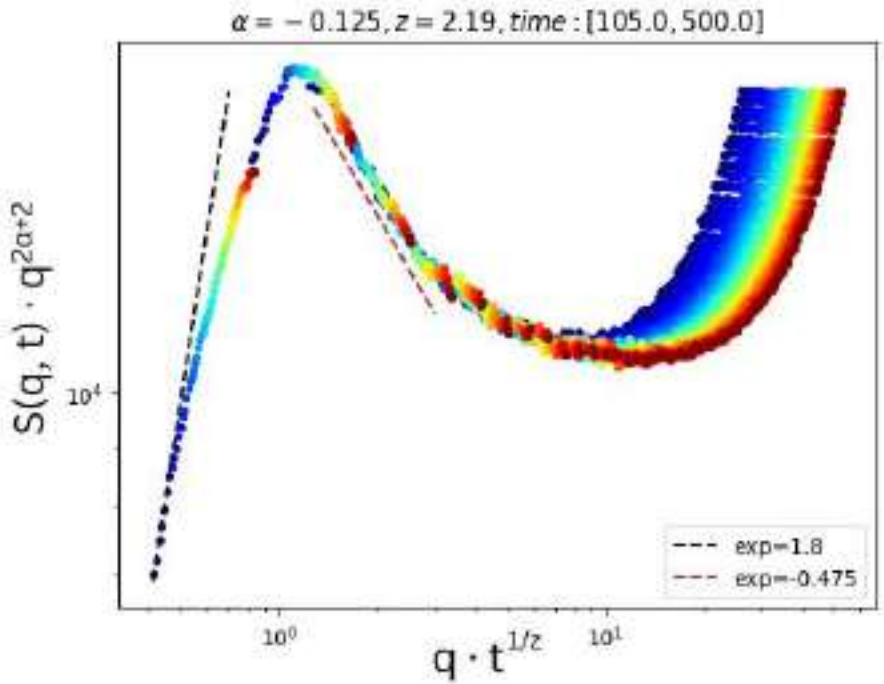
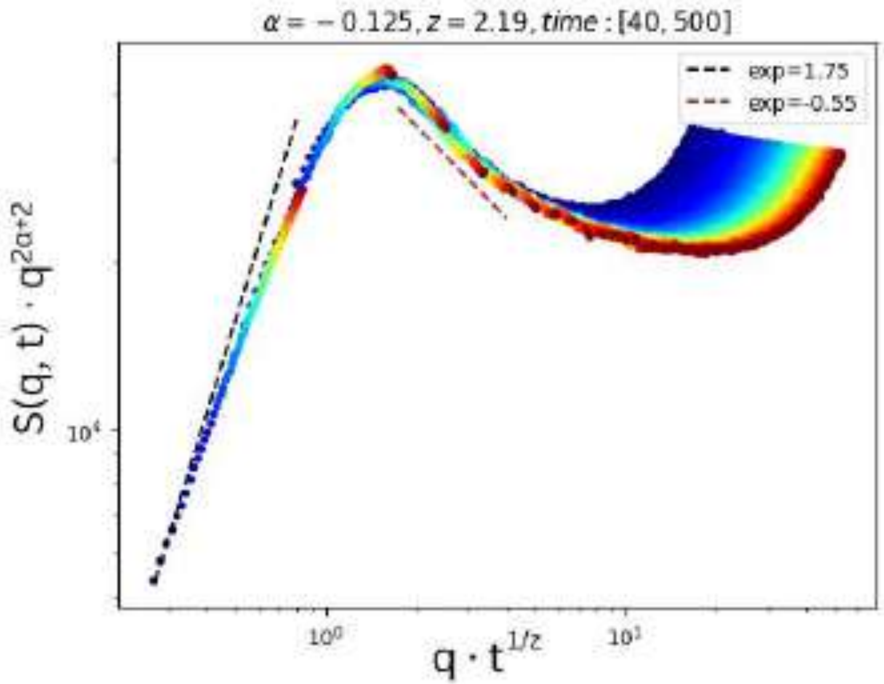
H. Vaquero del Pino & R. C. Phys. Rev. Res. **7**, 043192 (2025)

Quench crítico desde T=∞ (overgrowth): S(k,t)





