GRIFFITHS SINGULARITIES

Viktor Dotsenko (Paris 6): "Griffiths singularities in the random bonds and the random field Ising systems"

In this talk I'll discuss recent theoretical ideas about the so called Griffiths phase above the ferromagnetic phase transition point in the random bonds Ising systems, and its possible connection with the replica symmetry breaking phenomena. Besides, in terms of simple probabilistic arguments I'll demonstrate the origin of the Griffiths singularities in the free energy (and others thermodynamical functions) in the low temperature phase of the random field Ising model.

Alice Guionnet (Paris 11): "Long time behaviour for dynamics in random media"

In the last few years, it was shown that the speed of the decay to equilibrium for Glauber dynamics of short range interacting systems is closely linked with Griffiths singularities. However, the long time behaviour of the dynamics for mean field models as SK is far from clear. In order to get some feeling about it, G. Ben Arous, A. Dembo and myself have studied the spherical model. We get its exact asymptotic behaviour for any temperature. We show and explain the phenomenon of aging regime in this setting.

Frédéric Klopp (Paris 13): "Lifshitz singularities for the density of states of a random operator"

This talk is devoted to the Lifshitz tail behaviour of the density of states of random Schrödinger operators near their spectral edges. More precisely, let H be a $\mathbb{Z}^d$-periodic Schrödinger operator acting on $L^2(\mathbb{R}^d)$ and consider the random Schrödinger operator $H_\omega = H + V_\omega$ where

$$V_\omega(x) = \sum_{\gamma\in\mathbb{Z}^d} \omega_\gamma V(x-\gamma)$$

(here V is a positive potential and $(\omega_\gamma)_{\gamma\in\mathbb{Z}^d}$ a collection of positive i.i.d random variables). We prove that, at the edge of a gap of H that is not filled in for H_ω, the integrated density of states of H_ω has a Lifshits tail behaviour if and only if the integrated density of states of H is non-degenerate.

Raphael Lefévère (Louvain-la-Neuve): "What can be done about the Renormalization Group pathologies ?"

We explain that the so-called Renormalization Group pathologies in low-temperature Ising models are due to the fact that the renormalized Hamiltonian is defined only almost everywhere. The renormalized measures still satisfy the DLR equations and a variational principle.

Christian Maes (Leuven): "Dynamical aspects of Griffiths singularities"

We discuss the influence of the presence of random interactions on the convergence to the unique invariant measure for a class of spin flip dynamics. Both reversible and irreversible evolutions are considered.

Fabio Martinelli (L'Aquila): "Glauber dynamics in presence of Griffiths singularities"

In my talk I will consider what happens to the relaxational properties of a Glauber type dynamics when one removes the assumption of *translation invariance* of the interaction] and one considers in particular *random interactions* (joint work with F.Cesi and C.Maes). The simplest example of such a system is the (bond) dilute Ising ferromagnet. In this case the couplings between nearest neighbor spins become a collection of i.i.d random variables $\{J_{xy}\}$ that take only two values, $J_{xy} = 0$ and $J_{xy} = \beta$ with probability $1-p$ and p respectively, independently for each pair of nearest neighbors $x, y \in Z^d$. In a more pictorial form one starts from the standard Ising model and removes, independently for each bond $[x, y]$, the coupling J_{xy} with probability $1-p$.

Since the $\{J_{xy}\}$ are uniformly bounded, at sufficiently high temperatures (i.e. sufficiently small values of β) Dobrushin's uniqueness theory applies and detailed information about the unique Gibbs measure and the relaxation to equilibrium of an associated Glauber dynamics are available using the concept of complete analyticity. This regime is usually referred to as the *paramagnetic* phase and, at least for the two dimensional dilute Ising model, it is known to cover the whole interval $\beta < \beta_c$ where β_c is the critical value for the "pure" Ising system.

There is then a range of temperatures, below the paramagnetic phase, where, even if the Gibbs state is unique, certain characteristics of the paramegnetic phase like the analyticity of the free energy as a function of the external field disappear. This is the so called *Griffiths' regime* .