Continuous
STDGL equation

$\alpha_S \approx 0.11$

$\alpha \approx -0.125, \; z \approx 2.19$

Discrete
Glauber dynamics

$\alpha_S \approx 0.15$

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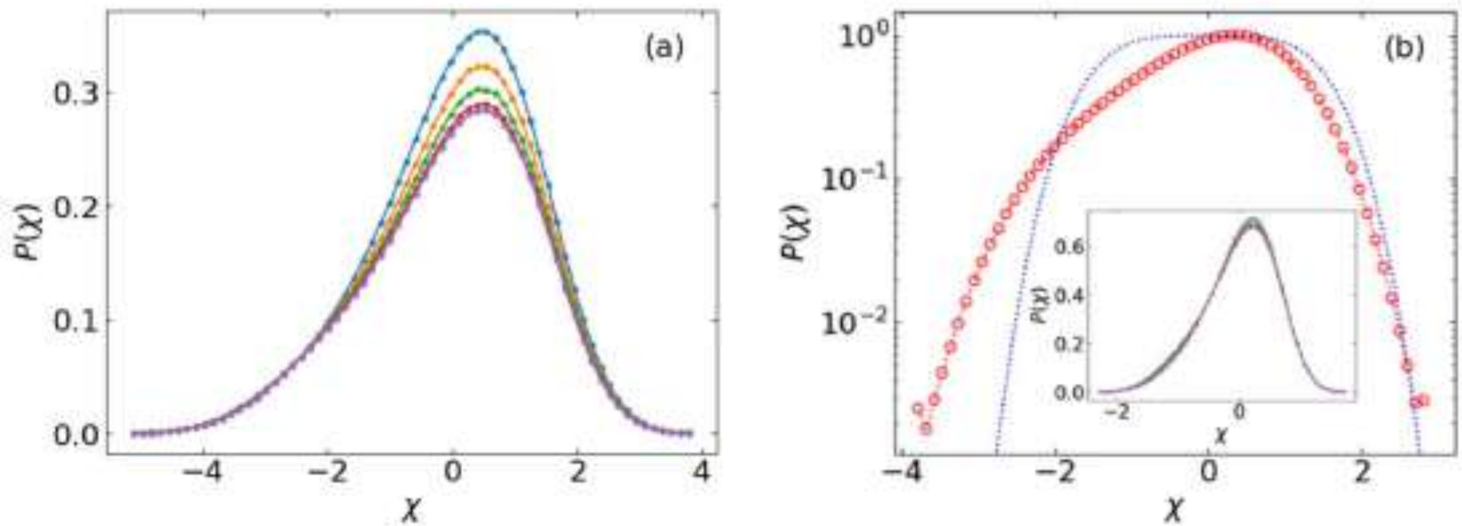
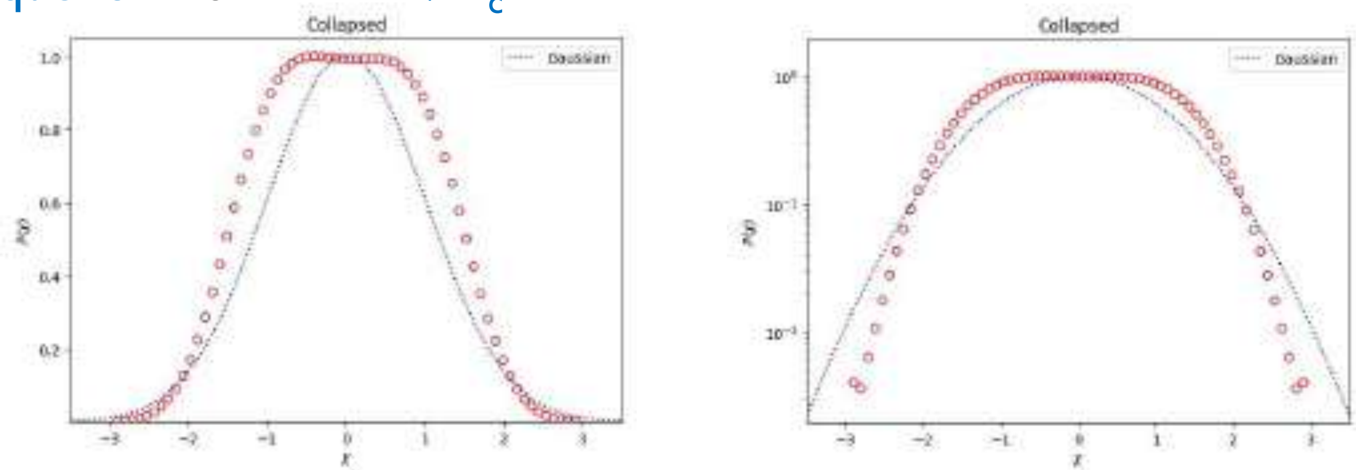
Prähofer-Spohn PDF collapse for critical Ising dynamics

Critical quench from $T=0 \rightarrow T_c$



Critical quench from $T=\infty \rightarrow T_c$



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H. Vaquero del Pino & R. C. Phys. Rev. Res. **7**, 043192 (2025)

Conclusions/Outlook

- Rich behavior wrt the traits of kinetic roughening universality classes
- Additional factors (e.g. noise type) may induce subclasses
- TW one and two-point statistics with non-KPZ exponents
- TW statistics under anomalous dynamic scaling ansatz
- Non-KPZ behavior (PDF, exponents, scaling ansatz) with KPZ nonlinearity
- Analogous phenomena for other nonlinearities and/or universality classes
- Analogous phenomena for equilibrium universality classes

- Deeper understanding of anomalous scaling required
- Role of symmetries
- Higher dimensions

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Thank you!

uc3m GISC Funding: Grant No. PID2021-123969NB-I00
Grant No. PID2024-159024NB-C21