This "anomalous behavior" is caused by the presence of arbitrarily large clusters of bonds associated with "strong" couplings J_{xy}, which can produce a long–range order inside the cluster. Even above the percolation threshold, i.e. when one of such clusters is infinite with probability one, there may be a Griffiths phase for values of $\beta \in (\beta_c, \beta_c(p))$, where β_c is the critical value for the Ising model on Z^d and $\beta_c(p)$ the critical value of the dilute model above which there is a phase transition. What happens is that for almost all realizations of the disorder J and for all site x there is a finite length scale $l(J,x)$, such that correlations between $\sigma(x)$ and $\sigma(y)$ start decaying exponentially at distances greater than $l(J,x)$.

The effect of the Griffiths' singularities on the dynamical properties are much more serious since, as we will see, the long time behaviour of any associated Glauber dynamics is dominated by the islands of strongly coupled spins produced by large statistical fluctuations in the disorder.

During my talk I will analyze in some detail the simple case of the dilute Ising model. Although at first sight such a model is a very special one, it is important to say that most of the results, particularily those concerning the dynamical behaviour inside the Griffiths phase, apply with minor changes to a much wider class of models.

Main Results

Consider the usual stochastic (or kinetic) Ising model associated to the model discussed above. It is a stochastic spin flip dynamics for which the (almost sure unique) Gibbs state is a reversible measure. Let us denote by $q(J,t)$ the absolute difference between the expectation at time t of e.g. the spin at the origin starting in some initial state (e.g. all pluses), and its equilibrium value.

Assume that in a cube of side length L, the correlation between $\sigma(x)$ and $\sigma(y)$ start decaying exponentially fast if, $|x-y|$ is greater than, say, $L/2$, with probability (w.r.t the disorder) at least $1-\exp(-cL)$.

Then

$$q(J,t) \geq c_1(J)\, \exp\Big[-t\, \exp\big[-k_1\, (\log t)^{1-\frac{1}{d}}\,\big]\Big]$$
$$q(J,t) \leq c_2(J)\, \exp\Big[-t\, \exp\big[-k_2\, (\log t)^{1-\frac{1}{d}}\, (\log\log t)^{d-1}\,\big]\Big]$$

We also analyze the average (over the disorder or spatially) $q(t)$ of $q(J,t)$ and prove both upper and lower bounds of the form

$$\exp[-\lambda_1(\log t)^{d/(d-1)}] \leq q(t) \leq \exp[-\lambda_2(\log t)^{d/(d-1)}(\log\log t)^{-d}]$$

for suitable constants λ_1, λ_2.

Francesca Nardi (Rome 2): "On the phase diagram of the Ising Model with alternating field"

We consider the two dimensional ferromagnetic nearest neighbor Ising model with alternating field: we take a positive magnetic field on the odd rows and a negative magnetic field on the even ones. We study the phase diagram and show that at any positive temperature some coexistence lines, that were present at zero temperature, disappear whereas others persist at low enough temperature. Using a particular cluster expansion we show that our system can be seen as a small perturbation of a set of independent one-dimensional Ising systems (on the even rows).

Leonid Pastur (Paris 7): "Lifshitz tails: one more fluctuation phenomenon in disordered systems"

We present first the heuristic arguments by I.Lifshitz showing the form of the density of states near the spectrum edges of the Schrödinger equation with a random potential and suggesting a natural classification of corresponding asymptotic regimes. We discuss also rigorous results on the Lifshitz tails for the integrated density of states and other quantities, certain physical applications and analogy with the zero distribution of the partition function according to Griffiths.

Senya Shlosman (Marseille): "Griffiths singularities and almost Gibbs states"

Gilles Tarjus (Paris 6): "Self-consistent Ornstein-Zernicke approximation for disordered systems"

We apply the formalism of liquid-state statistical mechanics, together with the replica method, to investigate the phase diagram of fluids (and magnetic systems) in the presence of quenched disorder. The correlation functions and the thermodynamic properties are derived within a self-consistent Ornstein-Zernike approximation. Preliminary results are presented, and the potential description of Griffiths singularities and replica symmetry breaking phenomena is briefly discussed.
